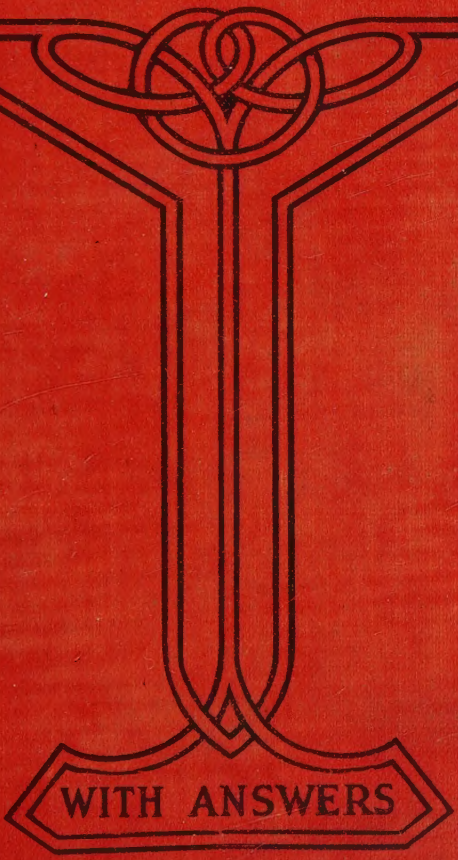
COMMERCIAL ARITHMETIC



WITH ANSWERS

OLIVER & BOYD'S
EDUCATIONAL SERIES

Robert Todd.
39 Friar St.

Commercial Arithmetic

A COMPLETE MANUAL OF

APPLIED ARITHMETIC

FOR SENIOR CLASSES

OLIVER AND BOYD
EDINBURGH: TWEEDDALE COURT
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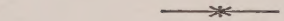
PREFACE.



THOUGH this little Manual has been specially prepared for Senior Classes in Elementary and Intermediate Classes in Secondary Schools, a knowledge of the four fundamental rules and reduction has alone been assumed. The application of arithmetical methods to business transactions has been made the specific aim of the book. Due prominence has been given to the brief up-to-date methods used in actual business, but the employment of all formulæ and short methods has been carefully based on an intelligent grasp by the pupil of the principles underlying these.

Decimals and the Decimalisation of Money have received that share of attention due to their importance, and the construction and use of tables of decimal values have been fully explained.

The spirit of the teaching throughout has been to make all arithmetical calculation as much a mental exercise as possible, and thus minimise the evils arising from a too frequent dependence on the mechanical aids of pencil and paper.

Care has been taken to make the exercises really practical, such as would be met with in the ordinary business of life, except where exercises of a more theoretic nature have been introduced to enable the pupil to acquire that manipulative dexterity so essential to correctness.

NOTE TO THIRD EDITION

In this, the Third, Edition, ten recent Examination Papers have been substituted for those which appeared in the previous edition, and thirty miscellaneous exercises have been added to the collection at the end of the book.

CONTENTS.

TABLES OF BRITISH MONEY, WEIGHTS, AND MEASURES.

1. TABLE OF MONEY.

4 farthings	=	1 penny (d.)
12d.	=	1 shilling (s.)
20s.	=	1 pound (£)

d., *s.*, and £ are contractions for *denarius*, *solidus*, and *libra*.

The diameter of a halfpenny is exactly 1 inch; 5 halfpennies or 3 pennies weigh 1 ounce avoirdupois.

Coins in Circulation.

Gold.—*Sovereign*, £1; *Half Sovereign*, 10s. Gold coins of £2 and £5 value are also issued, but are not used in business.

Silver.—*Crown*, 5s.; *Double Florin*, 4s.; *Half Crown*, 2s. 6d.; *Florin*, 2s.; *Shilling*, 1s.; *Sixpence*, 6d.; *Threepence*, 3d.

Other silver coins (the *Groat*, 4d.; *Twopence*, 2d.; and *Penny*, 1d.) are also coined, but their use is almost exclusively restricted to the **Maundy** (Thursday before Easter) **gift of money** to the poor.

Bronze.—*Penny*, 1d.; *Halfpenny*, ½d.; and *Farthing*, ¼d.

To avoid carrying large sums of gold, **Bank Notes** are issued and largely used.

The Bank of England issues notes for sums of £5, £10, £20, £50, £100, £200, £500, and £1000.

The **Scotch Banks** have their own issue of notes as above, but they also issue a **£1 note**, which to a very great extent takes the place of the sovereign in Scotland, but is not accepted in England.

2. TABLES OF WEIGHT.

By the Weights and Measures Act of 1878 the **Imperial standard of weight** is the **Pound Avoirdupois**; but gold, silver, and precious stones may be sold by the *ounce Troy*, and drugs by *Apothecaries' weight*. The General Medical Council recommended the withdrawal of the Troy ounce and its subdivisions in the dispensing of medicines, and the "British Pharmacopœia" employs the ounce avoirdupois ($437\frac{1}{2}$ grains) and not the ounce of 480 grains.

Avoirdupois Weight.

16 drams (dr.)	= 1 ounce (oz.)
16 oz.	= 1 pound (lb.)
14 lb.	= 1 stone (st.)
2 st. or 28 lb.	= 1 quarter (qr.)
4 qr. or 8 st.	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton.

The following should also be remembered—

1 Hundredweight (cwt.)	= 112 lbs.
1 Cental	= 100 lbs.
1 Ton	= 2240 lbs.
1 American Ton	= 2000 lbs.
1 Sack of Flour	= 280 lbs.
1 Firkin of Butter	= 56 lbs.
1 Barrel of Soap	= 256 lbs.

Troy Weight.

24 grains (gr.)	= 1 pennyweight (dwt.)
20 dwt.	= 1 ounce (oz.)
12 oz.	= 1 pound (lb.)

The grain weight was originally a dried grain of wheat.

1 lb. Troy	= 5760 grains
1 lb. Avoirdupois	= 7000 grains

The word "carat," as used ordinarily with reference to gold, indicates the fineness, that is, how many parts in 24 are of pure gold; but jewellers also use the word as an indication of weight. Diamonds, for instance, are said to be of *so many carats weight*. A diamond carat is about $3\frac{1}{6}$ grains Troy. Gold is sometimes also weighed by carats, the gold carat being 60 grains Troy.

Apothecaries' Weight.

According to the Pharmacopœia this table is—

437½ grains (gr.)	=	1 ounce (oz.)
16 oz.	=	1 pound (lb.)

but in some provinces medical practitioners still make use of the old weights which they indicate by the symbols set against them in the table.

20 grains (gr.)	=	1 scruple (℈)
3 scr.	=	1 dram (ʒ)
8 drs.	=	1 ounce (℥)
12 oz.	=	1 lb.

The grain and the pound are the Troy grain and pound.

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3. TABLES OF MEASURES.

Table of Length ("Lineal Measure").

The Imperial standard measure of length is the **Yard**. The inch, for tradesmen's purposes, is divided into *halves*, *quarters*, *eighths*, and *sixteenths* of an inch.

12 inches (in.)	=	1 foot (ft.)
3 ft.	=	1 yard (yd.)
5½ yd.	=	1 pole or rod (po).
40 po. or 220 yd.	=	1 furlong (fur.)
8 fur. or 1760 yd.	=	1 mile

The following measures of length should also be known :—

1 Knot or Geographical Mile	=	$1\frac{5}{13}$ ordinary miles
,, ,,	=	6080 feet
1 Fathom (used in measuring depths)	=	6 feet
1 Hand (used in measuring horses)	=	4 inches

Land Surveyors' Measure.

100 links	=	1 chain (22 yards)
10 chains	=	1 furlong

Cloth Measure.

9 inches	=	1 quarter (qr.)
4 qr.	=	1 yard (yd.)

Cloth is usually measured in yards, half yards, quarter yards, and eighths of a yard.

Table of Surface or Area ("Square Measure").

This table is formed by *squaring* the corresponding numbers in the table of length. There are *no square furlongs;* but two measures, *roods* and *acres*, which have no corresponding lineal measures, come in between square poles and square miles, instead.

144 square inches (sq. in.)	=	1 square foot (sq. ft.)
9 sq. ft.	=	1 sq. yard
30¼ sq. yd.	=	1 sq. pole
40 sq. po.	=	1 rood (ro.)
4 roods or 4840 sq. yd.	=	1 acre (ac.)
640 ac.	=	1 sq. mile

Joinery is sometimes estimated by the "*square of flooring*," which is 100 *sq. ft.;* brickwork by the *rood* of 272 *sq. ft.* (1½ bricks thick).

Land Surveyors' Square Measure.

10,000 sq. links	=	1 sq. chain
10 sq. chains	=	1 acre

Table of Solidity ("Cubic Measure").

This measure is used for the solid content or volume of solids, such as logs of wood, blocks of stone, rectangular cisterns, &c.

The table is formed by "cubing" the corresponding numbers in lineal measure.

1728 cubic inches (cub. in.)	=	1 cubic foot (cub. ft.)
27 cub. ft.	=	1 cub. yard (cub. yd.)

Rough timber is sold by the **load** of **40 cub. ft.**; Square or **Hewn timber** by the **load** of **50 cub. ft.**; **Firewood** and lathing by the **cub. fathom = 216 cub. ft.**

In shipping the following are used: —

5 cub. ft.	=	1 barrel bulk (B.B.)
8 B.B. (40 cub. ft.)	=	1 ton of shipping

The ton of shipping is a measure, *not* a weight.

Measure of Capacity.

The Imperial standard measure of capacity is the **Gallon**, which has a cubic content of 277·274 cub. inches, and holds 10 lb. avoirdupois of pure water. A pint of water weighs a pound and a quarter. The table is used both for liquids and dry goods, such as grain, but only within the limits indicated.

Ale and Beer	4 gills = 1 pint (pt.) 2 pt. = 1 quart (qt.) 4 qt. = 1 gallon (gall.)	}	Wines and Spirits
	2 gall. = 1 peck (pk.) 4 pk. = 1 bushel (bush.) 8 bush. = 1 quarter (qr.)	}	Corn and Dry Goods

Grain is now mostly sold by avoirdupois weight, and where the old names are retained the weight in lb. avoirdupois is also quoted in the price lists.

Though the variation is considerable, the most common weights per quarter are 480 and 504 lb. for wheat, 400 and 448 for barley, and 312 and 336 for oats.

Hay and straw are usually sold by the truss or by the load.

36 lb. avoirdupois	Straw	= 1 truss.
56 ,, ,,	Old* Hay	
60 ,, ,,	New Hay	

36 trusses = 1 load

* All hay is called old after the end of August.

Apothecaries' Fluid Measure.

60 minims (m)	= 1 drachm (f.ʒ)
8 drachms	= 1 ounce (f.ȝ)
20 ounces	= 1 pint (O)

The symbols are those used in the prescriptions of medical men.

Paper Measure.

24 sheets	= 1 quire
20 quires	= 1 ream
10 reams	= 1 bale

Table of Time.

60 seconds (sec.)	= 1 minute (min.)
60 min.	= 1 hour (hr.)
24 hrs.	= 1 day
7 days	= 1 week
4 weeks	= 1 lunar month (lun. mo.)
13 lun. mos.	= 1 year (yr.)

Table of Days.

52 weeks and 1 day	= 1 year
365 days	= 1 year
366 days	= 1 leap year

The year is divided into 12 *Calendar Months*:—

January	has 31 days	July	has 31 days
February	,, 28 ,,	August	,, 31 ,,
March	,, 31 ,,	September	,, 30 ,,
April	,, 30 ,,	October	,, 31 ,,
May	,, 31 ,,	November	,, 30 ,,
June	,, 30 ,,	December	,, 31 ,,

The Solar Year consists of 365 days, 5 hours, 48 mins., 48 secs. nearly. In four solar years, therefore, there will be 4×365 days + 1 *day, nearly*. To allow for this odd day, it is arranged that in every fourth year (that is, each year whose number is divisible by 4), the month of February shall contain 29 days instead of 28. Every fourth year will therefore contain 366 days, and it is known as **Leap Year**. Thus 1888 was a Leap Year.

This arrangement would be quite correct if the Solar Year contained 365 days, 6 hours, but this is rather more than its actual duration. The difference really amounts to rather more than 3 days in each 400 years. To allow for the error, only every fourth complete hundred is regarded as Leap Year. Thus, 1600 was a Leap Year (the 16 being exactly divisible by 4); but 1900 was not a Leap Year (19 not being exactly divisible by 4). The year completing the present century (2000) will be a Leap Year.

TABLE OF PRICES (Ready Reckoner).

Where similar calculations of prices have to be regularly made with varying numbers or at varying rates, Tables are constructed showing the total for all the commonly required numbers at different rates. The following is a specimen of such a Table, the principle of construction being quite evident:—

	At ¼d.	At ½d.	At ¾d.	At 1d.	At 2d.	At 3d.	At 4d.	At 5d.
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	£ s. d.
1	0 0¼	0 0½	0 0¾	0 1	0 2	0 3	0 4	0 0 5
2	0 0½	0 1	0 1½	0 2	0 4	0 6	0 8	0 0 10
3	0 0¾	0 1½	0 2¼	0 3	0 6	0 9	1 0	0 1 3
4	0 1	0 2	0 3	0 4	0 8	1 0	1 4	0 1 8
5	0 1¼	0 2½	0 3¾	0 5	0 10	1 3	1 8	0 2 1
6	0 1½	0 3	0 4½	0 6	1 0	1 6	2 0	0 2 6
7	0 1¾	0 3½	0 5¼	0 7	1 2	1 9	2 4	0 2 11
8	0 2	0 4	0 6	0 8	1 4	2 0	2 8	0 3 4
9	0 2¼	0 4½	0 6¾	0 9	1 6	2 3	3 0	0 3 9
10	0 2½	0 5	0 7½	0 10	1 8	2 6	3 4	0 4 2
20	0 5	0 10	1 3	1 8	3 4	5 0	6 8	0 8 4
30	0 7½	1 3	1 10½	2 6	5 0	7 6	10 0	0 12 6
40	0 10	1 8	2 6	3 4	6 8	10 0	13 4	0 16 8
50	1 0½	2 1	3 1½	4 2	8 4	12 6	16 8	1 0 10

COMMERCIAL ARITHMETIC

LONG TOTS AND CROSS TOTS.

An everyday requirement in all kinds of Bookkeeping is the addition of long columns of figures (**Long Tots**).

Such *thorough familiarity with the addition of every possible pair of numbers from* 1 *to* 9 (Addition Table) as makes the summing *practically automatic* is the best preparation for this, and indeed the only one which will make the accuracy of the summations reliable.

For rapid addition abundant practice should be had in taking together small groups of numbers whose sum is 10 or 11, these groups being added instead of the single figures.

As an example :

5	4	1
6	3	9
9	7	8
3	6	4
7	5	6
6	3	7
4	2	3
43	3	8

1st Column—10, 20, 28, 38.
2nd Column—13, 19, 29, 33.
3rd Column—13, 23, 32, 43.

Cross Tots is the same exercise, but with this difference, that the series of numbers to be added, instead of being placed in a vertical column, is arranged in one line *across* the page, *e.g.*, 37652 + 407 + 9386 + 1234567 + 84.

Care should be taken to add, first, all the figures at the *right hand of each number* : next, all those *second to the right*, and so on.

Long Tots and Cross Tots in actual accounts are used as a means of checking, because the totals of the various summations obtained by each process should be the same.

EXERCISE 1.

Add the following columns of figures.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
963279	768357	724937	356356	693256
567567	666666	568745	784577	34983
893258	768329	393939	694369	5247
732496	93493	732457	826782	23456
867386	896896	678356	469469	583245
846846	3246	926874	732487	65876

EXERCISE 2.

As in Exercise 1.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
763763	747474	273527	839783	846738
888888	326932	898989	767676	967396
516935	145671	846846	828348	874567
847258	932932	327327	567345	743474
693693	557955	987654	834568	567345
792357	843278	865865	763427	687568
898989	668668	737373	896748	743257
777777	793257	123456	736985	687435

EXERCISE 3.

Find the sum of the following without writing the numbers in columns.

(1) 85753 + 2748 + 32456 + 76932 + 89.
(2) 297326 + 7932 + 48567 + 987 + 669.
(3) 8945 + 548 + 673256 + 9998 + 4352.
(4) 6693 + 863 + 4692 + 77693 + 9896.
(5) 8674 + 288 + 36974 + 29698 + 77777.
(6) 7764 + 739 + 77645 + 6922 + 86453.
(7) 76723 + 3467 + 24367 + 291674 + 3216745.
(8) 999 + 99999 + 9999 + 99 + 999999.
(9) 987654321 + 12345 + 123456789 + 9879879879.
(10) 7634576345 + 987987 + 693693693 + 7639763.

EXERCISE 4.

Add the vertical columns (*a*), (*b*), (*c*), &c. Add the horizontal rows (1), (2), (3), &c. If the first set of answers is now added **the total should be the same** as that got by adding the second set of answers.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)
(1)	7924	3816	5204	768	1398	4265	5600	832
(2)	3872	1405	6234	1733	4265	836	1274	3508
(3)	138	5672	3088	5671	8824	5903	7800	1833
(4)	5243	739	1527	8933	1076	2694	330	4165
(5)	2756	4193	574	2707	385	7707	8042	5247

EXERCISE 5.

Work as in Exercise 4.

	(*a*)	(*b*)	(*c*)	(*d*)
(1)	296593	185482	1987	69284
(2)	26190	271645	253109	4703
(3)	3086	52080	64210	85296
(4)	758912	371	775378	163073
(5)	165807	6104	468253	407116
(6)	14765	9379	182679	789789
(7)	597134	188267	40307	987345

EXERCISE 6.

As in Exercise 4.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
(1)	123456	234567	123456	345678	321987	890123
(2)	789012	891234	789123	234567	654321	456789
(3)	345678	567891	123456	123456	987654	123456
(4)	901234	234567	789123	456789	321987	789012
(5)	567890	891234	456789	567890	654321	345678
(6)	123456	567890	123456	678912	234567	901234

EXERCISE 7.

As in Exercise 4.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
(1)	62345	31704	65372	46185	23469
(2)	71208	29186	14826	72903	87006
(3)	23796	52831	70293	87657	79664
(4)	51247	27460	89164	25009	31895
(5)	36983	91087	37866	72683	60023
(6)	12345	36825	59143	53746	98745
(7)	21787	74389	70088	86592	23456
(8)	79364	21603	25691	73465	91809
(9)	25873	78765	87325	12098	76345
(10)	12609	12987	46983	88723	23456
(11)	98073	81528	54169	63479	66138
(12)	62648	76479	78635	28634	20980

EXERCISE 8.

Add together lines 1 to 10 both inclusive in columns (*a*), (*b*), (*c*), (*d*), (*e*) of Exercise 7.

EXERCISE 9.

Add together lines 3 to 12 both inclusive of each of the columns of Exercise 7.

EXERCISE 10.

As in Exercise 4.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	Totals
(1)	32752	9228	9987	19425	7079	2657	159	13789	12578	987	
(2)	31725	3847	6105	13489	95483	7639	19843	1997	205	10195	
(3)	3891	3285	892	329	2356	32752	36734	13697	94321	4859	
(4)	38654	456	5943	25963	39642	20639	18090	3535	17449	4073	
(5)	16075	1488	457	9572	18919	74321	2432	16954	16789	4969	
(6)	13276	23473	2657	21555	2076	17218	98763	18763	17286	9876	
(7)	394	17955	38756	16391	18365	8520	5417	18764	375	19077	
(8)	9763	28765	17494	1243	47983	22279	16228	1689	4525	6537	
(9)	9687	12424	1405	8963	7485	1983	2324	13796	178	63287	
(10)	43986	19467	43958	17282	6772	897	2469	1643	1886	2489	
(11)	396	9678	16793	15945	34728	1096	2074	25793	673	2897	
(12)	58392	15587	220	14678	18215	7624	19998	26573	1234	32916	
(13)	6431	6590	34728	43172	13897	17963	6304	4689	24293	98756	
(14)	7238	56789	19845	9876	69840	13688	9576	770	7495	9505	
(15)	7790	615	2502	10075	1899	23209	1369	4689	18346	329	
(16)	28927	1678	17693	33369	39642	819	7400	3784	488	36095	
(17)	43897	963	36569	16596	11835	17632	12367	67935	19482	80632	
(18)	12120	5725	15698	1324	3305	6679	16821	14798	23478	9682	
Totals											Grand Total

EXERCISE 11.

As in Exercise 4.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	Totals
(1)	83159	9384	6315	58169	867	9261	3856	38469	8615	47489	
(2)	17268	8364	63915	92847	86493	46378	3987	19840	85438	76589	
(3)	36493	65748	278	37459	9846	17648	92768	56398	51735	65489	
(4)	674	19380	63984	3769	37659	81549	19056	58614	8739	10932	
(5)	9671	38485	85478	63594	2938	729	38974	9647	46387	15469	
(6)	10749	675	8615	674	95736	6159	6298	9836	8769	78547	
(7)	9875	8239	6472	8267	73916	8398	9689	49216	62847	38617	
(8)	98470	53637	90237	74639	9638	36859	42617	92648	9437	62549	
(9)	11179	84368	8135	76157	7684	984	8369	6478	86059	93684	
(10)	93875	69043	37258	19846	53869	72964	43861	25438	98432	37469	
(11)	86175	8741	58697	86	82379	9284	3849	3476	836	3685	
(12)	63947	53849	98432	96487	95973	738	67436	16279	9643	5918	
(13)	92617	93768	37618	14639	62590	72915	82678	29846	58398	62749	
(14)	9814	10670	76539	65397	9237	93649	82647	65498	7489	83564	
(15)	98	83619	36158	47394	94673	8759	85643	17385	58639	5849	
(16)	29460	295	29438	46074	19084	67395	98639	63984	91547	92467	
(17)	84376	35486	98157	93859	18649	83684	19358	17698	678	76938	
(18)	48739	23584	5893	82176	18769	16738	9375	13738	827	16385	
(19)	3859	46	9846	63958	8379	8465	7483	5367	8469	7386	
(20)	64538	2746	4639	7685	86497	37648	74683	74835	8649	9748	
Totals											Grand Total

MULTIPLICATION.

SHORT METHODS.

1. To multiply by 10, 100, 1000, &c., add to the multiplicand as many ciphers as there are in the multiplier.

Ex. $34581 \times 100 = 3458100$.

2. To multiply by any multiple of 10, 100, &c., *e.g.*, 30, 400, 7000, &c., multiply by the 3, 4, 7, &c., and add as many ciphers as follow it in the multiplier.

Ex. $26137 \times 400 = 10454800$.

3. To multiply by 9. Add to the right of the multiplicand a cipher, and subtract the multiplicand from the number thus formed.

Ex. $137 \times 9 = 1370 - 137 = 1233$.

When we add the cipher we have multiplied by 10, a number which is 1 greater than the multiplier 9. We therefore subtract the multiplicand, which leaves us with 9 times the multiplicand.

4. To multiply by a number consisting entirely of nines, add to the multiplicand as many ciphers as there are nines in the multiplier, and from the result subtract the original multiplicand.

Ex. $36125 \times 999 = 36125000 - 36125 = 36088875$.

5. To multiply by a number consisting entirely of nines except in the units, add to the multiplicand as many ciphers as there are figures in the multiplier, and from the result subtract *that multiple of the original multiplicand by which the unit figure of the multiplier falls short of* 10.

Ex. $7318 \times 9996 = 73180000 - (7318 \times 4)$
$73180000 - 29272 = 73150728$.

6. To multiply by 11. Add a cipher to the right of the multiplicand, and add the multiplicand to the number thus formed.

Ex. $521 \times 11 = 5210 + 521 = 5731$.

The reason for this will be obvious.

7. Similarly when the multiplier exceeds any power of 10 very slightly, proceed as for that power of 10, and to the result *add* as many times the multiplicand as the multiplier exceeds 10, 100, 1000, &c.

Ex. $7124 \times 1007 = (7124 \times 1000) + (7124 \times 7)$
$= 7124000 + 49868 = 7173868.$

8. To multiply by 5. Add a cipher to the right of the multiplicand, and divide by 2.

Ex. $273 \times 5 = 2730 \div 2 = 1365.$

When we add the cipher we are multiplying by 10, a number *twice* as great as 5. The answer, which will of course be twice as great, must therefore be divided by 2.

9. Similarly to multiply by 50, 500, &c., proceed as for 100, 1000, &c., and divide the result by 2.

Ex. $174 \times 50 = 17400 \div 2 = 8700.$

10. To multiply by 25, 125, 250, 625, proceed as for 100, 1000, 1000, 10,000, and divide by 4, 8, 4, and 16 respectively, for $25 = \frac{1}{4}$ of 100, &c.

11. With multipliers ending in 5, it is frequently convenient to double the multiplier and then either divide the multiplicand or the product by 2.

Ex. $329 \times 45 = 329 \times 90 = 29610 \div 2 = 14805.$

12. To multiply by any number from 11 to 19 in one line, multiply by the unit digit and at each step add the figure in the multiplicand *to the right* of that being multiplied, and when the last figure is reached add it to the last figure "carried."

Ex. $31654 \times 17 = 538118.$

The steps are—

7×4	...	...	...	...	28
$7 \times 5 + 2$ (carried)	$+ 4$ (figure to right of				$5) = 41$
$7 \times 6 + 4$ (,,)	$+ 5$ (,,	,,	,,		$6) = 51$
$7 \times 1 + 5$ (,,)	$+ 6$ (,,	,,	,,		$1) = 18$
$7 \times 3 + 1$ (,,)	$+ 1$ (,,	,,	,,		$3) = 23$
2 (,,)	$+ 3$	(last figure)			$= 5$

For numbers between 100 and 109, 1000 and 1009, &c., proceed as in above but add in the figure 2, 3, &c., places to the right instead of the one immediately to the right.

13. To multiply by any two figure multiplier in one line is a similar process to the last.

Ex. $23418 \times 56 = 1311408$.

The steps are—

$6 \times 8 \quad \ldots \quad \ldots = 48$
$6 \times 1 + 4$ (carried) $+ 8 \times 5 = 50$
$6 \times 4 + 5$ (,,) $+ 1 \times 5 = 34$
$6 \times 3 + 3$ (,,) $+ 4 \times 5 = 41$
$6 \times 2 + 4$ (,,) $+ 3 \times 5 = 31$
3 (,,) $+ 2 \times 5 = 13$

14. When both multiplier and multiplicand are a little greater than 100, 1000, &c., *e.g.*, 106×109.

1. Multiply together the numbers by which the quantities exceed 100, 1000, &c.

   $6 \times 9 = 54$ (these are the two right-hand figures of the answer).

2. Add to the first the excess of the other over 100, 1000, &c.

   $106 + 9 = 115$ (these are *hundreds* in the answer because the numbers dealt with slightly exceed 100).

Ans. 11554.

15. When both multiplier and multiplicand are a little less than 100, 1000, &c., *e.g.*, 994×991.

1. Multiply together the numbers by which the quantities fall short of 100, 1000, &c.

   $6 \times 9 = 54$ (these are the two right-hand figures of the answer).

2. From the first subtract the numbers by which the second falls short of 100, 1000, &c.

   $994 - 9 = 985$ (these are *thousands* in the answer because the numbers dealt with are a little less than 1000).

Ans. 985054.

Ex. 88×89.

$$(100 - 88) \times (100 - 89) = 12 \times 11 = 132$$

The 32 are the right-hand digits of the answer, the 1 is *one hundred* in the answer.

$88 - (100 - 89) = 88 - 11 = 77$ (hundreds)
77 hundred + 1 hundred = 78 hundred.

Ans. 7832.

16. The relation between the digits or groups of

digits in the multiplier is often such as allows the process to be shortened.

Ex. 31564 × 639 (630 = 70 times 9).

31564
639

284076 = 9 times 31564
19885320 = 70 times 284076

20169396

Freedom of Arrangement. The usual rule is to begin with the units figure of the multiplier, and having multiplied each digit of the multiplicand by that, to proceed to do the same again, using the tens figure as multiplier, being careful to place the first figure of the product so obtained under the tens figure of the previous product, and so on, using each figure of the multiplier in turn.

It should be understood that this method is only one of convenience, and that the multiplication may be done by taking the digits of the multiplier *in any order*, the essential point being that the digits so obtained are put in the proper columns.

This is often found useful in applying the contracted method last mentioned.

Ex. (1)

2973
856
23784
166488
2544888

Multiply first by 8, setting the first figure of the quotient in the hundreds place. Then instead of multiplying by 5 and by 6 in separate lines, we multiply by 56. We can do this readily as we have already multiplied by 8, and we have only to multiply that result by the other factor of 56, viz., 7. The placing of this result is determined by the place of the 6, the right-hand figure of the 56.

Ex. (2) 10973 × 37218

Multiply 10973 by 3, that result by 6, and that again by 4, taking care to put each line in its proper place.

10973
37218

32919 Multiplying by 3.
197514 ,, 18 (or 3 × 6).
790056 ,, 72 (or 18 × 4).

408393114

The correctness of a **product** may be tested by the process known as **casting out the nines.**

Add the digits of the *multiplicand*, divide the sum by 9, and set down the remainder.

Do the same as regards the *multiplier*. Multiply together these two remainders and divide the product by 9. The remainder so obtained should be the same as that got when the sum of the digits in the *product* is divided by 9.

Ex. 361472
13823

1084416 = 361472 × 3
722944 = 361472 × 20

8313856 = 361472 × 23
49883136 = 8313856 × 600 because 138 = 6 × 23

4996627456

Proof— $(3 + 6 + 1 + 4 + 7 + 2) \div 9 = 2$ with remainder **5**
$(1 + 3 + 8 + 2 + 3) \div 9 = 1$,, ,, **8**
$(5 \times 8) \div 9 = 4$,, ,, **4**
$(4 + 9 + 9 + 6 + 6 + 2 + 7 + 4 + 5 + 6) \div 9 = 6$,, ,, **4**

EXERCISE 12.

Work, as far as possible, *mentally*, the following :—

(1) 2374 × 10 ; 100 ; 1000 ; 100000
(2) 906 × 1000 ; 10 ; 10000 ; 1000000
(3) 3890 × 10 ; 1000 ; 100 ; 10000
(4) 71042 × 20 ; 60 ; 500 ; 7000
(5) 3590 × 30 ; 800 ; 70000 ; 9000
(6) 785 × 4000 ; 600000 ; 9000 ; 200

(7) 768395 × 9
(8) 274876 × 99
(9) 823548 × 999
(10) 657429 × 9999
(11) 528754 × 97
(12) 487651 × 994
(13) 348916 × 9995
(14) 475628 × 9980
(15) 123456 × 5
(16) 789012 × 50
(17) 345678 × 500
(18) 901234 × 25
(19) 567890 × 125
(20) 123456 × 625

EXERCISE 13.

Multiply, in *one* line, the following :—

(1) 3726954 × 11
(2) 4637845 × 12
(3) 5938621 × 13
(4) 6892147 × 14
(5) 5789214 × 15
(6) 6843297 × 16
(7) 5743296 × 17
(8) 6934578 × 18
(9) 8912456 × 19
(10) 7890123 × 23
(11) 4567890 × 34
(12) 1234567 × 45
(13) 8901234 × 56
(14) 5678901 × 67
(15) 2345678 × 78
(16) 9012345 × 89

EXERCISE 14.

Multiply, *in the shortest way*, the following :—

(1) 28674 × 11, 101, 125
(2) 31568 × 25, 99, 35
(3) 82751 × 17, 997, 250
(4) 36715 × 125, 1005, 13
(5) 75823 × 104, 25, 98
(6) 62174 × 250, 19, 1003
(7) 31768 × 25, 16, 97
(8) 47369 × 125, 15, 102
(9) 63971 × 250, 18, 45
(10) 36759 × 125, 35, 998

EXERCISE 15.

Multiply, *in two lines*, number 1 to number 16 ; and number 17 to number 30 *in three lines*.

(1) 37234 × 648
(2) 62089 × 287
(3) 12895 × 279
(4) 73985 × 255
(5) 84527 × 183
(6) 39874 × 568
(7) 58479 × 364
(8) 36874 × 1089
(9) 46789 × 13212
(10) 562875 × 945
(11) 253769 × 763
(12) 202842 × 324
(13) 632047 × 525
(14) 369875 × 832
(15) 527698 × 9108
(16) 78932 × 12111
(17) 361428 × 48126
(18) 463189 × 64164
(19) 236975 × 54273
(20) 529875 × 72369
(21) 693716 × 83296
(22) 387695 × 63672
(23) 141651 × 42856
(24) 816849 × 27309
(25) 729834 × 32864
(26) 329754 × 36448
(27) 698327 × 81936
(28) 731865 × 25525
(29) 78932612 × 5678109
(30) 78932612 × 567981

EXERCISE 16 (General).

(1) 93245678 × 16, 25, 18
(2) 31290427 × 14, 27, 48
(3) 35742394 × 15, 36, 22
(4) 32467894 × 21, 44, 45
(5) 69324863 × 72, 49, 55
(6) 46793256 × 24, 35, 99
(7) 34875694 × 54, 33, 56
(8) 29467328 × 32, 84, 121
(9) 72134567 × 63, 66, 96
(10) 74328943 × 64, 77, 108
(11) 94326789 × 28, 81, 132
(12) 45672398 × 42, 88, 144
(13) 729586 × 513
(14) 683792 × 568
(15) 719480 × 679
(16) 927059 × 729
(17) 876543 × 268
(18) 920786 × 794
(19) 379959 × 512
(20) 862479 × 5370
(21) 737870 × 9076
(22) 846897 × 6807
(23) 390867 × 5098
(24) 864297 × 6005
(25) 934685 × 38709
(26) 946587 × 608090
(27) 896385 × 69874
(28) 378600 × 750000
(29) 6879000 × 87400
(30) 8695720 × 975864
(31) 314159 × 78000
(32) 283600 × 76000
(33) 947000 × 89000
(34) 807000 × 97600
(35) 799600 × 87000
(36) 870000 × 79000
(37) 357927 × 35748
(38) 523469 × 87569
(39) 375698 × 76598
(40) 967483 × 84697
(41) 793684 × 70806
(42) 746786 × 96807
(43) 889688 × 60709
(44) 294673 × 76009
(45) 304096 × 700009
(46) 999999 × 940057
(47) 468567 × 780009
(48) 826826 × 956708
(49) 656565 × 867009
(50) 234567 × 800976
(51) 297792 × 978006
(52) 397856 × 900807

SQUARES.

When any **number is multiplied by itself** the product is called the **square** of that number.

Thus 25, the product of 5 × 5, is called the square of 5, usually written 5^2.

Similarly 125, the product of 5 × 5 × 5, is called the **cube** of 5, usually written 5^3.

The number which, when multiplied by itself, produces a given number is called the **Square Root** of that number.

Thus since $5 \times 5 = 25$, we say that 5 is the square root of 25. This is usually written $\sqrt{25} = 5$.

Similarly, since $5 \times 5 \times 5 = 125$, we say that 5 is the **Cube Root** of 125, usually written $\sqrt[3]{125} = 5$.

The application of the following principles will often save the labour of multiplication.

1. *The square of the sum of two numbers is equal to the sum of the squares of the numbers, plus twice the product of the numbers.*

Ex. $76^2 = 70^2 + 6^2 + 2\,(70 \times 6)$
$= 4900 + 36 + 840 = 5776$

2. *The square of the difference of two numbers is equal to the sum of the squares of the numbers, minus twice the product of the numbers.*

Ex. $93^2 = 100^2 + 7^2 - 2\,(100 \times 7)$
$= 10000 + 49 - 1400 = 8649$

3. *The difference of the squares of two numbers is equal to the product of the sum and the difference of the numbers.*

Ex. $53^2 - 49^2 = (53 + 49) \times (53 - 49)$
$= 102 \times 4 = 408$

EXERCISE 17.

Find *mentally* the **squares** of the following :—

(1) 21	(8) 101	(15) 68
(2) 41	(9) 203	(16) 85
(3) 33	(10) 1002	(17) 27
(4) 53	(11) 56	(18) 206
(5) 47	(12) 105	(19) 39
(6) 99	(13) 74	(20) 1010
(7) 999	(14) 89	

Give also *mentally* the answers to the following :—

(21) $46^2 - 45^2$	(26) 42×38
(22) $85^2 - 75^2$	(27) 53×47
(23) $139^2 - 39^2$	(28) 27×23
(24) $78^2 - 73^2$	(29) 550×450
(25) $37^2 - 35^2$	(30) 75×65

PRIME NUMBERS, FACTORS, MULTIPLES, TESTS OF DIVISIBILITY.

A number which cannot be divided *without remainder* except by itself and unity (that is, **1**) is called a **Prime Number** or Prime.

Thus 1, 3, 7 are prime numbers.

All numbers, other than prime, are called **Composite Numbers.**

The numbers which divide a composite number without remainder are called its **Factors**, and if these are prime, its **Prime Factors.**

Thus since $42 = 6 \times 7$, the numbers **6 and 7 are factors of 42**, but as 6 has as factors the primes 2 and 3, the **prime factors of 42 are 2, 3, and 7.**

The composite number is said to be a **Multiple** of each of its factors.

Thus 24 is a multiple of 2, 3, 4, 6, 8, and 12, since it is divisible by each of these without remainder.

The factors of any number can be found by actually dividing it by each of the prime numbers in turn—2, 3, 5, 7, &c.—and seeing whether or not there is a remainder, but there are certain **Tests of Divisibility**, and a knowledge of these will save this labour.

(1) **2** is a factor if the right hand or units digit of the number is exactly divisible by 2. Such numbers are called *even* numbers. Thus, 1398 is divisible by 2, because the units digit 8 is so divisible.

(2) **3** is a factor if the sum of the digits of the number is exactly divisible by 3. Thus, 315417 is divisible by 3, because $3 + 1 + 5 + 4 + 1 + 7 = 21$, is so divisible.

(3) **4** is a factor if the number formed by the two right-hand digits is exactly divisible by 4. Thus, 610524 is divisible by 4, because 24 is so divisible.

(4) **5** is a factor if the number has 0 or 5 for its right-hand digit.

(5) **6** is a factor if both 2 and 3 are factors, and so with all ***composite*** **numbers.**

(6) **8** is a factor if the number formed by the three right-hand digits is exactly divisible by 8. Thus, 7132504 is divisible by 8, because 504 is so divisible.

(7) **9** is a factor if the sum of the digits is exactly divisible by 9. Thus, 41922 is divisible by 9, because 4 + 1 + 9 + 2 + 2 = 18, is so divisible.

(8) **10** is a factor if the right-hand digit is 0.

(9) **11** is a factor if the sum of the digits in the *even* places equals the sum of the digits in the *odd* places, or if the difference of these sums is exactly divisible by 11. Thus, 11 is a factor of 3472095, for 3 + 7 + 0 + 5 = 15 and 4 + 2 + 9 = 15.

In finding prime factors. we may at any stage divide by a number which is not prime, provided that we write down its prime factors afterwards.

Ex. *Find the prime factors of* 570240.

10	570240	Divisible by 10, as last figure is 0.
8	57024	,, 8, as 24 is divisible by 8.
11	7128	,, 11, as 7 + 2 = 8 + 1.
9	648	,, 9, as 6 + 4 + 8 is divisible by 9.
9	72	
	8	

570240 = 10. 8. 11. 9. 9. 8
= 5. 2. 2. 2. 2. 2. 11. 3. 3. 3. 3. 2. 2. 2

which may be written, $2^7 . 3^4 . 5 . 11$.

EXERCISE 18.

1. Resolve into prime factors :—

(1)	(*a*) 6	(*f*) 42	(*k*) 704	(*p*) 1188
	(*b*) 12	(*g*) 55	(*l*) 756	(*q*) 1331
	(*c*) 15	(*h*) 66	(*m*) 792	(*r*) 1452
	(*d*) 21	(*i*) 70	(*n*) 1056	(*s*) 1584
	(*e*) 30	(*j*) 75	(*o*) 624	(*t*) 1728
(2)	(*a*) 68	(*e*) 252	(*i*) 715	(*m*) 3675
	(*b*) 78	(*f*) 315	(*j*) 854	(*n*) 4620
	(*c*) 102	(*g*) 525	(*k*) 1331	(*o*) 5250
	(*d*) 210	(*h*) 612	(*l*) 1456	(*p*) 11025

2. Write down all the prime numbers from 501 to 1000.

3. Tell at sight the remainders of the following dividends, the divisors being 2, 3, 4, 5, 8, 9, 11 :—

3175	18743	25567	9876541
14739	75262	70011	31054289

4. What is the smallest number above 1000 divisible by 2, by 3, by 4, by 5, by 6, by 7, by 8, by 9, by 10, by 11?

DIVISION.

SHORT METHODS.

1. To divide by 10, 100, 1000, &c., cut off as many digits at the right hand of the dividend as there are ciphers in the divisor. The figures so cut off form the *remainder*, and the others form the quotient.

Ex. 346273 ÷ 100 = 3462 quotient + 73 remainder.

2. To divide by any multiple of 10, 100, 1000, &c., *e.g.*, 30, 400, 7000, &c., cut off the ciphers from the divisor and an equal number of digits from the right hand of the dividend; divide the remaining number by the new divisor and put the cut-off digits to the right of the remainder for the full remainder.

Ex. 31528694 ÷ 400

400 | 315286$\frac{94}{294}$

78821 + 294

3. To divide by 5, 50, 500, &c., multiply both dividend and divisor by 2, and proceed as in dividing by 10, 100, 1000, &c., remembering to *divide the remainder* so obtained by 2.

Ex. 31627 ÷ 50 = (31627 × 2) ÷ (50 × 2)
= 63254 ÷ 100
= 632 + (54 ÷ 2)
= 632 quotient + 27 remainder.

4. To divide by 25, 125, 250, 625, multiply both dividend and divisor by 4, 8, 4, and 16 respectively, and proceed as in dividing by 100, 1000, 1000, 10,000, remembering to divide

the remainder, as in the last rule, by the multiplier used in each case.

Ex. $23485 \div 125 = (23485 \times 8) \div (125 \times 8)$
$= 187880 \div 1000$
$= 187 + (880 \div 8)$
$= 187$ quotient + 110 remainder.

5. Wherever the divisor can be broken up into factors, make use of continued *short* division, thus :—

3147025 ÷ 63 (Factors 9 × 7)

9 3147025 units
7| 349669 nines + 4 units (remainder)
49952 sixty-threes + 5 nines (remainder).

5 nines + 4 units = 49, therefore **Ans. 49952 + 49.**

To find the remainder after division by factors, multiply each separate remainder by all the divisors *preceding the one which produced it*, and add the products.

6. Contracted or Italian Long Division. This is a method by which the process may be shortened by omitting the products and setting down only the difference at each successive step.

Ex. Long Method.	**Ex.** Contracted Method.
268)2153749(8036	268)2153749(8036
2144	
... 974	0974
804	1709
1709	. 101
1608	
. 101	

In finding the difference "Complementary Addition" is the method usually followed. Thus in the above example we proceed, mentally, thus :—8 × 8 = 64 and **9** make **73**. Put down 9 as you say it and proceed, carrying 7. 8 × 6 = 48 + 7 = 55 and **0** make **55**. Put down 0 and proceed, carrying 5. 8 × 2 = 16 + 5 = 21 and **0** make 21. The 0 put in at the previous step is now seen to be useless and may be cancelled.

Where the division is long, that is, whether there will be many figures in the quotient, or where the same divisor is to be used over and over again, **a table of the multiples of**

the divisor should be made on a slip of paper. For divisor 365, for instance, it would be like this :—

365	730	1095	1460	1825	2190	2555	2920	3285
1	2	3	4	5	6	7	8	9

The slip is placed for each successive step at the right point under the dividend and the subtraction done without writing down the product.

Ex.

365)100000000(273972
2700
1450
3550
2650
950
220 remainder.

EXERCISE 19.

Divide, *mentally*, the following :—

(1) 3726497 ÷ 10, 1000, 100, 10000, 1000000
(2) 10736710 ÷ 100, 10000, 1000000, 10, 100000
(3) 73245993 ÷ 70, 300, 8000
(4) 15467105 ÷ 400, 60, 50000
(5) 48790300 ÷ 30, 200, 12000
(6) 82134740 ÷ 100, 11000, 120
(7) 27368 ÷ 5, 50, 500, 5000
(8) 311 ÷ 25, 125
(9) 723 ÷ 25, 125, 250
(10) 8141 ÷ 25, 125, 625

EXERCISE 20.

Divide by factors :—

(1) 8234567923 ÷ 16, 15, 18
(2) 7906234793 ÷ 24, 25, 27
(3) 6235792834 ÷ 14, 32, 36
(4) 9674325698 ÷ 44, 45, 48
(5) 8207634256 ÷ 54, 56, 63
(6) 7923457923 ÷ 64, 66, 72
(7) 8234506739 ÷ 75, 98, 216
(8) 7236784263 ÷ 125, 126, 128
(9) 8094237693 ÷ 135, 147, 192
(10) 8236792087 ÷ 154, 162, 252
(11) 7934256937 ÷ 168, 225, 343
(12) 8293476238 ÷ 324, 315, 704

EXERCISE 21.

(*a*) Work, by ordinary method, and also, as a second set of exercises, by the Italian method, the following exercises.

(*b*) Do number 10 to number 18 by means of **Tables of Multiples.**

(1) 8567435697 ÷ 31, 41, 51
(2) 7928354235 ÷ 52, 43, 53
(3) 6097432807 ÷ 53, 34, 43
(4) 7923406573 ÷ 62, 65, 67
(5) 8246735849 ÷ 92, 93, 97
(6) 7834562935 ÷ 83, 87, 69
(7) 8236745836 ÷ 83, 69, 74
(8) 7924563894 ÷ 73, 57, 47
(9) 8345672397 ÷ 47, 78, 65
(10) 6932458273 ÷ 513, 612
(11) 8246739846 ÷ 423, 432
(12) 7632834569 ÷ 522, 631
(13) 9435682435 ÷ 633, 235
(14) 7923567293 ÷ 832, 567
(15) 7935684279 ÷ 567, 765
(16) 8467569314 ÷ 864, 873
(17) 8267395423 ÷ 963, 987
(18) 8436725238 ÷ 879, 984

(19) 5693242854 ÷ 8234
(20) 3672364571 ÷ 7193
(21) 7934670585 ÷ 8243
(22) 1234671823 ÷ 8239
(23) 8092343538 ÷ 9426
(24) 6972354851 ÷ 5467
(25) 3672352944 ÷ 6396
(26) 3279358572 ÷ 5637
(27) 3245671233 ÷ 2763
(28) 7832672636 ÷ 8972
(29) 6932743673 ÷ 3947
(30) 8967238474 ÷ 7937
(31) 7934670579 ÷ 7923
(32) 6932745596 ÷ 6793
(33) 3672301248 ÷ 82799
(34) 6398684632 ÷ 45098
(35) 4683572256 ÷ 45672
(36) 6834226483 ÷ 79369

LONG TOTS AND CROSS TOTS (*Money*).

Rapid addition of columns of money is very necessary in commercial work. It is usual for those engaged in such work to add pence and farthings *at one operation*, and accountants are often expert enough to add shillings, pence, and farthings together.

The following method, known as the "dot and carry" method, is found, by many, to secure both speed and accuracy. In adding each column a dot is made whenever a sum is

reached at least equal to one of the next higher denomination, and the excess is carried forward to the further summation.

£	s.	d.
37	13·	5
2	6	9·
19	12·	4
6	17·	3·
24	15	10
91	5	7

Say 10, 13 = 1s. 1d.—make dot; carrying forward 1, say 5, 14 = 1s. 2d.—make dot; carrying forward 2, say 7 which put down in the answer. As there are 2 dots carry 2 to the shillings.

2, 17, 34 = £1, 14s.—make dot; carrying forward 14, say 14, 26 = £1, 6s.—make dot; carrying forward 6, say 6, 12, 25 = £1, 5s.—make dot and put down 5 in the answer. As there are 3 dots carry 3 to the pounds.

In Bookkeeping the **Balance**, *i.e.*, the difference between a certain total and the sum of a number of lines of pounds, shillings and pence, is usually found by **Complementary Addition**.

Ex.

	£	s.	d.
Sums to be added	26	13	$4\frac{1}{2}$
	2	0	8
	314	15	$6\frac{1}{2}$
	70	12	$3\frac{1}{4}$
Balance	*216*	*5*	$7\frac{3}{4}$
Total	630	7	6

Mentally—1, 3, 5 = $1\frac{1}{4}$ and $\frac{3}{4}$ (put down) = 2d. (carry);
2, 5, 11, 19, 23 = 1s. 11d. and 7d. (put down) = 2s. 6d. (carry 2);
2, 14, 29, 42 = £2, 2s. and 5s. (put down) = £2, 7s. (carry 2);
2, 6, 8, 14 and 6 (put down) = 20 (carry 2);
2, 9, 10, 12 and 1 (put down) = 13 (carry 1);
1, 4 and 2 (put down) = 6.

EXERCISE 22.

1. Add as they stand the rows (*a*), (*b*), (*c*), (*d*), (*e*). Find also total.

	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(*a*)	207	14	3	321	15	10	28	17	9	631	5	7	520	0	6	500	15	6
(*b*)	512	9	9	719	4	7	371	7	3	72	16	8	413	12	5	142	3	4
(*c*)	176	10	$4\frac{1}{2}$	445	13	8	414	13	$6\frac{3}{4}$	114	14	11	240	5	$7\frac{1}{2}$	97	10	$3\frac{1}{2}$
(*d*)	527	17	6	873	9	2	801	5	4	138	3	2	17	18	9	370	18	9
(*e*)	389	7	$9\frac{1}{4}$	96	19	$8\frac{1}{2}$	990	18	$0\frac{1}{2}$	501	0	9	125	3	$4\frac{3}{4}$	412	17	$7\frac{1}{2}$

2. Add lines 1 to 12 inclusive in columns (*a*), (*b*), (*c*), and (*d*).
3. Add lines 13 to 26 inclusive in each of the columns.
4. Add lines 1 to 18 inclusive in each of the columns.
5. Add lines 1 to 26 inclusive in each of the columns.

	(*a*)			(*b*)			(*c*)			(*d*)		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(1)	1	13	$7\frac{1}{4}$	0	0	$7\frac{1}{2}$	58	11	0	9	7	$6\frac{1}{2}$
(2)	0	7	8	0	15	$0\frac{3}{4}$	6	10	$11\frac{1}{2}$	10	19	$10\frac{1}{4}$
(3)	0	12	$0\frac{1}{2}$	1	10	$11\frac{1}{4}$	46	15	$10\frac{1}{4}$	11	18	$9\frac{3}{4}$
(4)	8	0	$11\frac{3}{4}$	16	16	$6\frac{1}{2}$	68	19	$11\frac{3}{4}$	12	17	$11\frac{1}{4}$
(5)	0	19	$10\frac{1}{2}$	85	14	$10\frac{3}{4}$	93	8	$7\frac{1}{2}$	13	16	$8\frac{3}{4}$
(6)	18	12	$5\frac{1}{4}$	60	17	$9\frac{1}{4}$	56	16	$11\frac{3}{4}$	14	15	$10\frac{1}{2}$
(7)	98	17	$7\frac{1}{4}$	94	19	$11\frac{3}{4}$	66	16	$6\frac{1}{2}$	254	14	$11\frac{1}{4}$
(8)	87	16	$10\frac{3}{4}$	70	16	$10\frac{1}{4}$	78	19	$10\frac{1}{4}$	715	18	$10\frac{1}{2}$
(9)	76	19	$11\frac{3}{4}$	59	17	$9\frac{1}{2}$	89	13	$9\frac{3}{4}$	916	15	$5\frac{3}{4}$
(10)	65	16	$9\frac{1}{2}$	47	18	$8\frac{1}{2}$	96	17	$11\frac{1}{4}$	175	10	$7\frac{1}{4}$
(11)	48	18	$10\frac{1}{2}$	65	11	$11\frac{1}{4}$	56	14	$7\frac{1}{2}$	89	13	$4\frac{1}{2}$
(12)	73	13	$7\frac{1}{4}$	88	17	$5\frac{1}{4}$	83	18	$10\frac{3}{4}$	7	19	$7\frac{3}{4}$
(13)	327	16	$11\frac{3}{4}$	674	18	$10\frac{1}{4}$	746	12	$3\frac{1}{2}$	548	19	$11\frac{1}{4}$
(14)	273	13	$9\frac{1}{2}$	476	12	$4\frac{1}{2}$	476	19	$11\frac{3}{4}$	485	12	$8\frac{3}{4}$
(15)	723	15	$8\frac{1}{4}$	647	19	$11\frac{3}{4}$	764	14	$9\frac{1}{4}$	584	17	$10\frac{1}{2}$
(16)	342	19	$10\frac{1}{2}$	597	13	$8\frac{1}{2}$	495	18	$10\frac{1}{2}$	999	17	$11\frac{3}{4}$
(17)	598	6	$4\frac{3}{4}$	975	17	$11\frac{3}{4}$	594	2	$8\frac{3}{4}$	666	11	$9\frac{1}{4}$
(18)	985	14	$11\frac{1}{4}$	759	13	$10\frac{1}{2}$	945	3	$11\frac{1}{2}$	707	16	$10\frac{1}{2}$
(19)	328	14	$7\frac{1}{2}$	476	16	$6\frac{1}{2}$	816	17	$8\frac{1}{2}$	568	18	$10\frac{1}{2}$
(20)	800	17	$5\frac{1}{4}$	567	18	$8\frac{1}{4}$	389	10	$10\frac{1}{4}$	374	16	$10\frac{1}{4}$
(21)	407	12	$8\frac{1}{2}$	678	19	$11\frac{3}{4}$	31	17	11	843	18	$8\frac{1}{4}$
(22)	670	18	$10\frac{1}{2}$	789	17	$10\frac{1}{2}$	346	18	$6\frac{1}{2}$	276	13	$7\frac{1}{2}$
(23)	598	10	$10\frac{3}{4}$	890	15	$4\frac{1}{4}$	407	13	$8\frac{3}{4}$	751	11	$9\frac{1}{4}$
(24)	742	18	$11\frac{1}{4}$	910	13	$3\frac{1}{2}$	748	11	11	208	10	$11\frac{3}{4}$
(25)	967	17	$11\frac{3}{4}$	678	18	$11\frac{3}{4}$	567	14	$4\frac{3}{4}$	540	19	$3\frac{1}{2}$
(26)	864	18	$11\frac{1}{4}$	497	17	$5\frac{1}{2}$	687	15	$10\frac{1}{4}$	115	17	$6\frac{1}{2}$

EXERCISE 23.

Add the columns (*a*), (*b*), &c., and rows (1), (2), &c., and find the *gross total*.

1.

	(*a*)			(*b*)			(*c*)			(*d*)		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(1)	324	7	4	7526	3	0	26	4	7	735	16	11
(2)	8136	14	3	343	16	4	93	16	2	189	4	2
(3)	4245	12	11	5127	12	5	17	4	8	6265	16	3
(4)	633	9	8	83	9	10	85	18	5	845	3	7
(5)	6215	14	2	417	0	5	35	2	5	176	12	3
(6)	73	0	6	3117	10	5	71	6	4	456	7	8
(7)	5831	10	10	27	15	6	15	12	8	95	17	3
(8)	243	4	7	83	3	6	34	8	2	6124	6	8
(9)	2717	11	3	7172	14	10	72	19	9	36	19	11
(10)	208	19	9	37	9	11	18	13	3	148	6	10
(11)	3	12	4	13	16	4	25	9	4	8224	12	2
(12)	65	11	2	4231	3	9	3	17	11	371	13	11
(13)	9138	4	1	145	9	10	73	2	9	29	9	8
(14)	85	17	8	36	4	2	12	6	4	9249	6	11
(15)	236	8	9	712	18	4	9	8	3	73	4	7

2.

	(*a*)			(*b*)			(*c*)			(*d*)		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(1)	824	9	6	26	13	9	4126	14	7	35	6	1
(2)	139	12	8	5342	14	1	93	16	8	89	14	2
(3)	4240	8	11	27	2	5	317	3	8	65	16	10
(4)	233	12	8	1784	9	1	85	14	4	45	13	7
(5)	15	14	4	417	10	5	1735	12	5	178	12	6
(6)	3173	1	0	183	0	8	71	6	6	56	7	8
(7)	831	0	10	827	15	6	215	10	8	95	14	9
(8)	4213	14	7	85	5	10	34	8	5	124	6	8
(9)	17	16	4	72	13	10	3572	12	9	39	12	11
(10)	5208	9	9	1937	9	0	18	13	2	148	6	10
(11)	3	12	8	316	12	4	825	19	4	24	16	2
(12)	665	15	2	31	3	4	3	17	11	77	13	4
(13)	138	4	7	3145	11	10	173	0	9	29	9	8
(14)	485	12	8	136	4	6	412	3	4	42	16	11
(15)	236	8	3	712	16	4	29	8	7	73	4	7

3. Add *lines* (1) to (18) (inclusive) of columns (*a*), (*b*), (*c*), and (*d*); *rows* (*a*), (*b*), (*c*), and (*d*) of lines (1) to (18); and also find gross total.

4. As in No. (3), using lines (4) to (21) (both inclusive) and columns (*b*), (*c*), (*d*), and (*e*).

5. As in Nos. (1) and (2).

	(*a*)			(*b*)			(*c*)			(*d*)			(*e*)		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(1)	328	18	4	726	10	5	615	8	10	826	13	9	126	14	7
(2)	773	6	7	417	3	6	440	16	7	73	4	7	18	13	8
(3)	363	14	11	95	16	8	89	14	0	342	14	11	93	16	8
(4)	82	14	8	37	10	11	467	0	5	712	16	4	937	0	10
(5)	627	2	5	319	17	8	865	18	10	27	2	5	317	3	8
(6)	29	3	11	62	9	10	70	16	11	326	8	11	208	9	9
(7)	784	9	7	885	14	4	45	3	7	784	9	7	85	14	4
(8)	869	19	6	139	2	6	925	14	4	29	9	8	39	12	11
(9)	477	16	5	35	12	5	78	12	6	417	10	5	735	12	5
(10)	142	10	10	578	16	9	754	19	9	173	0	9	572	14	9
(11)	173	0	8	779	9	5	352	4	8	183	0	8	71	6	6
(12)	12	4	7	172	13	10	17	3	11	412	3	4	172	13	10
(13)	827	15	6	15	0	8	796	19	10	827	15	6	215	10	8
(14)	283	8	11	234	18	11	134	16	8	85	5	10	34	8	11
(15)	272	13	10	577	15	9	99	2	9	273	13	10	572	12	9
(16)	37	19	8	918	3	9	48	6	10	937	9	9	18	13	8
(17)	319	2	6	25	19	8	123	17	7	316	12	6	825	19	4
(18)	931	8	3	63	17	10	678	3	9	31	3	8	3	17	11
(19)	45	1	10	174	10	9	29	19	5	145	11	10	178	0	9
(20)	236	14	9	12	18	4	442	16	11	136	4	6	412	3	7
(21)	722	18	7	429	8	7	179	8	3	712	16	7	29	8	7

EXERCISE 24.

Add the columns (*a*), (*b*), &c., *as they stand*; the rows (1), (2), (3), &c.; and find the Grand Total.

	(*a*)			(*b*)			(*c*)			(*d*)			(*e*)			(*f*)			Totals
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
(1)	3162	8	$3\frac{1}{2}$	9865	13	$9\frac{1}{4}$	7093	14	$11\frac{1}{2}$	9607	19	2	3851	17	$4\frac{1}{4}$	7408	13	9	
(2)	865	14	$7\frac{3}{4}$	859	11	$7\frac{1}{2}$	694	19	$4\frac{1}{4}$	8017	13	$6\frac{1}{2}$	7328	9	11	8016	14	$7\frac{1}{4}$	
(3)	9307	13	$6\frac{1}{2}$	7093	14	$5\frac{1}{4}$	8076	12	11	7329	14	8	697	13	10	783	12	$9\frac{1}{2}$	
(4)	739	16	$10\frac{1}{4}$	5628	1	$4\frac{1}{2}$	7326	19	$10\frac{1}{4}$	8620	5	$3\frac{1}{2}$	7013	12	$4\frac{1}{4}$	6270	19	$8\frac{3}{4}$	
(5)	8903	17	$11\frac{1}{4}$	986	12	$3\frac{1}{4}$	8016	14	$9\frac{1}{2}$	7302	11	10	86	14	4	5	17	$3\frac{1}{2}$	
(6)	719	16	8	3290	14	$10\frac{1}{2}$	876	12	$11\frac{1}{4}$	9008	14	$3\frac{1}{4}$	719	16	2	8932	13	$5\frac{1}{4}$	
(7)	2416	11	$4\frac{1}{2}$	876	12	$9\frac{3}{4}$	5432	6	$7\frac{1}{4}$	2415	7	$4\frac{1}{2}$	8623	9	$8\frac{1}{2}$	749	19	$11\frac{3}{4}$	
(8)	6027	13	$5\frac{1}{4}$	9046	13	$5\frac{1}{2}$	8026	18	$4\frac{1}{2}$	596	17	7	9701	5	$8\frac{1}{4}$	8072	13	$4\frac{1}{4}$	
(9)	5806	14	$11\frac{3}{4}$	2163	14	$10\frac{1}{4}$	2369	13	$11\frac{1}{2}$	8004	16	$4\frac{1}{2}$	899	18	4	731	4	$9\frac{1}{2}$	
(10)	981	2	$6\frac{1}{2}$	785	11	$9\frac{1}{4}$	864	14	$6\frac{1}{2}$	731	13	5	96	14	3	8201	15	$4\frac{1}{4}$	
(11)	7285	16	$9\frac{3}{4}$	1234	17	$11\frac{3}{4}$	5678	17	$4\frac{1}{2}$	965	16	$11\frac{1}{4}$	8032	17	$11\frac{1}{2}$	2098	17	$5\frac{3}{4}$	
(12)	6028	12	3	98	5	$10\frac{1}{4}$	2863	18	$6\frac{3}{4}$	1086	12	$7\frac{1}{4}$	119	4	$6\frac{1}{4}$	9007	19	$11\frac{1}{2}$	
(13)	864	3	$2\frac{1}{2}$	965	18	$4\frac{1}{2}$	9306	5	11	8721	18	$9\frac{1}{2}$	7068	13	10	1306	14	$7\frac{1}{4}$	
(14)	3208	18	11	1208	13	$9\frac{1}{4}$	1872	16	$3\frac{1}{2}$	7048	2	6	901	17	$3\frac{1}{2}$	940	7	$10\frac{1}{2}$	
Totals																			Grand Total

EXERCISE 25.

Add the columns and rows as they stand. Find the totals and the Grand Total.

	(*a*)			(*b*)			(*c*)			(*d*)			(*e*)			(*f*)			(*g*)			(*h*)			Totals
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
(1)	9032	14	$11\frac{1}{2}$	7314	13	$2\frac{1}{4}$	79	13	$10\frac{1}{2}$	890	2	$3\frac{1}{2}$	2916	14	$7\frac{1}{2}$	7398	4	$11\frac{1}{4}$	8136	14	$7\frac{1}{2}$	863	13	$11\frac{1}{2}$	
(2)	687	9	$1\frac{1}{4}$	286	14	$4\frac{3}{4}$	364	14	$6\frac{1}{4}$	7916	14	$7\frac{3}{4}$	359	2	$11\frac{1}{4}$	863	17	$5\frac{1}{2}$	1746	14	$8\frac{1}{4}$	9126	18	$3\frac{3}{4}$	
(3)	38	16	$7\frac{1}{2}$	7013	2	$8\frac{1}{2}$	2916	12	$3\frac{1}{2}$	8267	11	$3\frac{1}{2}$	8164	13	$5\frac{1}{2}$	9009	2	$4\frac{3}{4}$	9268	17	$10\frac{1}{2}$	8037	11	$2\frac{1}{4}$	
(4)	2916	12	$3\frac{1}{4}$	648	11	$2\frac{1}{2}$	3061	3	$4\frac{1}{2}$	948	17	$6\frac{3}{4}$	7928	1	6	7846	13	6	870	11	$7\frac{1}{4}$	960	18	$10\frac{1}{2}$	
(5)	18	16	0	7916	13	$5\frac{3}{4}$	976	19	11	2816	19	$11\frac{1}{2}$	874	12	$10\frac{3}{4}$	983	2	$11\frac{1}{2}$	1234	18	$11\frac{3}{4}$	8326	11	$2\frac{1}{4}$	
(6)	2986	11	$11\frac{1}{4}$	684	5	6	3084	6	$4\frac{1}{4}$	723	18	$10\frac{3}{4}$	8013	17	$9\frac{1}{2}$	87	19	$2\frac{1}{4}$	978	15	$6\frac{1}{2}$	9213	17	$6\frac{1}{4}$	
(7)	3261	13	$2\frac{1}{2}$	2081	3	$7\frac{1}{4}$	7936	17	$10\frac{1}{2}$	8067	13	$7\frac{1}{4}$	89	4	7	1648	18	$9\frac{1}{2}$	2516	12	$4\frac{1}{4}$	796	13	11	
(8)	948	1	$7\frac{3}{4}$	29	14	$11\frac{3}{4}$	382	14	6	753	14	$5\frac{1}{2}$	683	17	$5\frac{1}{2}$	785	12	$3\frac{3}{4}$	9028	3	$7\frac{1}{2}$	3647	18	$11\frac{1}{4}$	
(9)	5964	13	$10\frac{3}{4}$	675	12	$10\frac{1}{2}$	9017	12	$10\frac{1}{4}$	8632	17	$10\frac{1}{4}$	2816	13	$8\frac{1}{4}$	8723	17	4	5364	19	$11\frac{1}{4}$	2013	14	6	
(10)	2169	11	$3\frac{1}{2}$	9060	2	11	8796	17	$6\frac{3}{4}$	9286	12	$6\frac{1}{2}$	729	12	$9\frac{1}{2}$	2685	16	$8\frac{1}{2}$	2413	17	$4\frac{1}{2}$	972	11	1	
(11)	3084	5	9	853	14	$6\frac{1}{2}$	847	14	$10\frac{1}{2}$	798	17	$10\frac{1}{4}$	8013	16	$11\frac{1}{4}$	3015	12	$7\frac{1}{4}$	5279	4	$8\frac{1}{4}$	2376	14	$7\frac{1}{2}$	
(12)	16	11	11	7021	12	$10\frac{1}{4}$	7096	17	$4\frac{1}{2}$	8263	15	$4\frac{1}{2}$	976	19	8	963	18	$5\frac{3}{4}$	1374	18	$6\frac{1}{2}$	796	19	5	
(13)	947	12	$3\frac{1}{2}$	627	15	$3\frac{1}{2}$	3912	11	$11\frac{3}{4}$	7009	12	11	2813	16	$11\frac{3}{4}$	6084	14	$3\frac{1}{2}$	569	8	$9\frac{1}{4}$	3516	2	$7\frac{1}{4}$	
(14)	6581	14	$6\frac{3}{4}$	8902	17	9	9999	17	$3\frac{1}{4}$	745	18	$4\frac{1}{2}$	795	15	5	729	8	$10\frac{1}{4}$	3246	16	$4\frac{1}{2}$	2690	13	$6\frac{1}{2}$	
(15)	296	17	$2\frac{1}{4}$	846	12	4	865	4	$7\frac{1}{2}$	2368	17	$5\frac{1}{4}$	8067	19	$3\frac{1}{2}$	2684	3	$6\frac{1}{2}$	2085	17	11	943	15	$10\frac{1}{4}$	
(16)	3912	11	$3\frac{1}{2}$	5321	18	$6\frac{1}{2}$	8723	15	$4\frac{1}{4}$	726	13	$9\frac{1}{2}$	241	3	$8\frac{1}{4}$	4913	7	$8\frac{3}{4}$	6271	11	$6\frac{1}{2}$	3076	13	9	
(17)	49	12	$6\frac{1}{2}$	879	3	$5\frac{3}{4}$	986	17	6	2068	11	$3\frac{1}{4}$	7928	11	11	6528	4	$9\frac{1}{2}$	3016	18	$9\frac{1}{4}$	748	9	$10\frac{1}{2}$	
(18)	590	19	8	8026	4	$8\frac{1}{2}$	8072	13	$9\frac{1}{4}$	983	14	$6\frac{1}{4}$	86	14	$7\frac{1}{2}$	741	13	11	9273	15	10	8016	18	$7\frac{1}{4}$	
Totals																									Grand Total

EXERCISE 26.

Using complementary addition to find the balance, work the following in one operation :—

1.

£	s.	d.	
87	11	2	In hand
5	13	6	To be paid away
8	9	11	
23	14	9	
13	19	10	
8	18	8	
12	7	9	
			Balance.

2. Take £1, 14s. 2d. + £8, 12s. 5d. + £7, 16s. 11d. + £10, 18s. 4d. + £5, 19s. 8d. + £2, 14s. 6d. from £39, 3s. 6d.

3. Cash, £857, 10s. ; paid away, £37, 16s. 6d. + £25, 12s. 8d. + £143, 7s. 6d. + £107, 10s. + £59, 19s. 6d. + £83, 12s. 9d. + £60, 17s. 10d. + £19, 8s. 11d.

4. What is left of £50 after paying bills of £1, 15s. 3d., £1, 7s. 9d., £2, 13s. 10d., £1, 12s. 7d., 19s. 4d., and 15s. 2d. ?

5. The last balance shows that I had in the bank £547, 12s. 6d., but I have since drawn cheques for £39, 15s. 6d., £72, 12s. 9d., £14, 18s. 10d., £25, 10s. 6d., and £17, 19s. 4d. ; how much have I still to my credit?

6. Capital, £10,000 ; expenditure, £123, 16s. 8½d., £543, 19s. 6d., £78, 3s. 10½d., £46, 18s. 5d., £28, 9s. 6d., £573, 18s. 4d.

COMPOUND MULTIPLICATION.

By Factors. Where the multiplier does not exceed 12, or can be readily broken up into two factors each less than 12, the operation should be done *mentally*, and merely the figures of the answer set down.

Ex. Ordinary Method.

£	s.	d.	
235	16	$7\frac{1}{2}$	
		54	
		4 | 108	= 54 × 2f.
		27d.	+ $\frac{0}{4}$
		378	= 54 × 7d.
		12 | 405	
		33s.	+ 9d.
		864	= 54 × 16s.
		20 | 891	
		44£	+ 17s.
		270	= 54 × 5£
		162	= 54 × 30£
		108	= 54 × 200£
£12734,	17s.	$9\frac{0}{4}$d.	

Ex. By Factors.

£	s.	d.	
235	16	$7\frac{1}{2}$	(54 = 6 × 9)
		6	
1414	19	$9\frac{0}{4}$	= 6 times £235, 16s. $7\frac{1}{2}$d.
		9	
£12734	17	$9\frac{0}{4}$	= 9 × 6 times = 54 times £235, 16s. $7\frac{1}{2}$d.

Mentally—6 × 2 = 12 ÷ 4 = 3 and 0 over, put down $\frac{0}{4}$ carry 3:
6 × 7 = 42 + 3 = 45 ÷ 12 = 3 and 9 over, put down 9 carry 3:
6 × 6 = 36 + 3 = 39, put down 9 carry 3:
6 × 1 = 6 + 3 = 9 ÷ 2 = 4 and 1 over, put down 1 carry 4:
6 × 5 = 30 + 4 = 34:
6 × 3 = 18 + 3 = 21:
6 × 2 = 12 + 2 = 14:

Now proceed with multiplication by 9 in the same way.

EXERCISE 27.

(1) Multiply £37, 12s. 9¼d. by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

(2) Multiply £46, 18s. 11¾d. by 3, 5, 4, 2, 7, 8, 6, 10, 9, 12, and 11.

Multiply No. (3) to No. (10) *using factors* of the various multipliers.

(3) £18, 16s. 8¼d. × 22, 24, 25, 27

(4) £13, 11s. 9½d. × 18, 28, 32, 35

(5) £27, 18s. 11¾d. × 36, 40, 42, 45

(6) £32, 15s. 4¾d. × 48, 49, 55, 64

(7) £47, 0s. 7¾d. × 63, 66, 70, 84

(8) £53, 11s. 0½d. × 72, 77, 80, 81

(9) £65, 14s. 8¼d. × 88, 99, 120, 144

(10) £79, 19s. 1½d. × 121, 96, 108, 132

Do the following by ordinary method.

(11) £7, 11s. 3¾d. × 23, 29, 31, 37

(12) £10, 17s. 6½d. × 38, 41, 47, 53

(13) £0, 18s. 11¼d. × 51, 57, 67, 75

(14) £1, 2s. 8½d. × 59, 69, 61, 74

(15) £31, 11s. 8½d. × 79, 83, 85, 89

(16) £46, 5s. 0¾d. × 87, 93, 98, 101

(17) £53, 18s. 6¼d. × 91, 97, 103, 109

(18) £75, 0s. 8½d. × 95, 107, 117, 147

EXERCISE 28.

Practical Exercises.

1. The rent of a house is £8, 12s. 7½d. per calendar month; what is the yearly rent?
2. If one quarter of wheat cost £2. 16s. 8⅓d.; what will be the price of 56 qrs.?
3. If one ton of iron cost £23, 7s. 6½d.; what is the price of 96 tons?
4. If one score of lambs cost £7, 10s. 11½d.; what is the price of 121 score?
5. What cost 26 tons of coals, at 12s. 6d. per ton?
6. The rent of a shop is £2, 5s. 7½d. weekly; what is that per annum? (One year = 52 weeks.)
7. At 26s. 6d. per gallon what would 46 gallons of brandy cost?
8. A chest of tea contains 87 lb.; what will it cost at 3s. 11½d. per lb.?

9. What is the value of 98 yards of cloth, at 23s. 11¾d. per yard?

10. The rate for the poor in a parish is 1s. 11¼d. per £1 of rent; what must a person pay whose rent is £75?

11. How much does the pay of a regiment of 825 men amount to yearly at 1s. 1½d. each a day? (One year = 365 days.)

12. If the weekly forage of a horse be 14s. 6½d.; how much will it take to keep 857 horses for a year?

13. The rent of a shop is £2, 11s. 9½d. a week, and the annual taxes amount to £14, 7s. 6¼d.; what is the whole per annum?

14. The weekly receipts of a railway are £5742, 16s. 8d.; how much is that in a year?

15. If a labourer earns 18s. 6d. and spends 14s. 11¾d. per week; how much does he save a year?

16. If a man's income is £430 per annum, and his expenses 18s. 5½d. a day; what has he left over each year?

17. A merchant bought 25 pieces of cloth, each measuring 30 yards, for £656, 5s., and he sold it at £1, 1s. 7½d. a yard. Find his gain.

18. A fishmonger purchased 27 salmon for £26, 6s. 6d.; on an average each weighed 13 lb.; he sold the whole at 2s. 11¼d. per lb., but there were 7 lb. of waste; what did he gain?

19. A wine merchant imported 14 pipes of wine, which cost £1119, 0s. 7d.; the duty was £70; and he sold the whole at 32s. 9d. per dozen. If each pipe at first contained 60 dozen, but lost by leakage half a dozen bottles; what was his gain, allowing £75 for interest?

20. A butcher purchased 19 sheep for £1, 13s. 6½d. each; he sells the mutton, which amounts to 4½ stones each, at 5¼d. per lb.; the tallow, 9 lb. each, at 3d. per lb.; and the skin, &c., of each he sells for 8s. 3d. What does he gain by the bargain?

COMPOUND DIVISION.

By Factors. Where the divisor is less than 12, or can be readily broken up into factors less than 12, perform the operation *mentally*, putting down only the figures of the answer.

Care should be taken to keep **all the separate remainders**, when dividing by factors, **either in pence or in farthings**, and not some in each, for otherwise the correct result will not be obtained.

Ex.

	£	s.	d.	
9	12903	3	$9\frac{9}{4}$	$\div 756 = (9 \times 12 \times 7)$
12	1433	13	$9\frac{9}{4}$	
7	119	9	$5\frac{3}{4}$	
	17	1	$4\frac{1}{4}$	

Italian Method. Where the divisor cannot readily be broken into factors, use the **contracted** or **Italian method.**

Ex.

189)1674 18 $4\frac{1}{2}$ (8 17 $2\frac{3}{4}$

162

3258 shillings

1368

..45

544 pence

166

666 farthings

99 remainder.

The method of the table of multiples of the divisor can be used, as in simple division, where the exercise is a very long one.

Correct to the nearest Farthing. In *actual business* no account is usually taken of any fraction that cannot be paid, or, in other words, the calculation stops when farthings are reached. The result so obtained is not of course absolutely correct, but that it may be *as nearly accurate as possible* the following plan is followed :—

When the remainder is less than half the divisor no account is made of it, but where it is **as great as or greater than half the divisor**, increase the last figure of the quotient by 1.

Ex.

	£	s.	d.	
7	99	13	$11\frac{1}{2}$	$\div 21 (= 7 \times 3)$
3	14	4	$10\frac{1}{4}$	(a)
	4	14	$11\frac{1}{2}$	(b)

(a) $1\frac{1}{2}$d. = 6f. $\div 7 = \frac{6}{4} + 6$ remainder, but as 6 is greater than half the divisor 7, put down $\frac{1}{4}$d.

(b) $1\frac{1}{4}$d. = 5f. $\div 3 = \frac{1}{4} + 2$ remainder, but as 2 is greater than half the divisor 3, put down $\frac{1}{2}$d.

EXERCISE 29.

Work, by *short* division, using factors where necessary. Correct remainder is found as in simple division by factors.

	£	s.	d.			£	s.	d.	
(1)	798	15	6	÷ 5	(22)	380	9	0	÷ 42
(2)	195	15	$2\frac{1}{4}$	÷ 9	(23)	78	16	8	÷ 16
(3)	73	17	8	÷ 11	(24)	9134	1	8	÷ 55
(4)	17	1	7	÷ 12	(25)	529	4	0	÷ 63
(5)	75	12	0	÷ 84	(26)	56	14	0	÷ 72
(6)	93	16	$10\frac{1}{2}$	÷ 99	(27)	22	6	3	÷ 90
(7)	102	10	$6\frac{3}{4}$	÷ 108	(28)	37	5	$7\frac{1}{2}$	÷ 30
(8)	636	16	9	÷ 132	(29)	4	4	$4\frac{1}{2}$	÷ 45
(9)	484	15	$8\frac{1}{4}$	÷ 88	(30)	22	1	0	÷ 56
(10)	487	16	$10\frac{1}{2}$	÷ 64	(31)	135	0	0	÷ 50
(11)	375	11	6	÷ 45	(32)	38	8	0	÷ 36
(12)	892	10	$9\frac{3}{4}$	÷ 63	(33)	198	8	0	÷ 64
(13)	2389	16	$8\frac{1}{2}$	÷ 96	(34)	113	8	0	÷ 72
(14)	7480	11	$2\frac{3}{4}$	÷ 81	(35)	37	16	0	÷ 21
(15)	5480	12	$10\frac{1}{4}$	÷ 56	(36)	93	16	$10\frac{1}{2}$	÷ 99
(16)	6739	1	$1\frac{1}{4}$	÷ 121	(37)	75	12	0	÷ 84
(17)	3958	10	$11\frac{1}{2}$	÷ 144	(38)	45	0	0	÷ 120
(18)	8642	13	$8\frac{3}{4}$	÷ 49	(39)	9	2	0	÷ 96
(19)	7	6	0	÷ 24	(40)	36	6	0	÷ 132
(20)	64	19	0	÷ 36	(41)	81	4	0	÷ 336
(21)	37	14	8	÷ 48	(42)	21	4	8	÷ 112

Find result in No. 43 to No. 48, both inclusive, correct **to the nearest penny.**

	£	s.	d.			£	s.	d.	
(43)	45	10	4	÷ 32	(46)	307	11	2	÷ 64
(44)	63	12	6	÷ 35	(47)	210	10	0	÷ 77
(45)	116	10	10	÷ 54	(48)	316	13	4	÷ 108

EXERCISE 30.

Work, by *long* division, using contracted method.

	£	s.	d.			£	s.	d.	
(1)	18	7	6	÷ 17	(5)	38	2	$4\frac{1}{4}$	÷ 43
(2)	27	12	0	÷ 23	(6)	167	7	7	÷ 38
(3)	10	8	$6\frac{1}{2}$	÷ 26	(7)	418	13	4	÷ 65
(4)	17	1	$7\frac{3}{4}$	÷ 31	(8)	999	18	6	÷ 69

	£	s.	d.		
(9)	341	16	$8\frac{1}{2}$	÷	71
(10)	488	15	$8\frac{1}{4}$	÷	74
(11)	500	18	$1\frac{1}{2}$	÷	78
(12)	931	19	9	÷	87
(13)	478	13	$0\frac{3}{4}$	÷	89
(14)	118	15	7	÷	79
(15)	918	17	$5\frac{1}{2}$	÷	97

	£	s.	d.		
(16)	1118	13	6	÷	98
(17)	332	18	2	÷	104
(18)	102	10	$6\frac{3}{4}$	÷	109
(19)	207	8	$4\frac{1}{2}$	÷	231
(20)	9814	10	5	÷	758
(21)	215	12	$6\frac{1}{4}$	÷	317

EXERCISE 31.

When the divisor is compound, reduce both divisor and dividend to the lowest name in either; then divide as in Simple Division.

	£	s.	d.		£	s.	d.
(1)	301	2	$8\frac{1}{4}$	÷	7	0	$0\frac{3}{4}$
(2)	570	1	3	÷	13	11	$5\frac{1}{2}$
(3)	50	6	10	÷	0	17	$11\frac{3}{4}$
(4)	905	1	3	÷	8	7	$7\frac{1}{4}$
(5)	4349	1	$7\frac{1}{4}$	÷	6	3	$8\frac{3}{4}$

	£	s.	d.		£	s.	d.
(6)	8866	0	$0\frac{1}{2}$	÷	1	5	$0\frac{1}{2}$
(7)	2855	7	$0\frac{3}{4}$	÷	27	14	$5\frac{1}{4}$
(8)	4539	8	4	÷	0	18	$7\frac{1}{4}$
(9)	348	5	5	÷	0	19	1
(10)	2673	1	6	÷	7	13	$7\frac{1}{2}$

EXERCISE 32.

Practical Exercises.

1. If 36 cwt. of sugar cost £92, 14s.; what is that per cwt.?
2. If 49 tons of coal cost £28, 3s. 6d.; what is that per ton?
3. If a man's yearly income is £71, 7s. 10d.; how much is that per week?
4. If a man's yearly income is £72, 12s. $4\frac{3}{4}$d.; how much is that per day?
5. The pay of a first-class navy captain is £704, 8s. per annum; how much is that per lunar month?
6. The stock of a company of 384 partners is £429446; what is the share of each?
7. A navy consisted of 34090 ships, and the tonnage was 4144115 tons; how much was that on an average for each ship?
8. A farm of 372 acres is rented for £1334, 18s. 9d.; what is that per acre?
9. Bought $7\frac{1}{2}$ scores of sheep for £269, 13s. 9d.; what was the price of each?
10. A gentleman's annual income is £1800; he pays taxes and rent, £156, 16s. 8d., and he saves £436, 10s. 6d. annually; how much does he spend, monthly, weekly, and daily?
11. Divide £748, 16s. 8d. equally among 40 men.

12. Bought 785 lbs. of tea for £149, 12s. 9¾d.; how much was that per lb.?

13. A piece of cloth at 8s. 4d. per yard cost £18, 15s.; how many yards were in it?

14. A piece of gold cost £161, 17s. 6d., at £4, 7s. 6d. per oz.; what was its weight?

15. A gentleman gave £5, 13s. 4d. among some poor people, giving each 6s. 8d.; how many poor were there?

16. The revenues of a hospital amount to £5629, 10s.; how many patients will it maintain, when each patient requires £8, 13s. 9d.?

17. A servant having contracted a debt of £2, 6s. 8½d., allows 9½d. of his wages to lie in his master's hands every week; in what time will this pay the debt?

18. A labourer's house rent is £3, 13s. 9d. yearly; how much must he lay aside weekly, in order to pay it?

EXERCISE 33.

Questions on Simple Rules.

1. In 74867 crowns, how many dollars, each 4s. 6d.?

2. In 25104315 inches, how many yards?

3. How many francs of 9¾d. are in £2717?

4. How many dollars, each 4s. 3d., are in £1312, 3s. 9d.?

5. In £1939, 11s. 8d., how many florins?

6. A gentleman was robbed of the following sums, viz., 54 pieces of £3, 12s. each, 48 of 36s. each, 234 of 16s. 6d. each, 196 of 27s. each, 215 of 4s. 9d. each, 464 notes of £50 each; required the amount of his loss.

7. How many lb. of silver are in 2 dozen dishes, each dish weighing 25 oz. 15 dwt., and 2 dozen plates, each 15 oz. 15 dwt. 22 gr.?

8. How many hhds. sugar, each 11½ cwt., are in 141680 lb.?

9. How many canisters, each holding 12 lb., can be filled from 25 cwt. 2 qr. 24 lb. of tea?

10. From 1463645556 yards of cord, how many pieces could be cut, each measuring 1 yd. 1 in.?

11. How many inches will reach round the globe, it being 360 degrees, and each degree 69½ miles?

12. How many inches in 6817 ml. 2 fu. 7 po.?

13. In 20790 gallons how many pipes of 126 gal. each, puncheons of 84 gal., hhds. of 63 gal., and tierces of 42 gal., and of each an equal number?

14. There are 63 stacks of corn, each computed to contain 3 qr. 5 bu. 2 pk.; how much in the whole?

15. Mercury revolves 148 times round the sun in 35 years 249 days; required the time of one revolution.
16. The distance between London and Edinburgh is about 390 miles; how often will a coach-wheel of 15 ft. circumference revolve in performing the journey?
17. How many letters are in a book of 12 volumes, each 421 pages, each page 36 lines, and each line 38 letters?
18. Required the weight of 11 hhds. tobacco, each 5 cwt. 3 qr. 26 lb.
19. Divide £26, 3s. 11½d. among 4 men, 6 women, and 8 children, giving each man double a woman, and each woman triple a child.
20. The weekly wages of an equal number of men, women, and children amount to £170, 12s. 6d.; now each man receives 25s. 6d. per week, each woman 15s. 6d., and each child 4s. 6d.; how many are there of each?
21. If 6 chests of Bohea tea weigh 8 cwt. 1 qr. 18 lb.; what is the weight per chest?
22. A gentleman has 2 dozen tablespoons, each 2 oz. 14 dwt.; 3 dozen tea ditto, each 14½ dwt.; 4 mugs, each 13 oz. 17 dwt.; 2 tankards, each 23 oz. 13 dwt.; required the weight of the whole.
23. If 15 hhds. sugar weigh 52 cwt. 26 lb.; what is the weight of 1 hhd.?
24. What is the weight of 7 hhds. of sugar, each 3 cwt. 1 qr. 27 lb.?
25. What is the weight of 56 hhds., each 3 cwt. 8 lb.?
26. What is the net weight of 36 hhds., each 8 cwt. 3 qr., if 18 lb. 2 oz 12 dr. per hhd. be deducted as the weight of the case?
27. What is the net weight of 5 hhds., each 6 cwt. 2 qr. 14 lb., 3 lb. 4 oz. per hhd. being deducted?
28. A merchant paid £89, 6s. 5½d. for 5 pieces of cloth, each 25 yards; at what must he sell it per yard to gain £1, 11s. 3d. on the whole?
29. A merchant paid £64, 18s. for 288 yards of cloth, which getting damaged, he is content to lose £1, 18s. by it; at what must he sell it per yard?
30. A merchant bought 7 pieces of cloth, each 27 yards, for £55, 12s., and sold 56 yards of it at 5s. 3½d. per yard, and the rest at 6s. 8d.; whether did he gain or lose by it, and how much?
31. If a workman earns every week 12s. 6d., and spends 9s. 11¾d.; how much does he lay up in a year?
32. If a man earns 1s. 9d. per day, and spends 13½d.; how much does he lay up in a year?
33. A person's income is 4s. 6d. per day, and his expenses £71, 1s. 6d. yearly; how much does he save, or overspend, yearly, and how much does he spend weekly?
34. If 5 men, equally concerned in an adventure, clear each £114, 18s. 6d.; what was gained upon the whole?
35. Four men have an equal share of a voyage to Jamaica, whereby they lose £348, 15s.; what was the loss of each?
36. What was the stock of that trading company which consisted of 76 shares, each £784, 13s. 6d.?

VULGAR FRACTIONS.

We have often to make calculations which require us to deal with **parts of a unit** or whole. These parts are called **Fractions.**

In fractions two things have to be indicated :—

(1) The number of equal parts into which the whole is divided.

(2) How many of these equal parts we are to be concerned with.

One way of doing this is to place one number above and another below a horizontal line ; thus $\frac{3}{7}$ = three-sevenths.

When a fraction is so represented it is called a **Vulgar Fraction.** The number below the line indicates how many equal parts the whole has been divided into, and also therefore the *name* of the parts, and so is called the **denominator** (= namer). The upper number tells *how many* of these equal parts are taken, and is therefore called the **numerator** (= numberer).

From what has been said it follows that every vulgar fraction represents a *process of division*, the denominator being the *divisor*, and for practical purposes the numerator may be considered the *dividend*.

The numerator and denominator are called the **terms** of the fraction.

A **proper fraction** has its denominator *greater* than the numerator, as $\frac{3}{7}$.

An **improper fraction** is one in which the denominator is *not greater* than the numerator, as $\frac{5}{5}$, $\frac{7}{3}$.

A **mixed number** consists of a whole number + a proper fraction, as $3\frac{5}{6}$. An improper fraction can be expressed in the form of a mixed number, for $\frac{16}{5} = 16 \div 5 = 3\frac{1}{5}$; and a mixed number may be converted into an improper fraction ; thus, $3\frac{1}{7} = (3 \times 7) + 1$ sevenths $= \frac{22}{7}$.

A **whole number** may be expressed as a vulgar fraction with any given denominator ; thus $6 = \frac{6}{1} = \frac{12}{2} = \frac{18}{3} = \frac{24}{4} =$ &c.

A **simple fraction** has its numerator and denominator each formed of a whole number, as $\frac{5}{9}$.

A **complex fraction** has either its numerator or denominator, or both, formed of a fraction or a mixed number ; as $\frac{\frac{3}{7}}{5}$, $\frac{3}{2\frac{1}{2}}$.

A **compound fraction** is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{5}{6}$.

A fraction is not altered in value if both numerator and denominator are either multiplied or divided by the same number.

For if a 2-inch line be divided into 2 equal parts, each is $\frac{1}{2}$ of the whole line = 1 inch. If we divide it into 4 equal parts and take 2 of these, represented by $\frac{2}{4}$, this also measures 1 inch; therefore, $\frac{2}{4} = \frac{1}{2}$. But $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$, and $\frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$.

EXERCISE 34.

(*Mental.*)

1. Pick out the proper fractions, the improper fractions, and the mixed numbers: $\frac{5}{6}$, $\frac{5}{4}$, $5\frac{1}{4}$, $\frac{4}{7}$, $\frac{7}{4}$, $7\frac{4}{9}$, $\frac{13}{12}$, $\frac{12}{13}$, $\frac{9}{9}$.

2. Express as mixed numbers or integers :—

(1) $\frac{36}{9}$	(6) $\frac{2000}{43}$	(11) $\frac{2500}{81}$
(2) $\frac{23}{23}$	(7) $\frac{3197}{23}$	(12) $\frac{2000}{73}$
(3) $\frac{289}{17}$	(8) $\frac{4300}{47}$	(13) $\frac{3927}{1250}$
(4) $\frac{100}{31}$	(9) $\frac{2993}{35}$	(14) $\frac{6000}{133}$
(5) $\frac{355}{113}$	(10) $\frac{1000}{27}$	(15) $\frac{30000}{199}$

3. Express as improper fractions :—

(1) $7\frac{3}{4}$	(6) $15\frac{16}{17}$	(11) $924\frac{17}{18}$
(2) $11\frac{4}{5}$	(7) $90\frac{17}{50}$	(12) $89\frac{88}{89}$
(3) $13\frac{6}{7}$	(8) $79\frac{23}{70}$	(13) $928\frac{12}{89}$
(4) $8\frac{6}{11}$	(9) $23\frac{11}{36}$	(14) $3\frac{16}{113}$
(5) $8\frac{4}{13}$	(10) $329\frac{13}{15}$	(15) $23\frac{112}{331}$

4. How many eighths of a yard are in $7\frac{7}{8}$ yards?

5. How many twelfths of a penny are in $8\frac{5}{12}$d.

6. How many sixteenths of a yard has a draper sold who has disposed of $9\frac{3}{16}$ yards?

7. A grocer has sold 89 quarter-pounds of tea; how many lbs. has he sold?

8. A draper has sold 117 sixteenths of a yard; how many yards has he sold?

9. The average length of a year, according to the Gregorian Calendar, is $\frac{146097}{400}$ days. Express this as a mixed number.

10. Express 23 in fractional forms having as denominators, 3, 7, 2, 11, 5, 13, 25, 125.

HIGHEST COMMON FACTOR AND LEAST COMMON MULTIPLE.

The factors which two or more numbers have in common are called their **Common Factors** or **Common Measures**, and the product of the prime factors common to all the several numbers is called the **Highest Common Factor** (H.C.F.) or **Greatest Common Measure** (G.C.M.)

Thus, the G.C.M. of 45 and 120 is 15, for—

$$45 = 5 \times 9 = 5 \times 3 \times 3$$
$$120 = 10 \times 12 = 2 \times 5 \times 3 \times 2 \times 2$$

The prime factors common to both are 3 and 5, therefore G.C.M. $= 3 \times 5 = 15$.

It is not always necessary that the factors set down be prime. Any factor which is seen at once to be common may be set down whether prime or not.

Ex.
$$504 = 8 \times 7 \times 3 \times 3$$
$$264 = 8 \times 3 \times 11$$

The G.C.M. of 504 and 264 is 8×3 or **24.**

Another method of finding the G.C.M. which is sometimes followed is :—

Divide the larger by the smaller ; make the remainder a new divisor with the old divisor for dividend, and continue till an exact division occurs. The divisor at this stage is the H.C.F. or G.C.M.

```
36)63(1
   36
   --
27)36(1
   27
   --
 9)27(3
   27
   --
```

G.C.M. = 9

When the G.C.M. of *three* quantities is wanted, find the G.C.M. of two of them as above, and then find the G.C.M. of the answer and the third quantity in the same way. This gives the G.C.M. of the three. The process can be applied to any number of quantities.

The above shows the principle of the method better perhaps, but, as will be seen from the example below, to work by

arranging in columns and using contracted division shortens the process and is at the same time neater in appearance.

To find the G.C.M. of 1155 and 1464 proceed thus :—

1155	1464
228	309
66	81

The G.C.M. is 3.

Divide the greater by the less, and put down the remainder only, 309. Now divide 1155 by 309, and put down the remainder only, 228, and so on. *The work should stop as soon as the common factors of a pair are evident.*

EXERCISE 35.

Find the H.C.F. or G.C.M. of :—

(1) 48, 78
(2) 56, 98
(3) 121, 143
(4) 342, 665
(5) 448, 784
(6) 203, 261
(7) 375, 525
(8) 45, 27, 54
(9) 90, 84, 81
(10) 56, 84, 63
(11) 24, 36, 48, 216
(12) 32, 40, 64, 108
(13) 72, 84, 66, 176
(14) 198, 495, 209, 660
(15) 146, 730, 365, 219
(16) 924, 378, 612, 246

Any number, of which each of several given numbers is a factor, is said to be a **Common Multiple** of these numbers, and the least of such multiples is called their **Least Common Multiple** (L.C.M.).

To find the L.C.M. of two or more numbers—

(1) Find the prime factors of each.

(2) Find the product of each of the primes **taken as often as they occur** in any of the groups.

Ex. *Find the L.C.M. of* 4, 6, 18.

$4 \times 6 \times 18$ is a common multiple, but not the *least* one; for the given numbers have common factors.

$$4 = 2.2$$
$$6 = 2.3$$
$$18 = 2.3.3$$

A multiple of 4 must have 2.2 among its factors; of 6, 2.3; and of 18, 2.3.3. The number **2.2.3.3** fulfils each of these conditions *considered separately;* hence 36 is a multiple of 4, 6, and 18; and it is the least possible, for none of the factors 2.2.3.3 may be omitted without preventing some one of the conditions from being fulfilled.

The *L.C.M.* is also found thus :—

Find L.C.M. of 32, 18, 24, 12, 21, 40. Strike out each of the numbers which is a factor of another—in this case 12, which is a factor of 24—and having arranged the remaining ones in a line, divide by such prime numbers as are common factors of two or more of the numbers, carrying down such of the numbers as do not divide exactly. The process stops as soon as no two numbers in a line have a common factor.

2	32, 18, 24, 21, 40
2	16, 9, 12, 21, 20
2	8, 9, 6, 21, 10
3	4, 9, 3, 21, 5
	4, 3, 1, 7, 5

The L.C.M. is the product of the divisors and the numbers in the last row.

L.C.M. $= 2^3 \times 3^2 \times 4 \times 5 \times 7 = 10080$

EXERCISE 36.

Find the L.C.M. of :—

(1) 4, 6, 10, 12
(2) 8, 12, 15, 18
(3) 12, 16, 18, 20
(4) 12, 16, 18, 27
(5) 10, 6, 15, 12
(6) 12, 15, 20, 40
(7) 12, 28, 35, 21
(8) 32, 36, 49, 56, 42
(9) 20, 24, 25, 27, 45
(10) 28, 30, 32, 36, 42
(11) 35, 40, 42, 49, 28
(12) 8, 14, 18, 21, 32, 28
(13) 24, 27, 28, 32, 36, 56
(14) 15, 21, 24, 27, 28, 35
(15) 25, 32, 63, 40, 35, 56, 80
(16) 30, 36, 32, 48, 40, 54, 63

EXERCISE 37.

1. Explain what is meant by a prime number. Find all the prime numbers between 144 and 200.
2. Find the prime factors of 205975.
3. Resolve 7315 and 8415 into their prime factors, and find their G.C.M.
4. How many different divisors will a number have which is the product of 5 different prime numbers?
5. Find the least possible number of men in a regiment which can be arranged exactly in companies of 12, 20, 21, 24, 40, or 56.
6. What is the smallest sum of money which is payable either in crowns or in guineas?
7. The wheels of a bicycle are 80 and 84 inches respectively in circumference. A mark is put on the rims at the points nearest the ground when the rider mounts. How far has he ridden when the marks are next simultaneously at the lowest positions?
8. What is the smallest size of barrel which may be measured by 2, 3, 4, or 5 gallon jars?

9. Find the least number which when divided by 5, 7, 16, 12, leaves a remainder 3 each time.

10. A certain number lies between 4000 and 5000, and when it is divided by 8 the remainder is 5; by 9, 6; and by 11, 8. Find the number.

11. Find all the numbers of 3 digits which when divided by 2, 4, 6, 8, 10, leave remainders 1, 3, 5, 7, 9 respectively.

12. Find the G.C.M. of 4212, 2358, and 6138.

13. Find the G.C.M. of 116039, 122067, and 137137.

14. (1) Show that the numbers 111 and 11111 are prime to each other; (2) Find the L.C.M. of the even numbers from 2 to 20 inclusive.

15. Find the L.C.M. of 121, 132, 143, 154, and 165.

16. Give the tests by which it can be most readily found whether a given number is divisible by (*a*) 9, (*b*) 25; and prove their truth.

17. Resolve 18304, 20475, and 24255 into prime factors, and find the least whole number by which their product must be multiplied to make the resulting number a cube.

18. Four bells begin to toll simultaneously, and toll at intervals of 16, 18, 20, and 25 seconds respectively. After what time will they again toll simultaneously?

19. Four bicyclists start together to ride round a circular course, and take 30, 36, 40, and 45 seconds respectively to complete the circuit. After what interval will all, continuing to ride at the same rate, pass the starting-post together?

20. Find the least sum of money which contains 14s., 32s. 8d., and 21s. each an exact number of times.

Find also the G.C.M. of these three sums.

21. What two numbers, both greater than 29, have 29 for their G.C.M. and 4147 for their L.C.M.?

22. Three webs of cloth measure respectively 24, 39, and 57 yards. What is the least number of equal pieces which can be cut from the whole?

23. Write down as many numbers as possible such that the L.C.M. of the numbers is 72.

24. When the dollar is worth 4s. 2d., and the rupee 1s. 3d., what is the smallest number of dollars that can be exchanged for an exact number of rupees?

TO REDUCE VULGAR FRACTIONS TO THEIR LOWEST TERMS.

It has already been seen that the value of a vulgar fraction is not altered if we multiply or divide both numerator and denominator by the same number.

By applying the methods of finding the G.C.M. we may now find what is the largest number by which both terms can be divided and so *reduce the fraction to its lowest terms.*

Ex. Reduce $\frac{374}{969}$ to its lowest terms.

$$\frac{374}{969} = \frac{22 \times 17}{57 \times 17}$$

$$= \frac{22}{57}$$, for since 17 is a common factor of numerator and denominator, we may cut it out without altering the value of the fraction.

By applying the rules for finding the L.C.M. of numbers, we may also compare the relative values of different vulgar fractions. In the case of many fractions it is often almost impossible to do so by mere inspection.

To compare Vulgar Fractions. Introduce factors into the denominators to make them all equal. Then introduce into each numerator whatever factors have been put into the corresponding denominator.

Ex. 1. *Express with the same denominator,* $\frac{2}{3}$, $\frac{5}{7}$, $\frac{4}{5}$.

The common denominator must have 3, 7, and 5 as factors. Hence

$$\frac{2.7.5}{3.7.5}, \quad \frac{5.3.5}{7.3.5}, \quad \frac{4.3.7}{5.3.7}, \text{ or } \frac{70,\ 75,\ 84}{105}$$

If the common denominator is to be as small as possible, the L.C.M. of the given denominators should be taken.

Ex. 2. *Express* $\frac{5}{8}$, $\frac{7}{12}$, $\frac{11}{18}$ *as fractions with the least common denominator.*

The L.C.M. of 8, 12, 18, is 72. Hence

$$\frac{5 \times 9}{8 \times 9}, \quad \frac{7 \times 6}{12 \times 6}, \quad \frac{11 \times 4}{18 \times 4}, \text{ or } \frac{45,\ 42,\ 44}{72}$$

EXERCISE 38.

(*Do first six and last six mentally.*)

Reduce the following vulgar fractions to their lowest terms :—

(1) $\frac{24}{36}$ (9) $\frac{1152}{2016}$ (17) $\frac{1972}{9889}$

(2) $\frac{48}{54}$ (10) $\frac{1001}{1287}$ (18) $\frac{1517}{3700}$

(3) $\frac{96}{100}$ (11) $\frac{2424}{2727}$ (19) $\frac{1792}{3664}$

(4) $\frac{63}{81}$ (12) $\frac{2211}{2814}$ (20) $\frac{16502}{29436}$

(5) $\frac{72}{108}$ (13) $\frac{3584}{4096}$ (21) $\frac{511}{10001}$

(6) $\frac{324}{486}$ (14) $\frac{1435}{1722}$ (22) $\frac{10212}{11316}$

(7) $\frac{2639}{3393}$ (15) $\frac{812}{3045}$ (23) $\frac{11880}{13068}$

(8) $\frac{1391}{2675}$ (16) $\frac{999}{3552}$ (24) $\frac{32277}{40810}$

(25) Write fractions equal to $\frac{3}{4}$ with denominators 8, 20, 28, 36, 12.

(26) Write fractions equal to $\frac{2}{3}$ with numerators 12, 15, 21, 33, 48.

(27) Express 9 in fractional form with denominators 2, 9, 11, 13, 14.

(28) Write $\frac{5}{7}$ with denominators 21, 35, 63.

(29) Express with the same denominator $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{5}$, 4.

(30) Express with the same numerator $\frac{5}{6}$, $\frac{7}{9}$, $\frac{3}{4}$, 2.

EXERCISE 39.

(*Do Nos. 1 to 10 mentally.*)

A. Arrange the following fractions in order of magnitude :—

(1) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$

(2) $\frac{1}{8}$, $\frac{2}{13}$, $\frac{1}{7}$

(3) $\frac{4}{5}$, $\frac{24}{25}$, $\frac{5}{6}$

(4) $\frac{3}{5}$, $\frac{11}{15}$, $\frac{7}{10}$

(5) $\frac{3}{22}$, $\frac{1}{7}$, $\frac{2}{15}$

(6) $\frac{2}{11}$, $\frac{5}{33}$, $\frac{13}{66}$

(7) $\frac{11}{12}$, $\frac{9}{10}$, $\frac{14}{15}$, $\frac{13}{14}$

(8) $\frac{2}{3}$, $\frac{11}{15}$, $\frac{7}{12}$, $\frac{5}{9}$, $\frac{13}{21}$

(9) $\frac{3}{7}$, $\frac{4}{9}$, $\frac{9}{19}$, $\frac{7}{15}$, $\frac{5}{11}$

(10) $\frac{4}{5}$, $\frac{11}{15}$, $\frac{17}{20}$, $\frac{49}{60}$, $\frac{5}{6}$

(11) $\frac{15}{19}$, $\frac{13}{17}$, $\frac{14}{15}$, $\frac{19}{20}$

(12) $\frac{9}{10}$, $\frac{15}{16}$, $\frac{19}{20}$, $\frac{16}{17}$, $\frac{12}{13}$

(13) $\frac{10}{21}$, $\frac{7}{12}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{12}$, $\frac{5}{14}$

(14) $\frac{13}{30}$, $\frac{2}{5}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{19}{45}$, $\frac{9}{20}$

(15) $\frac{39}{76}$, $\frac{79}{152}$, $\frac{17}{38}$, $\frac{42}{95}$, $\frac{8}{19}$, $\frac{7}{17}$

(16) $\frac{2}{11}$, $\frac{7}{38}$, $\frac{6}{31}$, $\frac{9}{50}$, $\frac{11}{60}$, $\frac{3}{17}$

B. Find equivalent fractions with the least common denominator.

(Before finding the L.C.M. the pupil should see that each fraction is in its lowest terms.)

(1) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

(2) $\frac{3}{4}, \frac{5}{8}, \frac{11}{12}, \frac{7}{18}$

(3) $\frac{7}{15}, \frac{5}{18}, \frac{3}{25}$

(4) $\frac{2}{3}, \frac{3}{8}, \frac{5}{18}, \frac{13}{36}$

(5) $\frac{3}{7}, \frac{5}{8}, \frac{9}{14}, \frac{13}{28}$

(6) $\frac{4}{5}, \frac{3}{7}, \frac{13}{15}, \frac{16}{17}$

(7) $\frac{4}{16}, \frac{18}{48}, \frac{5}{80}, \frac{27}{45}$

(8) $\frac{4}{12}, \frac{6}{9}, \frac{17}{85}, \frac{56}{63}$

(9) $\frac{5}{6}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}$

(10) $\frac{4}{15}, \frac{11}{12}, \frac{3}{13}, \frac{1}{11}$

(11) $\frac{11}{12}, \frac{15}{16}, \frac{17}{18}, \frac{35}{36}, \frac{31}{32}, \frac{23}{24}$

(12) $\frac{3}{7}, \frac{8}{9}, \frac{2}{5}, \frac{4}{13}$

(13) $\frac{3}{5}, \frac{7}{15}, \frac{2}{9}, \frac{11}{24}, \frac{7}{8}, \frac{17}{45}$

(14) $\frac{7}{9}, \frac{8}{11}, \frac{5}{6}, \frac{3}{17}, \frac{280}{561}, \frac{7}{1122}$

ADDITION AND SUBTRACTION OF VULGAR FRACTIONS.

We cannot add or subtract quantities under different denominations, say shillings and half-crowns, or ounces and hundredweights, and just so we cannot add or subtract fractions *whose denominators are unlike.*

In both instances we must bring the quantities to be dealt with to the same denomination or name; or, as we have called it in fractions, the *Common denominator.* In fractions we then add or find the difference between the numerators.

In addition of *mixed numbers* deal as above with the fractional part, and to the result add the sum of the integers.

In subtracting *mixed numbers* the easiest plan is to reduce both to improper fractions, bring them to the same denominator and subtract the one numerator from the other.

This often causes unnecessary figuring. Where this is so it is better to proceed as in addition, viz., deal with the fractions and integers apart.

If the difficulty occurs of having to subtract a larger from a less number, we must borrow a unit from *the difference of the integers.*

Ex. How much greater is $13\frac{1}{8}$ than $9\frac{3}{4}$?

$$\begin{aligned} 13\tfrac{1}{8} - 9\tfrac{3}{4} &= 13 - 9 + \tfrac{1}{8} - \tfrac{3}{4} \\ &= 4 + \frac{1 - 6}{8} \\ &= 3 + \frac{8 + 1 - 6}{8} \\ &= 3\tfrac{3}{8} \end{aligned}$$

Here we met with the difficulty of subtracting $\frac{6}{8}$ from $\frac{1}{3}$, but by taking 1 from the 4 and expressing it as $\frac{8}{8}$, we got $\frac{9}{8}$ from which to take the $\frac{6}{8}$. The 4 of course was reduced to 3.

The addition or subtraction of **two** vulgar fractions **whose numerator is 1** can be worked mentally, thus—*Add or subtract the denominators for the numerator of the answer; multiply the denominators together for the denominator of the answer.*

Ex.

$$\frac{1}{3}+\frac{1}{5}=\frac{3+5}{3\times 5}=\frac{8}{15}$$

$$\frac{1}{2}-\frac{1}{7}=\frac{7-2}{7\times 2}=\frac{5}{14}$$

When the **numerators are other than 1** two fractions can still be added or subtracted mentally by "cross multiplying," thus—*Add or subtract the products of the numerator and denominator of the different fractions for the numerator of the answer; multiply the denominators together for the denominator of the answer.*

Ex.

$$\frac{2}{3}+\frac{4}{7}=\frac{(2\times 7)+(4\times 3)}{3\times 7}=\frac{26}{21}=1\frac{5}{21}$$

$$\frac{5}{6}-\frac{3}{4}=\frac{(5\times 4)-(3\times 6)}{6\times 4}=\frac{2}{24}=\frac{1}{12}$$

EXERCISE 40.

(To be worked mentally as far as possible.)

(*a*) (1) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$

(2) $\frac{5}{6}+\frac{7}{8}+\frac{5}{9}$

(3) $\frac{3}{4}+\frac{7}{8}+\frac{9}{10}$

(4) $\frac{8}{11}+\frac{7}{9}+\frac{3}{5}$

(5) $\frac{6}{7}+\frac{8}{9}+\frac{4}{63}$

(6) $\frac{3}{8}+\frac{4}{9}+\frac{17}{36}$

(7) $\frac{11}{12}+\frac{5}{7}+\frac{17}{42}+\frac{10}{21}$

(8) $\frac{9}{10}+\frac{3}{5}+\frac{8}{15}+\frac{7}{20}$

(9) $\frac{3}{7}+\frac{2}{49}+\frac{3}{14}+\frac{8}{21}$

(10) $\frac{11}{12}+\frac{3}{4}+\frac{7}{48}+\frac{11}{72}$

(11) $\frac{5}{8}+\frac{7}{9}+\frac{19}{36}+\frac{11}{144}$

(12) $\frac{13}{21}+\frac{3}{49}+\frac{16}{35}+\frac{11}{63}$

(13) $\frac{2}{3}+\frac{11}{27}+\frac{19}{54}+\frac{13}{108}+\frac{17}{81}+\frac{19}{243}$

(14) $\frac{7}{11}+\frac{3}{22}+\frac{5}{66}+\frac{9}{44}+\frac{13}{132}+\frac{79}{88}$

(*b*) (15) $\frac{29}{56}-\frac{3}{14}$

(16) $\frac{11}{17}-\frac{7}{15}$

(17) $\frac{8}{9}-\frac{4}{5}$

(18) $\frac{16}{19}-\frac{13}{18}$

(19) $\frac{9}{13}-\frac{5}{9}$

(20) $\frac{13}{17}-\frac{9}{13}$

(21) $\frac{13}{14}-\frac{19}{21}$

(22) $\frac{11}{21}-\frac{17}{42}$

(23) $3\frac{1}{2}-2\frac{1}{4}$

(24) $7\frac{1}{3}-5\frac{1}{5}$

(25) $17\frac{4}{5}-13\frac{3}{10}$

(26) $6\frac{4}{7}-3\frac{3}{14}$

(27) $18\frac{3}{4}-10\frac{2}{5}$

(28) $17\frac{2}{3}-10\frac{7}{12}$

(29) $23\frac{11}{12}-19\frac{17}{48}$

(30) $16\frac{19}{30}-14\frac{23}{90}$

(*c*) (31) $\frac{1}{2}+\frac{2}{3}-\frac{3}{4}-\frac{3}{8}$

(32) $\frac{3}{4}-\frac{1}{3}+\frac{5}{6}-\frac{1}{2}$

(33) $\frac{7}{8}+\frac{5}{6}-\frac{3}{4}-\frac{1}{48}$

(34) $\frac{8}{9}+\frac{3}{5}-\frac{4}{15}+\frac{1}{2}$

(35) $\frac{7}{8}+\frac{3}{4}+\frac{9}{10}-1\frac{4}{5}$

(36) $\frac{3}{7}+\frac{9}{14}-\frac{5}{21}+\frac{7}{8}$

(37) $2\frac{3}{4}+3\frac{1}{2}-\frac{7}{9}-\frac{11}{16}+\frac{3}{5}$

(38) $3\frac{1}{4}-\frac{2}{7}+2\frac{2}{5}-4\frac{1}{3}+\frac{2}{9}$

(39) $2\frac{4}{5}-\frac{3}{4}+2\frac{2}{3}-\frac{7}{8}+\frac{11}{12}$

(40) $\frac{11}{12}-\frac{7}{9}+\frac{4}{15}-\frac{13}{45}+1\frac{11}{60}$

(41) $5\frac{5}{6}-\frac{11}{20}+\frac{4}{9}-\frac{3}{8}-2\frac{1}{12}$

(42) $4\frac{1}{3}-\frac{2}{5}+3\frac{1}{8}-\frac{11}{20}+\frac{7}{15}$

EXERCISE 41.

1. A student spends $\frac{1}{4}$ of the day in teaching, $\frac{1}{12}$ in attending classes, $\frac{5}{24}$ in study, $\frac{5}{48}$ in recreation and meals, and $\frac{1}{24}$ in miscellaneous reading. What part of the day is he thus occupied?

2. $\frac{1}{5}$ of a pole is in sand, and $\frac{4}{15}$ of it in water. What part of the pole is thus below the surface of the water?

3. In an allied camp, $\frac{1}{5}$ of the soldiers are natives of England, $\frac{2}{15}$ of Scotland, $\frac{11}{120}$ of Ireland, and $\frac{1}{24}$ of Wales. What part of the camp is under British colours?

4. In 1685 the regular infantry and the regular cavalry of England were respectively $\frac{7}{130}$ and $\frac{17}{130}$ of the militia. What part were they together of the militia?

5. What part of a piece of cloth has a draper sold who has cut off $\frac{3}{16}$, $\frac{5}{32}$, $\frac{9}{64}$, and $\frac{7}{40}$ of it?

6. A treasurer has expended $\frac{1}{6}$, $\frac{7}{48}$, $\frac{1}{40}$, $\frac{7}{30}$, and $\frac{5}{24}$ of a given sum. What part of the whole has he laid out?

7. Of the number of one year's freshmen at Cambridge $\frac{139}{456}$ belonged to Trinity College, $\frac{65}{228}$ to St John's College, $\frac{8}{38}$ to Gonville and Caius College, and $\frac{7}{228}$ to Queen's College. What part of the whole did these together form?

8. $\frac{7}{8}$ of a pole is above the bottom of a pool, and $\frac{3}{16}$ is in the pool. What part of it is above the level of the water?

9. A retail draper who has bought $\frac{3}{4}$ of a piece of cloth, sells a portion equal to $\frac{1}{20}$ of the entire piece. What part of it has he left?

10. $\frac{2}{7}$ of a common is laid out as bleaching-ground. What part of it is left?

11. A sailor has spent $\frac{9}{13}$ of his life at sea. What part of his life has he spent on land?

12. The number of pear and apple trees in an orchard is $\frac{2}{3}$ of that of the whole, and that of the pear trees is $\frac{7}{25}$. What part of the whole is the number of apple trees?

13. A traveller has gone $\frac{1}{5}$ of a journey on foot, $\frac{2}{15}$ on horseback, $\frac{1}{4}$ by rail, and the rest by coach. What part has he gone by coach?

14. $\frac{1}{6}$ of the pole is blue, $\frac{2}{7}$ red, and the rest white. What part of it is white?

15. A student has in three weeks read respectively $\frac{5}{21}$, $\frac{2}{7}$, and $\frac{1}{3}$ of the first book of the *Æneid*. What part of it has he yet to read?

16. A soldier while in the army had spent $\frac{2}{5}$ of his life in the United Kingdom, $\frac{5}{27}$ in Canada, $\frac{1}{18}$ in Gibraltar, $\frac{1}{6}$ in India, and $\frac{1}{27}$ in the Crimea, where he died. What part of his life had he spent before enlisting?

17. A jeweller, having used $\frac{3}{10}$, $\frac{7}{20}$, and $\frac{11}{40}$ of an ingot of gold, wishes to know what part still remains.

18. When of the whole black and mulatto population of Cuba the free mulattoes were $\frac{3}{21}\frac{9}{35}\frac{4}{5}$, the free blacks $\frac{8}{6}\frac{7}{4}\frac{4}{17}$, and the mulatto slaves $\frac{3}{21}\frac{7}{35}$, what part was the number of black slaves?

MULTIPLICATION AND DIVISION OF FRACTIONS.

To multiply a fraction **by a whole number, multiply the numerator only by the given number** and retain the original denominator.

The denominator is only the **name** and remains unchanged; the numerator is the **number** and must be increased the required number of times

Ex. 1. $\frac{3}{8} \times 4 = \frac{12}{8} = \frac{3}{2}$

Ex. 2. $\frac{3}{4} \times 8 = \frac{24}{4} = \frac{6}{1} = 6$

Since 4 is common to both terms we reduce to lowest terms by dividing both by 4, and so reduce the fraction to its lowest possible terms.

These examples show us how we may shorten the process in certain cases.

(1) **When the multiplier is a factor of the denominator**, divide the denominator by the multiplier and retain the original numerator.

(2) **When the denominator is a factor of the multiplier**, divide the multiplier by the denominator and multiply the original numerator by the quotient.

In **dividing** a fraction by a whole number the above processes are just reversed.

(1) **When the divisor** (whole number) **is a factor of the numerator**, divide the numerator by the divisor and retain the original denominator.

(2) **When the numerator is a factor of the divisor** (whole number), divide the divisor by the numerator and multiply the original denominator by the quotient. This gives the new denominator whose numerator is 1.

When neither of the above holds good, **multiply the denominator only by the whole number**, and retain the original numerator.

Ex. $\frac{3}{7} \div 5 = \frac{3}{35}$.

To find the product of two or more fractions, multiply together all the numerators for the new numerator and multiply all the denominators for the new denominator.

Ex. $\frac{3}{4} \times \frac{5}{6} \times \frac{9}{10} = \frac{135}{240} = \frac{9}{16}$ (in lowest terms).

To save multiplying and then reducing, the process of *cancelling*, *i.e.*, removing from the terms of both numerator and denominator any factors which are common to both, is adopted in the multiplication and division of fractions.

Taking the above example again,

$$\frac{\overset{1}{\cancel{3}}}{4} \times \frac{\overset{1}{5}}{\underset{2}{\cancel{6}}} \times \frac{9}{\underset{2}{\cancel{10}}} = \frac{9}{16}$$

we strike out of 3 in numerator and 6 in denominator the common factor 3, leaving 1 and 2 respectively; so 5 and 10.

When all the cancelling possible has been done, apply the rule for multiplication of fractions.

Division of Fractions is worked as a process of multiplication in which the terms of the dividing fraction are reversed, the numerator becoming the denominator, and the denominator the numerator of the multiplying fraction.

Ex. $\frac{5}{7} \div 3 = \frac{5}{21}$ By rule for dividing by a whole number.

But $3 = \frac{3}{1}$, therefore $\frac{5}{7} \div \frac{3}{1} = \frac{5}{21}$ Same exercise as above by rule

and $\frac{5}{7} \times \frac{1}{3} = \frac{5}{21}$ for multiplication of fractions.

After the divisor has been inverted the exercise is merely ordinary multiplication of fractions.

When any of the quantities are in the form of *mixed* numbers we must see that they are first reduced to *improper* fractions.

EXERCISE 42.

Work first ten sums mentally.

(1) $\frac{5}{18} \times 5, 6, 36$

(2) $\frac{11}{125} \times 5, 25, 125$

(3) $\frac{5}{256} \times 4, 16, 64$

(4) $\frac{19}{336} \times 42, 48, 56$

(5) $\frac{3}{4} \times \frac{5}{6}$

(6) $\frac{3}{7} \times \frac{7}{11}$

(7) $\frac{5}{8} \times \frac{4}{11}$

(8) $\frac{3}{7} \times \frac{14}{27}$

(9) $\frac{11}{13} \times \frac{26}{77}$

(10) $\frac{4}{15} \times \frac{45}{64}$

(11) $\frac{18}{19} \times \frac{76}{81}$

(12) $1\frac{3}{11} \times \frac{11}{15}$

(13) $2\frac{9}{11} \times \frac{4}{7}$

(14) $\frac{5}{8} \times 6\frac{4}{7}$

(15) $\frac{3}{11} \times 7\frac{1}{7}$

(16) $9\frac{1}{7} \times 8\frac{3}{84}$

(17) $19\frac{1}{2} \times 16\frac{11}{1925}$

(18) $23\frac{1}{2} \times 3\frac{11}{47}$

(19) $\frac{1}{2} \times 2\frac{2}{5} \times \frac{3}{4}$

(20) $\frac{1}{4} \times 3\frac{1}{2} \times \frac{7}{8}$

(21) $3\frac{3}{4} \times \frac{16}{29} \times \frac{15}{32}$

(22) $3\frac{2}{9} \times \frac{9}{58} \times 6\frac{1}{2}$

(23) $\frac{3}{5} \times \frac{10}{33} \times 8\frac{1}{4}$

(24) $6\frac{3}{4} \times \frac{8}{27} \times 6\frac{2}{9}$

The word "Of" in Vulgar Fractions. To find the fraction *of* any number, as $\frac{3}{4}$ of 5, means to multiply 5 by 3 and divide the result by 4. This is just multiplication by $\frac{3}{4}$; hence $\frac{3}{4}$ of 5 may be written $5 \times \frac{3}{4}$.

The word *of* following a fraction corresponds to the word *times* following an integer.

3 times $4 = 4 \times 3$; $\quad \frac{3}{5}$ of $4 = 4 \times \frac{3}{5}$;

$\frac{2}{3}$ of $\frac{5}{7} = \frac{5}{7} \times \frac{2}{3} = \frac{2}{3} \times \frac{5}{7}$

There is one distinction to be made between *of* and × which should be carefully observed. If $\frac{2}{3}$ of $\frac{4}{5}$ occurs in a series of operations, the sign before $\frac{2}{3}$ refers not to the $\frac{2}{3}$ alone, but to the result of $\frac{4}{5} \times \frac{2}{3}$; in other words, the *of* has the same effect as a bracket round $\frac{4}{5} \times \frac{2}{3}$.

For example, $\frac{1}{2} \div \frac{2}{3}$ of $\frac{1}{4}$ is not the same as $\frac{1}{2} \div \frac{2}{3} \times \frac{1}{4}$; the former is $\frac{1}{2} \div (\frac{2}{3} \times \frac{1}{4})$, or $\frac{1}{2} \div \frac{2}{3} \div \frac{1}{4}$.

EXERCISE 43.

(1) $\frac{2}{5}$ of $\frac{26}{27}$ of $\frac{5}{13}$

(2) $\frac{3}{4}$ of $\frac{5}{9}$ of $\frac{11}{18}$

(3) $\frac{7}{9}$ of $\frac{3}{4}$ of $8\frac{1}{2}$

(4) $\frac{2}{9}$ of $\frac{11}{12}$ of $\frac{7}{9}$ of $4\frac{1}{2}$

(5) $\frac{8}{9}$ of $2\frac{1}{4}$ of $\frac{3}{7}$ of $5\frac{1}{2}$

(6) $\frac{2}{9}$ of $\frac{7}{13}$ of $\frac{18}{19}$ of $3\frac{1}{4}$ of $\frac{13}{18}$

(7) $\frac{9}{59}$ of $\frac{8}{9}$ of $4\frac{11}{12}$ of $3\frac{2}{3}$

(8) $\frac{3}{4}$ of $5\frac{1}{2}$ of $\frac{2}{33}$ of $\frac{16}{27}$ of 5

(9) $\frac{22}{29}$ of $16\frac{2}{3}$ of $\frac{3}{29}$ of $70\frac{1}{12}$

(10) $\frac{13}{15}$ of $\frac{30}{169}$ of $\frac{17}{88}$ of $\frac{11}{120}$ of $307\frac{3}{11}$

(11) $\frac{1}{2}$ of $\frac{4}{17}$ of $\frac{8}{33}$ of $8\frac{1}{4}$ of $8\frac{1}{2}$

(12) $\frac{1}{3}$ of $\frac{1}{9}$ of $\frac{1}{81}$ of $\frac{1}{729}$ of 6561

(13) $\frac{2}{11}$ of 19 of $\frac{4}{35}$ of $4\frac{1}{4}$ of $\frac{11}{17}$ of $8\frac{3}{4}$

(14) $\frac{3}{17}$ of $\frac{35}{36}$ of $7\frac{1}{5}$ of $7\frac{1}{12}$ of $\frac{60}{91}$

(15) A soldier was in hospital $\frac{5}{48}$ of the time he served in India, which was $\frac{6}{55}$ of his life. What part of his life was he in hospital?

(16) A sailor's share of prize-money is $\frac{7}{50}$ of a midshipman's, whose share is $\frac{5}{24}$ of a lieutenant's. What part of a lieutenant's share does a sailor get?

(17) Jack, who gets $\frac{3}{7}$ of a plum-pudding, gives $\frac{2}{3}$ of his share to Tom, who gives $\frac{1}{5}$ of his to Harry. What part of the plum-pudding does Harry get?

(18) A schoolboy prepares his lessons at home in $\frac{1}{5}$ of the time he plays, which amounts to $\frac{11}{120}$ of $\frac{5}{8}$ of a day. During what part of a day does he prepare his lessons?

(19) On a colonial railway the fare per mile by the third class is $\frac{7}{12}$ of that by the second, which is $\frac{4}{5}$ of that by the first, which is $3\frac{3}{4}$d. Find the fare per mile by the third class.

(20) Find the receipts of a railway for a week which amount to $\frac{751}{768}$ of £6384.

(21) The number of registrars employed in a census was $\frac{161}{1937}$ of that of the enumerators, of whom there were 38740. Find the number of registrars.

(22) If a train runs a mile in $\frac{3}{5}$ of $3\frac{3}{4}$ minutes, in what time will it run $\frac{7}{10}$ of $23\frac{1}{2}$ miles?

(23) 24 flagstaffs are placed on a road at the distance of $\frac{2}{3}$ of $73\frac{1}{2}$ yards between each. How many yards are between the first and the last?

(24) If a train runs $\frac{4}{5}$ of a mile in a minute, how many miles will it run in $\frac{3}{8}$ of $43\frac{1}{2}$ minutes?

EXERCISE 44.

(1) $\frac{12}{25} \div 4, 6, 36$

(2) $\frac{18}{19} \div 6, 9, 54$

(3) $\frac{128}{1385} \div 16, 64, 384$

(4) $\frac{25}{36} \div 5, 75, 150$

(5) $\frac{14}{31} \div \frac{15}{62}$

(6) $\frac{17}{19} \div \frac{34}{39}$

(7) $\frac{11}{12} \div \frac{23}{24}$

(8) $\frac{15}{17} \div \frac{10}{51}$

(9) $\frac{4}{13} \div \frac{5}{26}$

(10) $\frac{17}{91} \div \frac{34}{1069}$

(11) $\frac{10}{21} \div \frac{13}{42}$

(12) $\frac{19\frac{3}{8}}{\frac{31}{32}}$

(13) $\frac{17\frac{3}{11}}{\frac{38}{39}}$

(14) $\frac{4\frac{2}{3}}{5\frac{1}{4}}$

(15) $\frac{5\frac{2}{9}}{11\frac{3}{4}}$

(16) $\frac{21\frac{7}{19}}{7\frac{6}{7}}$

(17) $\frac{31}{32} \div \frac{3}{4}$ of $10\frac{1}{3}$

(18) $\frac{11}{14} \div \frac{4}{5}$ of $8\frac{7}{16}$

(19) $\frac{1}{4}$ of $\frac{14}{15} \div \frac{7}{26}$

(20) $\frac{1}{6}$ of $2\frac{3}{4} \div \frac{7}{9}$

(21) $\frac{2}{7}$ of $1\frac{7}{16} \div \frac{27}{43}$ of $\frac{53}{81}$

(22) $\frac{4}{7}$ of $\frac{23}{30} \div \frac{8}{73}$ of 4

(23) $\frac{9}{10}$ of $\frac{111}{112} \div \frac{3}{4}$ of $1\frac{1}{11}$

(24) $\frac{3}{7}$ of $\frac{45}{49} \div \frac{2}{3}$ of $\frac{7}{12}$

EXERCISE 45.

1. How many pieces, each $30\frac{3}{4}$ yards, are contained in $114\frac{4}{5}$ yards?

2. If a piece of cloth is $29\frac{5}{4}$ yards in length, and a remnant $1\frac{55}{64}$ yard; how many times is the former as long as the latter?

3. How many squares, each $\frac{1}{9}$ of a square inch, are contained in $132\frac{1}{4}$ square inches?

4. How many postage-stamps, each containing $\frac{18}{25}$ of a square inch, are in a sheet of $172\frac{4}{5}$ square inches?

5. How many times can a measure containing $\frac{7}{8}$ of a pint be filled out of a vessel containing $63\frac{7}{8}$ pints?

6. How many times will a coin $2\frac{1}{5}$ inches in circumference turn round in traversing 30 inches?

7. Mercury is $13\frac{3}{5}$ times as heavy as water, and gold is $19\frac{3}{5}$ times. How many times is gold as heavy as mercury?

8. A book of 240 leaves without boards is $1\frac{1}{10}$ inch thick, and another of 180 leaves without boards is $\frac{7}{10}$ inch thick. How many times is the paper of the former as thick as that of the latter?

9. How many men are in a regiment of which $\frac{3}{1\,0}$ = 255 men?

10. Find the distance from London to Karachi, that from the head of the Red Sea to Karachi, which is 1700 miles, being $\frac{4\,0}{1\,4\,3}$ of it.

11. Of an army $\frac{1}{3}$ is English, $\frac{7}{2\,4}$ Scotch, $\frac{1}{1\,2\,0}$ Welsh, and the remainder numbers 4796 Irish. How many are there in all?

12. Of the distance from Edinburgh to London by rail *via* Carlisle, that from Edinburgh to Carlisle is $\frac{1}{4}$, from Carlisle to Preston $\frac{9}{4\,0}$, while from Preston to London is 210 miles. Find the distance from Edinburgh to London.

13. A labourer can do a piece of work in $12\frac{1}{2}$ days. How much of it can he do in a day?

14. If $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{7}$ of a ship be worth £1500; what is its whole value?

15. The product of two numbers is $5\frac{1}{4}$ and the greater is $7\frac{3}{8}$; find the less.

16. Of a pole $\frac{1}{1\,2}$ is painted white, $\frac{7}{3\,0}$ green, $\frac{1}{6}\frac{3}{0}$ red, and the remaining 5 feet, black. Find the length of the pole.

TO SIMPLIFY COMPLEX FRACTIONS.

By the definition given of a vulgar fraction the line separating numerator from denominator in a complex fraction, is equal to the sign of division, and therefore the numerator and denominator must each be first expressed as a simple fraction, and then the former divided by the latter.

Ex. $$\frac{3\frac{1}{8} - 1\frac{1}{4} \text{ of } \frac{2}{5}}{7\frac{1}{2} + \frac{2\frac{1}{3}}{6 \div \frac{1}{2}}} = \frac{\frac{25}{8} - \left(\frac{5}{4} \times \frac{2}{5}\right)}{\frac{22}{3} + \frac{7}{\frac{6}{1} \times \frac{2}{1}}} = \frac{\frac{25}{8} - \frac{1}{2}}{\frac{22}{3} + \left(\frac{7}{3} \times \frac{1}{12}\right)}$$

$$= \frac{\frac{25 - 4}{8}}{\frac{22}{3} + \frac{7}{36}} = \frac{\frac{21}{8}}{\frac{264 + 7}{36}} = \frac{21}{8} \times \frac{9}{271} = \frac{189}{542}$$

Simplify both numerator and denominator simultaneously step by step, keeping the fraction in complex form to avoid confusion of parts, till each is a simple fraction.

EXERCISE 46.

Simplify—

(1) $\dfrac{(2\frac{3}{4}+\frac{1}{4}) \text{ of } 2\frac{4}{9}}{1\frac{1}{4} \text{ of } (\frac{3}{2}+2\frac{1}{3})}$

(2) $\dfrac{3\frac{1}{3}+8\frac{4}{9}}{6\frac{1}{12}-3\frac{7}{8}} \div \dfrac{3\frac{1}{7} \text{ of } 9\frac{7}{9}}{3\frac{13}{14} \text{ of } 4\frac{2}{5}}$

(3) $\dfrac{\frac{1}{20}+\frac{8}{9}-\frac{1}{2}(1\frac{2}{3}-\frac{3}{10})}{2\frac{2}{35}\div 10\frac{2}{7}} \times \dfrac{779}{1161} \text{ of } \dfrac{252}{451}$

(4) $\dfrac{(\frac{5}{12}+\frac{9}{20})\div\frac{2}{15}}{(\frac{5}{12} \text{ of } \frac{9}{20})-\frac{2}{15}} \div \dfrac{\frac{5}{12}+\frac{9}{20}+\frac{2}{15}}{\frac{5}{12} \text{ of } \frac{9}{20} \text{ of } \frac{2}{15}}$

(5) $\dfrac{\frac{1}{2}-\frac{1}{3}-\frac{1}{12}}{\frac{1}{2}-(\frac{1}{3}-\frac{1}{12})} \div \dfrac{\frac{1}{2}\times\frac{1}{3}-\frac{1}{12}}{\frac{1}{2}\times(\frac{1}{3}-\frac{1}{12})}$

(6) $\dfrac{2\frac{1}{2}-1\frac{1}{3} \text{ of } 2\frac{1}{4}+1\frac{7}{9}}{(2\frac{1}{2}-1\frac{1}{3}) \text{ of } (2\frac{1}{4}+1\frac{7}{9})} \text{ of } \left(1+\dfrac{1}{3\frac{5}{8}}\right)$

(7) $\dfrac{17\frac{6}{7}}{9\frac{1}{11}}+2\frac{3}{7}\times(9\frac{1}{2}-2\frac{1}{17})-\dfrac{5}{1-\frac{3}{31}}$

(8) $\dfrac{\dfrac{1}{7}+\dfrac{2}{7+8}}{\dfrac{1}{6}+\dfrac{2}{6\times7}+\dfrac{3}{6\times7\times8}} \div \dfrac{5\frac{2}{3} \text{ of } 3\frac{14}{17}}{2\frac{3}{4} \text{ of } 5\frac{10}{11}}$

FRACTIONAL VALUES AND FRACTIONAL REDUCTION.

To find the value of a fraction of a concrete quantity we now know, from the definition of the symbol "of," to be **an exercise in Compound Multiplication and Division in which the numerator is the multiplier and the denominator the divisor.**

Ex. Find the value of $\frac{7}{8}$ of £4, 13s. 6d.

£	s.	d.
4	13	6
		7
8)32	14	6
4	1	$9\frac{3}{4}$

If instead of a *simple* fraction we have to deal with a mixed number, we may either reduce it to an improper fraction and proceed as above; or, having found the value of the fractional part, find by compound multiplication the value of the required number of times of the concrete quantity and add the results.

To express one concrete quantity as a fraction of another concrete quantity, reduce both to the same denomination, make the *quantity to be expressed as a fraction*, the **numerator**; and the quantity *of which it is to be expressed as a fraction*, the **denominator.**

Ex. Reduce £1, 2s. 7d. to the fraction of £1, 13s. 5d.

£1, 2s. 7d. = 271d. £1, 13s. 5d. = 401d.

£1, 2s. 7d. = $\frac{271}{401}$ of £1, 13s. 5d.

Or, $\frac{\text{£1, 2s. 7d.}}{\text{£1, 13s. 5d.}} = \frac{271\text{d.}}{401\text{d.}} = \frac{271}{401}$.

EXERCISE 47.

Find the value of—

(1) $\frac{5}{12}$ of £1.
(2) $\frac{5}{9}$ of £4, 1s.
(3) $\frac{4}{5}$ of 18s. 7d.
(4) $\frac{7}{19}$ of £8, 12s. $11\frac{3}{4}$d.
(5) $\frac{1}{2}$ of $\frac{3}{5\frac{1}{2}}$ of £1.
(6) $\frac{5}{19}$ of 7s. $8\frac{1}{4}$d.
(7) $\frac{5}{8}$ of 1 ton 2 cwt.
(8) $\frac{7}{6}$ of 11 yds. 2 ft. 7 in.
(9) $\frac{5}{8}$ of 3 tons 13 cwt. 2 qr.
(10) £3, 3s. $3\frac{3}{4}$d. ÷ $3\frac{3}{4}$.
(11) £4, 17s. $6\frac{3}{4}$d. × $2\frac{4}{7}$.
(12) £5, 17s. $6\frac{5}{9}$d. × $14\frac{2}{7}$.
(13) Find the sum of $\frac{7}{16}$ of £5, $\frac{3}{7}$ of £9, 13s. $2\frac{3}{4}$d., and $\frac{5}{12}$ of 2s. 6d.
(14) $1\frac{1}{3}$ of £1, 1s. + $\frac{5}{8}$ of £1 + $\frac{5}{16}$ of 2s. 6d. + $29\frac{2}{20}$ of $7\frac{1}{2}$d.

Find the difference between—

(15) $2\frac{3}{4}$ of 11s. 8d. and $1\frac{1}{2}$ of 17s. $2\frac{1}{2}$d.
(16) $\frac{2\frac{4}{9} \text{ of } 5\frac{1}{8}}{27\frac{1}{2}}$ of £1, 2s. 6d. and $\frac{5\frac{1}{12}}{2\frac{1}{2}+3\frac{3}{4}}$ of 12s. 6d.
(17) $1\frac{2}{3}$ of 1 ton 12 cwt. 3 qr. and $\frac{3}{7}$ of 3 tons 13 cwt. 1 qr.
(18) $3\frac{1}{27}$ of 2 qr. 25 lb. and $\frac{1}{4}$ of 4 cwt. 2 qr. 20 lb.

(19) Reduce 11s. 6d. to the fraction of £1.
(20) ,, 2s. $2\frac{1}{2}$d. ,, ,, £1.
(21) ,, 2 ft. 8 in. ,, ,, 1 yd.
(22) ,, 3 ro. 15 po. ,, ,, 1 ac.
(23) ,, 6 fur. 15 po. ,, ,, 1 ml.
(24) ,, 6 oz. 3 dwt. ,, ,, 1 lb. tr.

(25) Reduce 4s. 4d. to the fraction of 13s. 8d.
(26) ,, 7s. $8\frac{1}{4}$d. ,, ,, 13s. $3\frac{3}{4}$d.
(27) ,, £1, 15s. 3d. ,, ,, £3, 13s. 9d.
(28) ,, $2\frac{1}{4}$ of 3 tons ,, ,, $\frac{1}{7}$ of 1 qr.
(29) ,, $2\frac{2}{5}$ of $1\frac{1}{2}$ ton ,, ,, $1\frac{3}{4}$ of 5 cwt.
(30) ,, $1\frac{1}{5}$ of 5 oz. ,, ,, 1 lb. 8 oz.
(31) ,, $\frac{3}{4}$ of 9 miles ,, ,, 20 fur.
(32) ,, $3\frac{1}{3}$ of $\frac{9}{20}$ mile ,, ,, $4\frac{1}{2}$ miles.
(33) ,, $\frac{8}{9}$ of 3 fur. ,, ,, 1 mile 20 po.
(34) ,, $\frac{4}{5}$ of 8d. ,, ,, $\frac{2}{5}$ of 9d.
(35) ,, $\frac{3}{10}$ of 2s. ,, ,, $1\frac{1}{7}$ of 5s.
(36) ,, $1\frac{1}{6}$ of $\frac{2}{3}$ of £2, 10s. ,, ,, $2\frac{1}{4}$ of $\frac{1}{2}$ of £1.

EXERCISE 48.

Miscellaneous Exercises in Vulgar Fractions.

1. A can do a piece of work in 4 days, which B can do in 5 days, C in 6 days, and D in 8 days; what part could the four do in one day?

2. A's share of a ship was $\frac{7}{8}\frac{7}{8}$, of which he sold $\frac{3}{4}$; how much of the ship remained to him?

3. $\frac{3}{8}$ of $\frac{5}{8} + \frac{2}{5}$ of $\frac{5}{7}$ of a gallon of wine was sold; how much of the gallon was left?

4. Simplify $(4\frac{1}{3}$ of $\frac{2}{3}\frac{6}{7} \div {}_{1}{}^{4}{}_{1}$ of $5\frac{1}{2}) \div (7\frac{7}{1}\frac{7}{8}$ of $\frac{2}{5}\frac{7}{6} - \frac{1}{2}\frac{4}{1}$ of $\frac{3}{7}\frac{5}{2})$.

5. A man performed a piece of work in $6\frac{3}{1}\frac{3}{8}$ days; what part did he do per day?

6. What number is that $\frac{4}{1}\frac{4}{1}$ of which is 76?

7. What number diminished by $\frac{2}{5}$ of itself becomes 942?

8. A father's age is 66 years; 6 years ago his son's age was $\frac{5}{1}\frac{5}{9}$ of his; what is the son's age now?

9. A can do a piece of work in 4 days, which B can do in 5, and C in 6 days; how much more can B and C together do in 1 day than A can?

10. A father left $\frac{7}{1}\frac{7}{5}$ of his estate to his only daughter, and the remainder to his only son; if the difference of the two legacies was £3696, what was the value of the estate?

11. A person has $\frac{7}{9}$ of a property worth £4500; he sells $\frac{1}{3}$ of his share to A and $\frac{2}{9}$ of $\frac{3}{4}$ of the remainder to B; what part of the whole property has he left, and what is it worth?

12. A number is divided by $1\frac{9}{1}\frac{9}{1}$ of $7\frac{1}{3}$: the quotient is multiplied by $\frac{5}{7}$ of $16\frac{2}{3}$; the product is $\frac{5}{9}$ of $4\frac{1}{6}$ of 36. What is the number?

13. A starts from a certain place at 20 miles an hour; after $2\frac{1}{2}$ hours B starts after him at $26\frac{2}{3}$ miles an hour. When will B overtake A, and where?

14. A captain sends out 20 more than $\frac{1}{3}$ of his men, and retains 20 more than $\frac{1}{2}$; how many men are under his command?

15. A, B, and C can do a piece of work in 8 days which A and B can do in 12 days, and B and C can do in 16 days. What part can each do in a day?

16. Simplify $\frac{2}{3}$ of $\frac{3}{5} \div \frac{1}{4} - \frac{2}{3} \div \frac{3}{5}$ of $\frac{1}{4} \div 40$.

17. Which is the greater, $2\frac{1}{2}$ of $\frac{3}{5}$ of $\dfrac{2}{\frac{2}{5}}$ or $\dfrac{4}{3 + \dfrac{2}{1\frac{1}{4}}}$?

18. A man loses $\frac{5}{8}$ of his capital, and afterwards loses $\frac{7}{5}$ of the remainder; what fraction of his capital has he left?

DECIMAL FRACTIONS.

We call our system of notation **decimal** because the value of any figure is **increased tenfold** by its position, *relative to the units place*, being changed **one place to the left**; and so, of course, **decreased to one-tenth of its former value** by being moved one place nearer to the units, *i.e.*, **towards the right.**

We are now to consider the case where we place figures *to the right of units*. We must first mark units in some way, and this is done by making a dot (·) a little above the line, called **the decimal point.** The decimal point is placed immediately to the right of units.

The *places* to the left of units are, in order, *tens*, *hundreds*, *thousands*, &c., and so to the right of units they become, in order, *tenths*, *hundredths*, *thousandths*, &c.

Thus 3 3 3 3 · 3 3 3 represents

3	Thousands
3	Hundreds
3	Tens
3	Units
3	Tenths
3	Hundredths
3	Thousandths

As with whole numbers, if there are no units of any of the denominations, ciphers must be put in to keep the given figures in their proper places relatively to the *point.*

Figures placed to the right of the decimal point are therefore fractions, but fractions with no denominator expressed. **The denominator**, however, is indicated by the position of the figures relatively to the *point*, and **is always some power of ten.**

We can, just as with whole numbers, read off the values not only of single digits in a row, but of sets of digits taken from a row.

Thus, in 231·5706 we can read 3 tens, 7 hundredths, 23 tens, 15 tenths, 706 ten thousandths, and so on; **the denomination of any set being that of its right hand figure**, and the chosen set being read without regard to the point. That 31·5 may be read three-hundred-and-fifteen tenths can be verified by taking the value of the separate digits and adding them. Thus:—

$$31{\cdot}5 = 31 + \frac{5}{10} = \frac{310 + 5}{10} = \frac{315}{10}$$

When we read the whole row we say:—Two hundred and thirty-one, point five, seven, nought, six. The learner should not read 4·16 four point sixteen; for if he were then to read 4·2 properly as four point two, the hearer would get the impression that 4·16 is greater than 4·2. If sixteen is to be said, the word hundredths should follow.

Any digit in a row (except 0) is of greater value than all the digits on its right. Thus, **1** is greater than ·999.

In comparing numbers, begin with the digits of highest order. Thus, 4·26 is greater than 4·19, for the first place where they differ, beginning on the left, gives 2 in the one and 1 in the other. The figures which follow cannot alter this relation.

The following property of decimals must be noticed:—

$$\cdot760 = \tfrac{7}{10} + \tfrac{6}{100} + \tfrac{0}{1000} = \tfrac{76}{100} = \cdot76$$

$$\text{and } \cdot7600 = \tfrac{7}{10} + \tfrac{6}{100} + \tfrac{0}{1000} + \tfrac{0}{10000} = \tfrac{7}{10} + \tfrac{6}{100} = \cdot76.$$

Hence *we may place any number of* **ciphers** *at the end of a decimal, or we may remove them from the end,* **without altering the value of the decimal.**

We may similarly place a decimal point followed by any number of ciphers at the end of a whole number without affecting its value, thus 3 = 3·0000..., and 126 = 126·00000.....

A vulgar fraction whose denominator is a power of ten can be at once written in the decimal notation by placing the numerator figures so that the right-hand one is in the place of similar name to the denominator. Thus:—

$$\frac{14}{100} = \cdot14\,;\ 2\frac{3}{1000} = 2{\cdot}003$$

If the *denominator* is a factor of 10, 100, 1000, &c. (any power of 10), we multiply numerator and denominator by the

number of times it is contained in the power of 10 of which it is a factor, and then write it in decimal notation as above.

Ex. $\frac{3}{25} = \frac{3 \times 4}{25 \times 4} = \frac{12}{100} = \cdot 12$

Fractions whose value can be thus exactly expressed are called **Finite Decimals.**

Where, however, the denominator is not a factor of *some* power of 10, we are not able to express the value *exactly* in decimal notation. Were we to try it we would find a certain digit or set of digits continually repeated. Such decimal fractions are called **Recurring,** or **Repeating,** or **Circulating** decimals, and the repeated digit or set of digits is called the **Period.**

The period is marked by placing a dot over the repeated digit, if there is only one, or over the first and last if there are more than one.

Thus $\frac{2}{3} = \cdot 66666$, &c., written $\cdot\dot{6}$
$\frac{2}{22} = \cdot 9545454$, &c., ,, $\cdot 9\dot{5}\dot{4}$
$\frac{3}{7} = \cdot 4285714285714$, &c., ,, $\cdot\dot{4}2857\dot{1}$

The first and third examples where all the figures of the fraction recur are called **pure repeating decimals**; and the second, and others like it, where only some of the figures of the fraction recur, are called **mixed repeating decimals.**

EXERCISE 49.

1. Write as decimals
$\frac{3}{10}$, $\frac{1}{100}$, $\frac{12}{100}$, $1\frac{2}{10}$, $\frac{12}{10}$, $\frac{246}{1000}$, $109\frac{9}{100}$, $\frac{47}{10000}$, $\frac{512}{10}$, $4\frac{2}{1000}$
2. Write as decimals $\frac{1}{2}$, $\frac{3}{5}$, $\frac{21}{25}$, $\frac{3}{4}$, $\frac{19}{20}$, $\frac{11}{50}$, $\frac{3}{200}$.
3. Write as vulgar fractions ·9, ·09, ·19, 1·1, 30·003, 10·01, 203·0007.
4. Write as a sum of a series of vulgar fractions ·2103, ·05106, 3·2715.

To multiply or divide a decimal by 10, 100, 1000, &c. Since 36·5 means 3 tens, 6 units, 5 tenths; and 365· means 3 hundreds, 6 tens, 5 units; without alteration of the digits and merely by **shifting the position of the point one place to the right,** we have **increased the value** of each separate digit—and consequently **of the whole number—tenfold.**

If we move the point one place farther to the right—to do this we must insert a 0—we change 365· into 3650·, or 365 into 3650. We have again increased the number tenfold or made

it 100 times as large as it was at first by moving the point two places to the right.

Therefore **to multiply by 10, 100, 1000, &c., we move the decimal point 1, 2, 3, &c., places to the right**; or in other words we move it as many places to the right (adding 0's if need be) as there are ciphers in the multiplier.

Division by 10, 100, 1000, &c., is exactly the reverse process. To **divide by 10, 100, 1000, &c., move the point as many places to the left** (adding 0's if need be) **as there are ciphers in the divisor.**

EXERCISE 50.

Mental.

A. Multiply each of the following by 10, 100, 1000 successively.

B. Divide each of the following by 10, 100, 1000 successively.

(1) 4·6	(9) 17·05706	(17) 1·1003
(2) 4·19	(10) 43·76201	(18) 5·372
(3) 1·05	(11) 30·0012	(19) ·5
(4) ·628	(12) 83·00156	(20) ·87
(5) 2·006	(13) 4·19	(21) ·628
(6) ·0005	(14) 1·05	(22) ·04
(7) 37·0007	(15) 5·345	(23) ·015
(8) 28·03205	(16) 2·006	(24) ·005

ADDITION AND SUBTRACTION OF DECIMALS.

Addition. Addition is performed by the same method as is employed for whole numbers. Perhaps a little more care is necessary to secure that figures of the same order stand in the same columns.

Ex. *Add* 34·92, 169·091, 7016·5, 9·01025

```
  34·92
 169·091
7016·5
   9·01025
-----------
7229·52125
```

Place the units figures in the same column, and the others fall into their places as a matter of course. Then add as with whole numbers, and mark the place of the units by a point on its right.

EXERCISE 51.

(1) ·30103 + ·47712 + ·60206 + ·69897
(2) ·096 + ·0096 + 96·0096 + ·96
(3) 7·0096 + ·314 + ·326 + 81·093 + 325·73
(4) ·7146 + ·003 + 94·216 + ·314 + 95·279
(5) 93·423 + ·875 + ·329 + 4·326 + 57·916
(6) 373·912 + 37·3912 + 3739·12 + 3·73912
(7) 247·35 + 9·168 + ·709 + 82·361 + 18·017
(8) ·73 + ·0073 + ·073 + ·00073 + ·000073
(9) ·716 + ·00716 + 716·0716 + ·0000716

Subtraction. Arrange figures of the same order in the same column, and proceed as with whole numbers, marking the units figure by a point on its right.

Ex. 1. *From* 276·1 *take* 93·248

276·1
93·248
182·852

The learner will remember that if we choose we can write 00 after the 1 in the upper line without altering the value.

Ex. 2. *How much greater is* 17·026 *than* ·97 ?

17·026
·97
16·056

We may, if we please, write 0 to the right of 97.

EXERCISE 52.

(1) ·5475 − ·4212
(2) ·875 − ·525
(3) ·275 − ·198
(4) 5·25 − 3·875
(5) 3·125 − 1·9375
(6) 8·425 − 5·3875
(7) 1·25 − ·175
(8) 2·834 − 2·786
(9) 3·245 − 1·2375
(10) 1·1 − ·0009
(11) 8·75 − 7·00009
(12) 9·03 − ·90003

MULTIPLICATION AND DIVISION OF DECIMALS.

Here again the processes are just the same as in dealing with whole numbers, the only question being as to the *placing of the point in the answer.*

Multiplication. Place the digits of the multiplier so that **the units digit will come under the right-hand figure of the multiplicand.** As in simple multiplication, we then at each step put the first figure of the product under the figure by which we are multiplying. In each separate product, and also in the answer, the point will come exactly under the point in the multiplicand.

Ex. Multiply 62·75 by 93·814.

$$
\begin{array}{r}
62{\cdot}75 \\
93{\cdot}814 \\
\hline
5647{\cdot}5 \\
188{\cdot}25 \\
50{\cdot}200 \\
{\cdot}6275 \\
{\cdot}25100 \\
\hline
5886{\cdot}82850
\end{array}
$$

If the multiplier has no units figure, supply its place with 0.

The number of figures to the right of the point in the answer is always equal to the sum of the numbers after the point in multiplicand and multiplier.

Division. In division of decimals, unless the divisor is a *whole number*, we must multiply both dividend and divisor by that power of ten which will make the divisor a **whole number.**

This we have already seen is merely a question of moving the point, and, as we saw in vulgar fractions, to multiply both dividend and divisor by the same number does not alter the value of the resulting quotient.

Ex. Divide 73·056 by ·08.

The divisor is $\frac{8}{100}$, and to make it a whole number we must therefore multiply by 100. Do the same to the dividend. To do this, we know, we move the point two places to the right. The sum now reads—

$$7305{\cdot}6 \div 8$$

$$
\begin{array}{r|l}
8 & 7305{\cdot}6 \\
\hline
 & 913{\cdot}2
\end{array}
$$

Work by short division as with whole numbers and place the point under the point.

This mechanical aid to the placing of the point can also be got in long division if we place the answer above the dividend, figure for figure, as if we were working by short division.

Ex. Divide ·416065 by ·0325.

Multiplying both by 10000 we have 4160·65 ÷ 325.

```
      12·802 Ans.
325)4160·65
    325
    ----
     910
     650
    ----
    2606
    2600
    ----
      650
      650
```

The two ciphers to the left of the whole number are of no use and may be cut out. If there is a remainder when all the figures of the dividend have been used, we may suppose any number of ciphers to follow, for the addition of these at the right of a decimal does not alter its value.

Divide by *factors* (short division) where the divisor can be readily broken up; or employ the Italian method of long division in division of decimals as elsewhere.

EXERCISE 53.

(1) 5·27 × 4·83
(2) ·436 × 2·19
(3) 1·89 × ·76
(4) 2·38 × 3·47
(5) 5·62 × ·213
(6) ·278 × ·547
(7) 5·27 × ·00483
(8) ·0436 × ·00219
(9) 18·9 × ·000076
(10) ·238 × ·0347
(11) ·0562 × ·0000213
(12) ·00278 × ·000547
(13) 52·7 × 48300
(14) 4·36 × 219000
(15) ·189 × 7600
(16) ·00238 × 347000
(17) ·00562 × 21300
(18) 27800 × ·000547
(19) 98·7654 × ·983427
(20) ·123456 × ·654321
(21) 5·78934 × ·000763
(22) ·007639 × 763900
(23) 87·6591 × 684000
(24) ·000009 × ·000983

EXERCISE 54.

(1) 175·03 ÷ 76100
(2) ·403858 ÷ 63400
(3) 39538 ÷ 5300
(4) ·39237 ÷ 3190
(5) ·110925 ÷ 153000
(6) ·52441 ÷ 22900
(7) ·0156366 ÷ ·0042
(8) ·03486 ÷ 4·98
(9) ·378816 ÷ 5·919
(10) 20973·6 ÷ ·8739
(11) 9110·64 ÷ 2900
(12) 7·127577 ÷ 1·0053
(13) 1·7503 ÷ 7·61
(14) 40·3858 ÷ 6·34
(15) 39·538 ÷ ·53
(16) 392·37 ÷ 31·9

(17) 110·925 ÷ 1·53
(18) 5·2441 ÷ 22·9
(19) 1750·3 ÷ ·0761
(20) 4038·58 ÷ ·0634
(21) 3953·8 ÷ ·053
(22) 39·237 ÷ ·319
(23) 1109·25 ÷ ·0153
(24) 524·41 ÷ ·0229

CONTRACTED METHODS IN MULTIPLICATION AND DIVISION OF DECIMALS.

Multiplication. When two decimals are to be multiplied together, especially if there are many figures after the decimal point, it is usually sufficient for all practical purposes to get the product *approximately* correct, or as it is usually put, *correct to a certain number* (3 or 4 as the case may be) *of decimal places.*

A very slight alteration in our rule for multiplication of decimals is all that is required.

Place the units digit of the multiplier under that decimal place of the multiplicand to which the product is to be correct, and, with this digit so placed, reverse the position of the others relatively to it.

Ex. 314·62875 × 23·618. Answer to be correct to three decimal places.

We place the units digit of multiplier under the third decimal place of multiplicand—8—because that is the last place wanted in the answer. The reason for reversing will be spoken of below.

Ordinary Method		Contracted Method
314·62875		314·628 75
23·618		816·3 2
943·88625	Multiplication by 3	943·886
6292·5750	,, ,, 2	6292·575
188·777250	,, ,, 6	188·777
3·1462875	,, ,, 1	3·146
2·51703000	,, ,, 8	2·517
7430·90181750		7430·901

By placing the units digit as explained above, and reversing the digits of the multiplier, we so arrange matters that each digit when multiplied into the digit of the multiplicand immediately above it gives a figure to be placed *in the last place required.* The pupil ought, however, to multiply in each case mentally into one or two digits to the right so as to get the correct carrying figure. Thus, in multiplying by 6, we say, *mentally,* 6 × 7 = 42, 6 × 8 = 48 + 4 = 52, carry 5; 6 × 2 = 12 + 5 = 17, put down 7.

In order to secure greater accuracy it is usual to work for a result with *one decimal place more than the number asked for.*

Division. For contracted division of decimals begin in the ordinary way.

The number of digits we can get in the quotient after the contracted working begins is one less than the number of digits in the divisor. Where the contracted method will begin depends therefore on the number of decimal places required in the answer.

Ex. Divide 16683·952 by 1462·5 correct to 6 places.

This means 166839·52 ÷ 14625, and as there are five digits in the divisor we can only get 4 figures in the quotient by the contracted method. We must work by the ordinary method then till we get $(6 - 4) = 2$ decimal places in the answer, adding ciphers if necessary at the right hand of the dividend. When that point has been reached instead of taking down a figure from the dividend at each step we leave the remainder unaltered and **strike off the last digit of the divisor.**

Remember at the subsequent steps, in multiplying, to get the carrying figure from the digit so struck off.

```
                    000011·407832 Ans.
1'4'6'2'5 ) 166839·52
              14625
              -----
               20589
               14625
               -----
                59645
                58500
                -----
                11452
                10237
                -----
                 1215
                 1169
                 ----
                   46
                   43
                   --
                    3
                    2
                    -
```

The contracted method begins here. With the 5 struck off we say $7 \times 5 = 35$, carry 3; $7 \times 2 = 14 + 3 = 17$ (put down 7); and so, at succeeding steps, when multiplying.

The full setting down is given above to show the principle better, but it saves figuring and space and is neater too, to use the Italian method and subtract as you multiply without setting down the product.

EXERCISE 55.

Find, with the least possible working, the results of the following :—

A. (1) $7{\cdot}384 \times 9{\cdot}6754$ to the nearest third-place decimal.
(2) $28{\cdot}7106 \times 19{\cdot}3196$ to the nearest fourth-place decimal.
(3) $\cdot 314159 \times 9{\cdot}5746$ to the nearest fifth-place decimal
(4) $392{\cdot}7148 \times 25{\cdot}098$ to 7 significant figures.
(5) $917{\cdot}098 \times 376{\cdot}95$ to the nearest unit.
(6) $(38{\cdot}2375)^2$ to 5 significant figures.
(7) $4975{\cdot}06 \times 312{\cdot}196$ to 5 significant figures.
(8) $32{\cdot}1598 \times {\cdot}562089$ to 3 decimal places.
(9) $\cdot 23045 \times 15{\cdot}8476$ to 3 decimal places.
(10) $3{\cdot}72 \times {\cdot}0005962$ to 4 decimal places.
(11) $\cdot 005762 \times 3{\cdot}20876$ to 4 decimal places.
(12) $\cdot 156208 \times {\cdot}36208$ to 4 decimal places.

B. (1) $6{\cdot}75 \div 3{\cdot}25$ to 4 decimal places.
(2) $10 \div 4{\cdot}75$ to 3 decimal places.
(3) $20{\cdot}6 \div 3{\cdot}3125$ to 3 decimal places.
(4) $6{\cdot}23475 \div {\cdot}04875$ to 6 significant figures.
(5) $4{\cdot}12189 \div {\cdot}04763$ to 4 significant figures.
(6) $\cdot 004365 \div {\cdot}71215$ to 5 decimal places.
(7) $\cdot 0007 \div 3{\cdot}125$ to 6 decimal places.
(8) $\cdot 00034625 \div 631{\cdot}247$ to 4 significant figures.
(9) $436{\cdot}3205 \div 279{\cdot}152$ correct to 3 places of decimals.
(10) $185{\cdot}37612 \div {\cdot}08764032$ correct to 4 places of decimals.
(11) $\cdot 5361085 \div 23{\cdot}450842$ correct to 4 places of decimals.
(12) $17{\cdot}1890865 \div {\cdot}13249672$ correct to 5 places of decimals.

To express Vulgar Fractions as Decimals, and Decimals as Vulgar Fractions. We have already seen that, where a vulgar fraction has, for its denominator, some power of ten ; or can be expressed as a fraction with some power of ten for denominator, it is only an exercise in notation to express it as a decimal.

Ex. $\frac{3}{100} = {\cdot}03$; $2\frac{7}{1000} = 2{\cdot}007$
$\frac{2}{5} = \frac{4}{10} = {\cdot}4$; $2\frac{4}{125} = 2\frac{32}{1000} = 2{\cdot}032$

We can now, however, express any vulgar fraction as a decimal ; for, since every vulgar fraction indicates a division process, **to convert a vulgar fraction into the corresponding decimal is merely to carry out the division indicated by its terms.**

Ex. 1. $\frac{7}{8} = 7 \div 8$

$$8 \,|\, \underline{7{\cdot}000} \\ \underline{{\cdot}875}$$

Insert point after units of dividend and add as many ciphers as you may require.

Ex. 2. $\frac{3}{16} = 3 \div 16$

$$2 \,|\, \underline{3{\cdot}0} \\ 8 \,|\, \underline{1{\cdot}5000} \\ \underline{{\cdot}1875}$$

Ex. 3. $\frac{8}{37} = 8 \div 37$

$$37 \,)\, 8{\cdot}000000 \quad \text{Q } {\cdot}216216, \text{ \&c., Ans.} \\ \underline{74} \\ 60 \\ \underline{37} \\ 230 \\ \underline{222} \\ 80 \\ \underline{74} \\ 60 \\ \underline{37} \\ 230 \\ \underline{222} \\ 8$$

In the third example it will be noticed we are not able to express the value of the vulgar *exactly* in decimal form; for the set of digits—216—in the quotient, keep continually recurring, and this is shown in writing down the answer as mentioned before, by the placing of dots above the repeating figures.

Thus $\frac{8}{37} = {\cdot}\dot{2}1\dot{6}$ **Ans.**

We may with advantage make a table of the decimal equivalents of the commonly occurring vulgar fractions, such as $\frac{1}{2}$, $\frac{1}{4}$, &c.

$\frac{1}{2} = {\cdot}5$ $\qquad$ $\frac{1}{4} = {\cdot}25$; $\frac{3}{4} = {\cdot}75$

$\frac{1}{8} = {\cdot}125$; $\frac{3}{8} = {\cdot}375$; $\frac{5}{8} = {\cdot}625$; $\frac{7}{8} = {\cdot}875$

$\frac{1}{16} = {\cdot}0625$; $\frac{3}{16} = {\cdot}1875$; $\frac{5}{16} = {\cdot}3125$, &c.

The decimal equivalents of $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{11}$ should be remembered, for all fractions with these same denominators have for their decimal equivalents the easily remembered multiples of these indicated by their numerators.

Thus $\frac{1}{3} = {\cdot}\dot{3}$; $\frac{2}{3} = {\cdot}\dot{6}$

$\frac{1}{9} = {\cdot}\dot{1}$; $\frac{2}{9} = {\cdot}\dot{2}$; $\frac{4}{9} = {\cdot}\dot{4}$; $\frac{5}{9} = {\cdot}\dot{5}$; $\frac{7}{9} = {\cdot}\dot{7}$; $\frac{8}{9} = {\cdot}\dot{8}$

$\frac{1}{11} = {\cdot}0\dot{9}$; $\frac{2}{11} = {\cdot}1\dot{8}$; $\frac{3}{11} = {\cdot}2\dot{7}$; $\frac{4}{11} = {\cdot}3\dot{6}$; $\frac{5}{11} = {\cdot}4\dot{5}$; $\frac{6}{11} = {\cdot}5\dot{4}$, &c.

$\frac{1}{6} = {\cdot}1\dot{6}$; $\frac{5}{6} = {\cdot}8\dot{3}$ $\qquad$ $\frac{1}{7} = {\cdot}\dot{1}42857\dot{}$

The same digits, in the same order, but beginning each time with the next lowest digit, give the decimal equivalents for $\frac{2}{7}$, $\frac{3}{7}$, &c.

$$\tfrac{2}{7} = \cdot\dot{2}85714\dot{2}\,;\ \tfrac{3}{7} = \cdot\dot{4}28571\dot{4}\,;\ \tfrac{4}{7} = \cdot\dot{5}71428\dot{5}$$
$$\tfrac{5}{7} = \cdot\dot{7}14285\dot{7}\,;\ \tfrac{6}{7} = \dot{8}57142\dot{8}$$

Another group which should be noted because of the frequent occurrence of one or other of them is that where 13 is the denominator.

Here there are two groups of digits which always keep the same order beginning with a different digit increasingly large as the numerator increases.

$$\tfrac{1}{13} = \cdot\dot{0}76923\dot{0} \qquad \tfrac{2}{13} = \cdot\dot{1}53846\dot{1}$$
$$\tfrac{3}{13} = \cdot\dot{2}30769\dot{2} \qquad \tfrac{5}{13} = \cdot\dot{3}84615\dot{3}$$
$$\tfrac{4}{13} = \cdot\dot{3}07692\dot{3} \qquad \tfrac{6}{13} = \cdot\dot{4}61538\dot{4}$$
$$\tfrac{9}{13} = \cdot\dot{6}92307\dot{6} \qquad \tfrac{7}{13} = \cdot\dot{5}38461\dot{5}$$
$$\tfrac{10}{13} = \cdot\dot{7}69230\dot{7} \qquad \tfrac{8}{13} = \cdot\dot{6}15384\dot{6}$$
$$\tfrac{12}{13} = \cdot\dot{9}23076\dot{9} \qquad \tfrac{11}{13} = \cdot\dot{8}46153\dot{8}$$

To express Decimals as Vulgar Fractions. When there is no recurring digit or set of digits in the decimal to be converted, the figures of the given decimal are used as the numerator, and the denominator is that power of ten which expresses the denomination of the right-hand digit of the decimal.

Thus, as we have already seen—

$$3{\cdot}616 = 3\tfrac{616}{1000} \text{ or } \tfrac{3616}{1000}$$
$$74{\cdot}0025 = 74\tfrac{25}{10000} \text{ or } \tfrac{740025}{10000}$$
$$\cdot01278 = \tfrac{1278}{100000}$$

When the decimal to be converted has a recurring digit or set of digits, we get the numerator by subtracting from the whole the non-repeating part; and for the denominator put as many 9's as there are repeating figures, and to the right of these as many 0's as there are non-repeating digits *in the decimal*.

Thus
$$\cdot\dot{2}\dot{3} = \tfrac{23}{99}$$
$$6{\cdot}1\dot{6} = 6\tfrac{16}{99} \text{ or } \tfrac{610}{99} \ (= 616 - 6 \text{ non-repeating})$$
$$17{\cdot}60\dot{1}\dot{3} = 17\tfrac{5953}{9900} \ (= 6013 - 60 \text{ non-repeating})$$
$$\text{or } \tfrac{174253}{9900} \ (= 176013 - 1760 \text{ non-repeating})$$

As will be seen from the above examples, the rules hold good whether the fractions are *kept as mixed numbers* or *reduced to improper fractions*.

EXERCISE 56.

A. Reduce to decimals :—

(1) $\frac{5}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{3}{4}$ (5) $\frac{7}{8}$ (6) $\frac{1}{3}$ (7) $\frac{1}{9}$

(8) $\frac{5}{6}$ (9) $\frac{1}{6}$ (10) $\frac{9}{16}$ (11) $\frac{1}{5}$ (12) $\frac{1}{75}$ (13) $\frac{1}{1875}$ (14) $\frac{7}{11}$

(15) $\frac{8}{11}$ (16) $\frac{4}{9}$ (17) $\frac{9}{11}$ (18) $\frac{1}{7}$ (19) $\frac{3}{11}$ (20) $\frac{1}{13}$ (21) $\frac{1}{999}$

(22) $\frac{11}{54}$ (23) $\frac{2}{3}$ (24) $\frac{16}{17}$ (25) $\frac{11}{13}$ (26) $\frac{34}{33}$ (27) $\frac{1541}{21}$ (28) $\frac{107}{333}$

(29) $\frac{5}{37}$ (30) $1\frac{29}{54}$ (31) $2\frac{5}{28}$ (32) $6\frac{1}{18}$ (33) $1\frac{9}{14}$ (34) $1\frac{17}{198}$ (35) $6\frac{43}{74}$ (36) $2\frac{3}{28}$

B. Reduce to vulgar fractions :—

(1) ·5
(2) ·25
(3) ·75
(4) ·125
(5) ·625
(6) ·34
(7) ·375
(8) ·005
(9) ·078
(10) ·15625
(11) ·03125
(12) ·01875
(13) 3·24
(14) 5·128
(15) 7·512
(16) 6·075
(17) ·00525
(18) ·015625
(19) ·078125
(20) ·006225
(21) $\cdot\dot{7}$; $\cdot\dot{3}$; $\cdot\dot{6}$
(22) $\cdot\dot{2}\dot{7}$; $\cdot\dot{5}\dot{4}$; $\cdot\dot{0}\dot{6}\dot{3}$
(23) $\cdot\dot{1}\dot{8}\dot{5}$; $\cdot\dot{0}\dot{7}\dot{4}$; $\cdot\dot{9}\dot{6}\dot{2}$
(24) $\cdot\dot{4}2857\dot{1}$; $\cdot\dot{0}08\dot{1}$
(25) $\cdot\dot{6}15384\dot{6}$; $\cdot\dot{0}9523\dot{8}$
(26) $\cdot\dot{5}7142\dot{8}$; $\cdot\dot{7}2\dot{9}$
(27) $\cdot\dot{3}7\dot{2}$; $\cdot\dot{2}2\dot{5}$
(28) $\cdot58\dot{3}$; $\cdot29\dot{5}\dot{4}$; $\cdot1\dot{6}$
(29) $\cdot0\dot{4}\dot{5}$; $\cdot65\dot{2}9\dot{6}$; $\cdot00\dot{6}$
(30) $\cdot63\dot{7}2\dot{9}$; $\cdot01\dot{4}87\dot{3}$
(31) $\cdot8\dot{3}$; $\cdot14\dot{9}$; $\cdot57\dot{6}3\dot{9}$
(32) $\cdot00\dot{5}7\dot{6}$; $\cdot0948\dot{9}1\dot{0}$
(33) $\cdot000\dot{2}5\dot{9}$; $\cdot3\dot{3}7\dot{8}$
(34) $\cdot80\dot{8}\dot{1}$; $5\cdot5\dot{6}\dot{3}$
(35) $6\cdot23\dot{4}76\dot{1}9\dot{0}$; $\cdot31\dot{1}53846$
(36) $27\cdot39\dot{0}9523\dot{8}$; $\cdot81\dot{6}1904\dot{7}$

ADDITION AND SUBTRACTION OF RECURRING DECIMALS.

Arrange the digits of the various lines in the proper columns as for addition and subtraction of finite decimals. Before beginning to add or subtract, draw a perpendicular line to the right of the last non-repeater in any line. Find L.C.M. of the number of figures in the different recurring periods; carry out the periods in each line to this extent beyond the vertical line already drawn; erect another vertical

line here, and add or subtract as the case may be. The recurring period in the answer will be between the vertical lines. The figures should be carried one place beyond the second vertical line to allow for carrying.

Ex. Add $3\cdot14\dot{2}$, $26\cdot0625$, $\cdot5\dot{9}$, and $7\cdot3\dot{4}1\dot{2}$.

$3\cdot1422\,\vert\,222222\,\vert\,2$	Here **5** in the second line is the non-repeater farthest from the decimal point. L.C.M. of recurring periods 1, 2, and 3 = 6; carry out periods six places beyond line. We may add 0's in the second line if we choose.
$26\cdot0625\,\vert\,\quad\quad\,\vert$	
$\cdot5959\,595959\,5$	
$7\cdot3412\,412412\,4$	
$37\cdot1419\,230594\,\vert$	

To adhere strictly to the above rule in dealing with long recurring periods would in most cases give us far more decimal places than are required for the purpose in hand. We are now to consider how to find the number of places required without incurring the labour of finding useless figures.

In every approximate decimal the last digit retained should be the nearest in its place.

Approximate Sum or Difference. In general, work out two more places than are required, and then strike out the last two, increasing the last figure retained by 1 if the next figure is 5 or more.

This rule is not absolute, for when the rows to be added are very numerous, the effect of the columns neglected might extend two places to their left, and so the last digit retained might be doubtful.

Ex. *Find the sum to the nearest third-place decimal of—*

$31\cdot2514914$	Neglect the 6th and 7th columns of decimals.
$26\cdot1874426$	
$3\cdot6132519$	Strike out 83, and increase the 9 by 1, since 8 is greater than 5. The sum required is **62·040**. In such a case the last 0 must not be omitted.
$\cdot9876558$	
$62\cdot03983$	

EXERCISE 57.

A. (1) Add together $3\cdot4\dot{5}$, $64\cdot\dot{9}$, $\cdot3475$, $6\cdot8\dot{2}3\dot{6}$, $17\cdot34\dot{5}$.

(2) Add together $\cdot21975$, $373\cdot\dot{1}\dot{8}$, $\cdot00\dot{9}3\dot{4}$, $\cdot5\dot{2}\dot{6}$, $\cdot2\dot{9}7\dot{4}$.

(3) From $275\cdot63\dot{8}2\dot{7}$ take $96\cdot912\dot{3}4\dot{5}$.

(4) Take $\cdot39\dot{5}17\dot{4}$ from $\cdot40\dot{8}3\dot{5}$.

(5) $6\cdot3\dot{4}\dot{5} + 28\cdot92\dot{7} + 156\cdot473\dot{6} - 99\cdot72\dot{3}6\dot{4}$.

(6) $42\cdot7\dot{8}\dot{4} + \cdot009817 + 296\cdot51\dot{3} + \cdot\dot{7}24\dot{8} - 309\cdot34\dot{7}\dot{8}$.

(7) Find the sum of $2\cdot3\dot{4} + 65\cdot\dot{8}\dot{1} + \cdot3\dot{4}\dot{7}\dot{8} + \cdot517 + 825\cdot\dot{6}\dot{7}$.

(8) Add together $\cdot\dot{7}17\dot{5} + 4\cdot\dot{7}\dot{2} + \cdot937 + 853\cdot\dot{6} + \cdot345\dot{6}$.

(9) From $34\cdot62\dot{3}\dot{8}\dot{5}$ take $29\cdot\dot{5}\dot{7}$.

(10) Take $173\cdot\dot{6}\dot{8}$ from $219\cdot7\dot{2}39\dot{6}$.

(11) $\cdot2\dot{3}5\dot{6} + 41\cdot73\dot{8} + 846\cdot\dot{9}\dot{4} + \cdot3\dot{5}7\dot{9} - 779\cdot3\dot{4}56\dot{7}$.

(12) $47\cdot\dot{5} + 289\cdot\dot{7}\dot{2} + \cdot50\dot{3}7\dot{9} + 65\cdot00\dot{4}\dot{6} - 319\cdot40\dot{0}5\dot{7}$.

(13) Add together $5\cdot\dot{6}\dot{4} + \cdot\dot{6}3\dot{8} + 274\cdot05\dot{6} + 74\cdot50\dot{3}\dot{8} + \cdot7594$.

(14) Find the sum of $197\cdot\dot{3} + \cdot9093 + 14\cdot8\dot{5}\dot{6} + \cdot\dot{5}73\dot{8}\dot{4} + 2\cdot16\dot{7}\dot{8}$.

(15) Find the difference between $615\cdot38\dot{4}7\dot{9}$ and $700\cdot\dot{2}\dot{1}$.

(16) Take $529\cdot\dot{4}17\dot{6}$ from $1000\cdot\dot{2}1\dot{3}$.

B. Find the result of—

(1) $7\cdot\dot{2}\dot{7} + 9\cdot291\dot{6} + 8\cdot\dot{3}\dot{6}$ to 3 decimal places.

(2) $\cdot\dot{0}3\dot{6} + \cdot\dot{0}\dot{3}\dot{6} + \cdot03\dot{6}$ to 4 places.

(3) $\cdot\dot{0}243\dot{9} + \cdot\dot{0}0\dot{3} + 3\cdot141592$ to 4 places.

(4) $\cdot91908\ldots + \cdot72428\ldots + \cdot72607\ldots$ to 3 places.

(5) $\frac{2}{17} + \frac{3}{19} + \frac{4}{23}$ to 6 places.

(6) $\frac{6}{13} + \frac{3}{7} + \frac{8}{11}$ to 4 places.

(7) $3\frac{1}{6} + 7\frac{5}{13} + 2\frac{2}{37}$ to 3 places.

(8) $\frac{2}{3}$ of $\frac{15}{17} + \frac{3}{4}$ of $\frac{16}{19} + 7\frac{1}{39}$ to 3 places

(9) $\cdot\dot{7} - \cdot7291\dot{6}$ to 4 places.

(10) $\cdot2\dot{5}\dot{9} - \cdot\dot{0}027\dot{1}$ to 5 places.

(11) $\cdot\dot{9}6\dot{2} - \cdot\dot{9}\dot{0}$ to 4 places.

(12) $\cdot0625 - \cdot041\dot{6}$ to 3 places.

(13) $\cdot2 - \cdot008\dot{3}$ to 3 places.

(14) $1\cdot041393\ldots - \cdot698970\ldots$ to 5 places.

(15) $1\cdot41497\ldots - 1\cdot32222\ldots$ to 4 places.

(16) $\frac{1}{259} - \frac{1}{271}$ to 5 places.

(17) $\frac{1}{77} - \frac{1}{333}$ to 5 places.

(18) $2\frac{1}{4} - \frac{6}{13} + \frac{2}{813} - \frac{1}{407}$ to 5 places.

MULTIPLICATION AND DIVISION OF RECURRING DECIMALS.

When one of the two decimals to be multiplied together is a terminating one, make the recurring one the multiplicand, and having extended the period one or two places for carrying purposes proceed as in multiplication of finite decimals.

Ex. $\cdot 21\dot{7}\dot{2} \times \cdot 675$.

$$
\begin{array}{ll}
\cdot 21727272 & \\
0\cdot 675 & \\
\hline
\cdot 13036363 & 6 \\
\cdot 01520909 & 0 \\
\cdot 00108636 & 3 \\
\hline
\cdot 14665909 & 0
\end{array}
$$

Ans. $\underline{\cdot 1466590}$

When the divisor is a terminating decimal and the dividend a recurring one, the period should be extended as far as is necessary instead of adding 0's. The exercise is otherwise the same as division of decimals.

In any case other than the above we can only get a correct result by converting the decimals to vulgar fractions, carrying through the required exercise, and then converting the vulgar fraction of the answer back again to a decimal.

Ex. $\cdot 3\dot{4}7\dot{4} \div 1\cdot\dot{6}$.

$$\cdot 3\dot{4}7\dot{4} = \frac{3474-3}{9990} = \frac{3471}{9990}$$

$$1\cdot\dot{6} = \frac{16-1}{9} = \frac{15}{9}$$

$$\frac{3471}{9990} \div \frac{15}{9} = \frac{1157}{5550} = \textbf{Ans.}\ \underline{\cdot 20\dot{8}4\dot{6}}\ (1157 \div 5550)$$

EXERCISE 58.

A. (1) $\cdot 1\dot{6} \times 7$
(2) $27\cdot 08\dot{3} \times 9$
(3) $\cdot 40\dot{6} \times 52$
(4) $27\cdot 88\dot{3} \times 8\cdot 75$
(5) $\cdot 625 \times \cdot\dot{3}$
(6) $87\cdot\dot{2} \times \cdot 0\dot{7}$
(7) $\cdot 845 \times \cdot 4\dot{8}$
(8) $87\cdot 34 \times 2\cdot 6\dot{7}$
(9) $\cdot 62\dot{3} \times \cdot 0\dot{6}$
(10) $\cdot 3\dot{0}\dot{9} \times \cdot 4\dot{6}$
(11) $\cdot 825 \times \cdot 3\dot{6}$
(12) $64\cdot 35 \times \cdot 7\dot{2}\dot{2}$
(13) $16\cdot\dot{1}4\dot{5} \times \cdot 5729$
(14) $\cdot 7\dot{2}1\dot{6} \times \cdot\dot{5}\dot{4}$
(15) $\cdot 94\dot{3} \times \cdot 21\dot{7}\dot{2}$
(16) $\cdot 63\dot{5} \times 9\cdot 48\dot{1}$

B. (1) $5\cdot 1\dot{6} \div 4$
(2) $3\cdot\dot{3}7\dot{0} \div 7$
(3) $4\cdot 1966\dot{2} \div 37$
(4) $73\cdot 41\dot{6} \div 6\cdot 25$
(5) $15\cdot 0\dot{6} \div 1\cdot 5$
(6) $169\cdot\dot{3} \div \cdot 05$
(7) $75\cdot 2\dot{6} \div 5$
(8) $\cdot 1134 \div \cdot\dot{7}$
(9) $23\cdot 5 \div \cdot\dot{4}$
(10) $8\cdot\dot{9}6\dot{2} \div \cdot\dot{3}$
(11) $363\cdot 5\dot{4}0\dot{7} \div 48\cdot\dot{3}$
(12) $9 \div \cdot\dot{4}\dot{5}$
(13) $\cdot 12693 \div 19\cdot 3\dot{9}$
(14) $17\cdot\dot{4}\dot{5} \div \cdot\dot{8}\dot{4}$
(15) $\cdot 3\dot{4}7\dot{4} \div \cdot\dot{6}$
(16) $\cdot 24835 \div \cdot 29\dot{6}$

VALUATION OF DECIMALS OF CONCRETE QUANTITIES.

In the case where the decimal is terminating and the **concrete quantity of one denomination** the method is practically that of ordinary Reduction.

Ex. 1. Find the value of ·4375 of £1.

Ex. 2. Find the value of 7·0625 of 20 tons.

£ ·4375	7·0625 × 20 tons
20	20
s. 8·7500	tons 141·2500
12	20
d. 9·00	cwt. 5·00
Ans. 8s. 9d.	**Ans.** 14 tons 5 cwt.

Where the decimal is terminating but the **concrete quantity is of more than one denomination**, we must first express the concrete quantity in terms of one denomination and then proceed as above.

Ex. 1. Find the value of ·2896 of £15, 12s. 6d.

Ex. 2. Find the value of 12·3125 of 5 cwt. 2 qr. 14 lb.

£15, 12s. 6d. = 312½s. = 312·5s.

·2896	
312·5	
86·88	
3·4752	
14480	
s. 90·50000	
12	
d. 6·0	
Ans. £4, 10s. 6d.	

5 cwt. 2 qr. 14 lb. = 22½ qr. = 22·5 qr.

22·5	
12·3125	
6·75	× 3 tenths
270·0	× 12 units
·2700	× 12 thousandths
·01125	× 5 ten thousandths
qr. 277·03125	
28	
·25000	
·6250	
lbs. 0·87500	
16	
oz. 14·000	
Ans. 3 ton 9 cwt. 1 qr. 14 oz.	

Should the decimal be non-terminating (recurring) we must convert it to a vulgar fraction, and then find the value of that vulgar fraction of the concrete quantity.

EXERCISE 59.

Find the value of—

(1) £·0375.
(2) £·0416.
(3) £·875.
(4) £·634375.
(5) £·9989583.
(6) £·714583.
(7) £·16.
(8) £·3.
(9) £·6.
(10) £·04583.
(11) £·01875.
(12) £·003125.
(13) 6·74 of 6s. 3d.
(14) 7·875 of £6, 10s. 4d.
(15) ·0035 of £25.
(16) 1·15 of £29, 3s. 4d.
(17) 67·005 of £10.
(18) 3·00005 of £416, 13s. 4d.
(19) ·005 of 1 ton.
(20) 6·25 of 3 qr. 16 lb.
(21) ·375 of a furlong.
(22) 7·0625 of 20 tons.
(23) ·00075 of $1\frac{1}{2}$ miles.
(24) 3·35 of 5 gall.
(25) 57·29$\dot{1}\dot{6}$ of 12s. 6d.
(26) ·169$\dot{4}$ of 13s. 4d.
(27) 50·$\dot{9}\dot{0}$ of 5s. 6d.
(28) 1·2$\dot{2}\dot{4}$ of £3, 17s. 11d.
(29) 2·$\dot{4}$ of £·0625.
(30) 4·3$\dot{0}\dot{9}$ of £2, 15s.
(31) ·06$\dot{2}\dot{5}$ of £495.
(32) 3·3$\dot{4}$28571 of £·4$\dot{9}$.
(33) ·3571428 of 3s. 6d.
(34) 2·3$\dot{4}\dot{5}$ of £6, 17s. 6d.
(35) 6·$\dot{3}\dot{9}$ of 8 ft. 3 in.
(36) 6·1$\dot{6}$ of 5 lb. 10 oz.

To express one concrete quantity as a decimal of another we must proceed to express the one as a **vulgar** fraction of the other *(see page 52)*, and then convert the vulgar fraction in the answer to a decimal.

EXERCISE 60.

Reduce :—

(1) 9d. to the dec. of £1.
(2) 10d. to the dec. of £1.
(3) 17s. 6d. to the dec. of £1.
(4) 12s. $8\frac{1}{4}$d. to the dec. of £1.
(5) 19s. $11\frac{3}{4}$d. to the dec. of £1.
(6) 14s. $3\frac{1}{2}$d. to the dec. of £1.
(7) 3s. 4d. to the dec. of £1.
(8) 6s. 8d. to the dec. of £1.
(9) 13s. 4d. to the dec. of £1.
(10) 11d. to the dec. of £1.
(11) $4\frac{1}{2}$d. to the dec. of £1.
(12) $\frac{3}{4}$d. to the dec. of £1.
(13) 7 lbs. to the dec. of a cwt.
(14) 8 lbs. to the dec. of a cwt.
(15) 17 lbs. to the dec. of a ton.
(16) 18 cwt. 13 lbs. to the dec. of a ton.
(17) 11 oz. 17 dwt. to the dec. of a lb.
(18) 13 dwt. 16 gr. to the dec. of a lb.
(19) 5 dwt. 12 gr. to the dec. of an oz.

(20) 7 oz. 14 dr. to the dec. of a cwt.
(21) 10 oz. 12 dr. to the dec. of a lb.
(22) 3 oz. 14 dwt. 8 gr. to the dec. of a lb.
(23) 3 qrs. to the dec. of a cwt.
(24) 6 fur. 5 pls. to the dec. of a mile.
(25) 2 ro. 11 pls. to the dec. of an acre.
(26) 11d. to the dec. of a shill.
(27) 10d. to the dec. of a shill.
(28) $7\frac{1}{2}$d. to the dec. of a shill.
(29) 12s. $4\frac{3}{4}$d. to the dec. of a guinea.
(30) 4 bush. 3 pks. $1\frac{1}{4}$ gal. to the dec. of a quarter.
(31) 5 yds. 2 ft. 11 in. to the dec. of a pole.
(32) 7 days 6 hrs. to the dec. of a year.
(33) 7 hrs. 9 mins. to the dec. of a day.
(34) 22 mins. 3 secs. to the dec. of an hour.
(35) 1 ro. 22 pls. to the dec. of an acre.
(36) 20 pls. 8 yds. to the dec. of an acre.
(37) 18 yds. 4 ft. to the dec. of a rood.
(38) 3 days 12 hrs. to the dec. of a year.
(39) What dec. of 3 ro. 12 sq. po. is 4 acres?
(40) What dec. of £2, 10s. added to 12s. 6d. will make £1?

DECIMALISATION OF MONEY.

In very many business calculations where *correctness to the nearest penny is all that is wanted*, the conversion of the money to be dealt with into the decimal of £1 will both simplify and shorten the work.

Correctness to 3 places of decimals, that is, to the $\frac{1}{1000}$ part of £1 (which is less than a farthing), **is what is usually aimed at.**

Ex. 1. Express £7, 11s. $8\frac{1}{2}$d. as the decimal of £1.

£7, 11s. $8\frac{1}{2}$d. = £7·585

The £7 of course remains a whole number.

The first digit to the right of the point is *tenths*, so we must find how many tenths of £1, or *florins*, are in the sum: 11s. ÷ 2 = 5 fl. + 1s.

Reduce now the 1s. left over and $8\frac{1}{2}$d. to farthings: 1s. $8\frac{1}{2}$d. = 82 farthings; adding to this 1 for every 24 (82 ÷ 24 = 3+) we have 82 + 3 = 85: these give the correct figures in the second and third places of decimals.

The third place of decimals is *thousandths*, but farthings are *960ths* of £1 ; so it is necessary to change 960ths to thousandths before putting down as decimals.

$$960 \div 40 = 24$$
$$1000 \div 40 = 25$$

so every 24 farthings must be increased to 25, or the number of farthings increased by $\frac{1}{24}$.

Conversely in expressing third place decimals as farthings we must change thousandths to 960ths—that is, we make 25 thousandths 24 farthings, or decrease the thousandths by $\frac{1}{25}$.

If when dividing the number of farthings by 24 the remainder is more than 12, it will be less of an error to *add 1 to the number of times* than to take the number found in dividing, as we have already seen in dealing with other approximations.

Ex. 2. Express £7, 11s. 9d. as the decimal of £1.

£7, 11s. 9d. = £7 + 5 fl. + (1s. 9d. =) 84 farthings
= 7·5875 ($84 \div 24 = 3\frac{12}{24}$)

When a calculation has been made by decimalising the money, it remains, of course, **to express the decimal result as £ s. and d.**

This **is exactly the converse of the last.**

Ex. Find the value of £15·196.

The **1** nearest the decimal point is *florins* = 2s.

The **96** (2nd and 3rd places of decimals) represents the farthings + 1 for every 24. We take from 96 therefore 1 for every 25 ; $96 \div 25 = 3 + 21$ over. This is nearer 4 than 3 times, so we take 4 away (96 − 4) = 92 = 23d. = 1s. 11d.

Ans. £15·196 = £15, 3s. 11d. correct to nearest farthing.

EXERCISE 61.

To be worked mentally.

Express in £ s. d. to the nearest farthing :—

1. £·25, £·125, £·75, £·775.
2. £3·9176, £2·538, £1·066, £8·888.
3. £·123, £2·345, £·89176, £8·177.
4. £15·9876, £159·876, £1·59876, £·1598.

Express in £ to the nearest third-place decimal :—

5. £3, 11s. 6d., 8s. 6d., 2s. 6d., 9d., 1s.
6. £1, 14s. 7½d., £2, 10s. 3d., £4, 1s. 1d., £9, 9s. 9¼d.
7. 10s., 5s., 2s. 6d., 1s. 3d., 7½d., 2s., 1s., 6d., 3d.
8. £5, 17s. 8d., £4, 14s. 5¾d., £1, 9s. 6¼d., £1, 0s. 5¾d.

9. Explain how to express a given sum of money at sight as a decimal of £1, taking as examples 13s. 5¼d. and 1s. 0¼d., and writing down each as a decimal, correct to the third place.

10. Express each of the following in £ to 3 decimal places, and then add them together: £307, 2s. 9½d.; £5, 0s. 1¼d.; ·74 of a shilling; £200; 3s. 3½d.; 2s. 6½d.; ·003 of £1.

11. Find the value of £7·134225 + 8·903s. + 10·2d.

12. Add together ·2625 of £1, ·0625 of 13s. 4d., and ·825 of 9d.

When dealing with the decimalising of money correct to three places it will be remembered we had to divide by 24 at one stage.

If in dividing by 24 the remainder is 12, a **5** in the fourth place expresses it exactly; if 6 is the remainder **25** are the correct digits for the fourth and fifth places, and **75** in these places is the exact equivalent when the remainder is 18. So also 3 over gives **125**, 9 over gives **375**, 15 over gives **625**, and 21 over gives **875** in the fourth, fifth, and sixth places.

In many cases it will be sufficient to remember these, but it is found helpful also to make use of **a table of decimals** for all the remainders from 1 to 23.

1	for remainder	= ·000041$\dot{6}$	13	for remainder	= ·000541$\dot{6}$
2	,,	= ·0008$\dot{3}$	14	,,	= ·00058$\dot{3}$
3	,,	= ·000125	15	,,	= ·000625
4	,,	= ·001$\dot{6}$	16	,,	= ·000$\dot{6}$
5	,,	= ·00208$\dot{3}$	17	,,	= ·00070$\dot{8}\dot{3}$
6	,,	= ·00025	18	,,	= ·00075
7	,,	= ·00291$\dot{6}$	19	,,	= ·00079$\dot{1}\dot{6}$
8	,,	= ·000$\dot{3}$	20	,,	= ·0008$\dot{3}$
9	,,	= ·000375	21	,,	= ·000875
10	,,	= ·00041$\dot{6}$	22	,,	= ·00091$\dot{6}$
11	,,	= ·00458$\dot{3}$	23	,,	= ·00095$\dot{8}\dot{3}$
12	,,	= ·0005			

COMPOUND MULTIPLICATION BY DECIMALS.

It must be remembered that it is **the answer that is to be correct to three places**, and a very small error may grow to be a very decided one when we multiply it by a large multiplier.

Ex. Find the cost of 715 articles at £3, 8s. 9½d. each.

£3, 8s. 9½d. = £3·439583

Suppose we take £3·4396 as being *very nearly correct*, it will be seen how the error has increased in the answer.

	3·4396	3·439583 | 3
Reverse multiplier	517	517
and work as for *four* places correct, to get carrying figure.	17·1980 34·3960 2407·7200	17·1979 34·3958 2407·7083
	2459·314	2459·302
Ans.	£2459, 6s. 3½d.	**Ans.** £2459, 6s. 0½d.

The answer in the second working is exactly what would be got by doing the exercise by compound multiplication; but it will be seen that the *very small* error started with in the first working has grown to 3d. in the answer.

EXERCISE 62.

Find the answers to the following by applying the rules and table for the decimalisation of money; and using contracted multiplication (correct to 3 places) of decimals.

1. What will 146 tons cost at £20, 7s. 1½d. a ton?
2. Find price of 3725½ tons at £7, 13s. 4½d. a ton.
3. I bought 9124 tons at £84, 19s. 1¼d. per ton; what should I pay?
4. What is the cost of 5625 yards of silk at £1, 17s. 8½d. a yard?
5. What is the value of 383 butts of wine at £905, 12s. 3¼d. each?
6. Required the value of 1147⅝ yards of cloth at £1, 2s. 7½d. and £1, 4s. 8¼d. a yard.
7. Find the cost of 1957 articles at £5, 3s. 11½d. each.
8. Find the cost of 2265 ounces of gold at £3, 10s. 4¾d. per ounce.
9. What is the cost of 3107 lambs at £1, 14s. 7½d. each?
10. What is the total value of 7425 ponies at £31, 6s. 5½d. each?
11. If one article cost £15, 19s. 9¾d., what will 8109 articles cost?
12. What is the value of 9213 tons at £20, 0s. 10½d. per ton?
13. If a bankrupt can only pay 14s. 5¾d. in the £1, and his debts amount to £4809, what were his assets?
14. Find the cost of 6090 articles at 19s. 0½d. each.
15. Find the cost of 8009 articles at 10s. 10½d. each.
16. If one ton of goods cost £91, 1s. 9d., what is the cost of 3871¾ tons?
17. What is the value of 5467⅜ tons at £19, 0s. 10½d. each?
18. Find the cost of 7215$\frac{9}{10}$ articles at £5, 12s. 11¾d. each.

COMPOUND DIVISION BY DECIMALS.

For division, decimalise the money as for multiplication and then use contracted division of decimals.

Ex. Divide £731, 10s. 7¼d. by 85·643.

If we wish to get the answer *exact* we must work for correctness to four places; if correctness to pence only is wanted, then three places will be enough.

£731, 10s. 7¼d. = £731·530

£8, 10s. 10d. = 000008·5416 **Ans.**

Moving the point we have 8′5′6′4′3·) 731530·

46386
3565
140
54
3

There are 5 digits in the divisor, so 4 places will be got after the contracted method begins. Begin the contracted method, therefore, in this case as soon as the decimal point is reached.

EXERCISE 63.

Decimalise the money and use contracted division of decimals in the following:—

(1) £713, 5s. 10d. ÷ 53, 73, 83

(2) £376, 7s. 5d. ÷ 73, 65, 59

(3) £282, 5s. 6½d. ÷ 43, 52, 67

(4) £2076, 13s. 5¼d. ÷ 87, 93, 95

(5) £2124, 8s. 2¾d. ÷ 89, 76, 78

(6) £246, 17s. 6¼d. ÷ 259, 333

(7) £3672, 14s. 4¾d. ÷ 273, 493

(8) £4681, 10s. 6¼d. ÷ 934, 987

(9) £5624, 16s. 8¼d. ÷ 497, 873

(10) £5019, 13s. 0¼d. ÷ 1979, 2435

(11) £10835, 12s. 3¾d. ÷ 1773, 5867

(12) £9450, 14s. 6¾d. ÷ 2279, 3981

(13) If 11 yards of cloth cost £4, 5s. 0¼d., what is the price of 1 yard?

(14) A person sold 9 pieces of lawn, each 11 yards, for £38, 5s. 2¼d.; what was the price per yard?

(15) If 705 lb. tea cost £34, 10s. 3¾d., what is it per lb.?

(16) If a farm of 57 acres is let at £55, 4s. 4½d., what is the rent per acre?

(17) Divide £3, 10s. among 5 men and 6 women, and give each man thrice the share of a woman.

(18) If 37 yards cloth cost £23, 13s. 3½d., what is the price of 1 yard?

RULES FOR MENTAL OR SHORT CALCULATION OF PRICES.

1. To find the price of **a dozen** when the price of one is given, reduce the price to pence, call these shillings, and for every farthing in the price add 3d.

Ex. 1 doz. at 2s. 3½d. each.
2s. 3½d. = 27½d. **Ans. £1, 7s. 6d.**

2. To find the price of **2 dozen** when the price of one is given, reduce the price to pence, call these florins, and for every farthing more add 6d.

Ex. 2 dozen at 1s. 4¼d. each.
1s. 4¼d. = 16¼d. **Ans.** 16fl. 6d. = £1, 12s. 6d.

3. To find the price of **4 dozen** when the price of one is given, reduce the price to farthings and call these shillings.

Ex. 4 dozen at 1s. 2½d. each.
1s. 2½d. = 14½d. = 58f. **Ans. £2, 18s.**

4. To find the price of a **gross** when the price of one is given, reduce the price to pence, multiply by 12 and call the result shillings, and to this add 3s. for each farthing in the price given.

Ex. 1 gross at 10½d. each.
10d. × 12 = 120d. **Ans.** 120s. + 6s. = £6, 6s.

5. To find the price of a **score** when the price of one is given, reduce the price to shillings, call these pounds, and for every 3d. in the price add 5s.

Ex. A score at £2, 3s. 7½d. each.
£2, 3s. 7½d. = 43s. + 2½ threepences. **Ans. £43,** 12s. 6d.

6. To find the price of **100** when the price of one is given, reduce the price to farthings, call these pence, and take twice as many shillings; for $100 = 4 + ((12 \times 4) \times 2)$.

Ex. 100 at 3½d. each.
3½d. = 14f. **Ans.** 28s + 14d. = £1, 9s. 2d.

7. To find the price of **240** when the price of one is given, reduce the price to pence, call these pounds, and for every farthing in the price add 5s.

Ex. 240 at 3s. 6½d. each.

3s. 6½d. = 42½d. **Ans.** £42, 10s.

8. To find the price of **480** when the price of one is given, reduce the price to halfpence, call these pounds, and for the farthing, should there be one, add 10s.

Ex. 480 at 2s. 1¾d. each.

2s. 1¾d. = 25d. + 3f. = 51 halfpence + 1f. **Ans.** £51, 10s.

9. To find the price of **960** when the price of one is given, reduce the price to farthings and call these pounds.

Ex. 960 at 2s. 1½d. each.

2s. 1½d. = 25½d. = 102f. **Ans.** £102.

10. To find the price of **252** when the price of one is given, reduce the price to pence, call these guineas, and for each farthing in the price add 5s. 3d.; for 252 pence = 1 guinea.

Ex. 252 at 3s. 4½d. each.

3s. 4½d. = 40½d.

Ans. 40 guin. + 10s. 6d. = £42, 10s. 6d.

11. To find the price of **any power of 10** when the price of one is given, express the price as the decimal of £1, and for every cipher in the number move the point one place to the right; convert the answer to £ s. d.

Ex. 1000 at 9s. 6d. each.

9s. 6d. = ·475 **Ans.** £475.

12. To find the price of any number **near any of the above mentioned,** find the price as shown for that number and add or subtract the price of the odd articles.

Ex. 11 at 1s. 2½d. each.

1 dozen at 1s. 2½d. each cost 14s. 6d.

14s. 6d. − 1s. 2½d. = **13s. 3½d. Ans.**

13. To find the price of **1 cwt.** when the price of 1 lb. is given, find, as shown, the price of 100 lb. and of 1 doz. lb. and add results for the answer.

Ex. 1 cwt. at 3½d. a lb.

100 lb. at 3½d.	=	£1, 9s. 2d.
1 doz. lb. at 3½d.	=	3s. 6d.
1 cwt. at 3½d. a lb.	=	**£1, 12s. 8d. Ans.**

14. To find the price of **1 lb.** when the price of 1 ounce is given, reduce the price to farthings, multiply by 4, and the result gives the answer in pence.

Ex. 1 lb. at 2¾d. an ounce.
2¾d. = 11f. ; 11 × 4 = 44d. = **3s. 8d. Ans.**

15. To find income for a **year** given the income for a day, reduce the daily income to pence, take that number of sovereigns and of half-sovereigns and 5 times as many pence, add these for the answer. For, 365 pence = 240d. + 120d. + 5d. = £1 + 10s. + 5d.

Ex. What is my yearly income at 7s. 6d. a day?
7s. 6d. = 90d.
Ans. = £90 + 90 half-sovs. (£45) + 450d. (£1, 17s. 6d.) = £136, 17s. 6d.

16. To find wages for a year given the wage earned each working day (**313**, excluding Sundays), reduce the daily wage to pence, take that number of sovereigns, of crowns, of shillings, and of pence, and add these for the answer. For, 313 pence = 240d. + 60d. + 12d. + 1d. = £1 + 5s. + 1s. + 1d.

Ex. A labourer earns 4s. 6d. a day, what will he earn a year?
4s. 6d. = 54d.
Ans. = £54 + 54 crowns (£13, 10s.) + 54s. (£2, 14s.) + 54d. (4s. 6d.) = £70, 8s. 6d.

17. To find the price of a number which is a **multiple of any of the above mentioned**, proceed as for that number, and multiply the result by the given multiple for the answer.

Ex. 5 score at £1, 3s. 4d. each.
£1, 3s. 4d. = 23⅓s., therefore 1 score costs £23, 6s. 8d.
£23, 6s. 8d. × 5 = **Ans.** £116, 13s. 4d.

18. To find the price of a number which is a **factor of any of the above mentioned**, proceed as for that number, and divide the result by the number of times the factor is contained in the number.

Ex. 120 at 2s. 3d. each.
2s. 3d. = 27d. therefore price of 240 is £27
£27 ÷ 2 = £13, 10s. **Ans.**

19. To find the price of any number of articles when the price of one is an **aliquot part of £1** (that is, is contained in £1 an exact number of times), divide the number of articles

by the number of times the price is contained in £1 and call the result pounds.

The pupil should make and learn a table of the aliquot parts of £1, like this—

10s.	$= \frac{1}{2}$ of £1	4s.	$= \frac{1}{5}$ of £1
6s. 8d.	$= \frac{1}{3}$ of £1	3s. 4d.	$= \frac{1}{6}$ of £1
5s.	$= \frac{1}{4}$ of £1	&c.	&c.

Ex. 865 articles at 3s. 4d. each.

£1 ÷ 3s. 4d. = 6; 865 ÷ 6 = £144$\frac{1}{6}$ **Ans.** £144, 3s. 4d.

20. To find the price of any number of articles when the price of one is a **multiple of some aliquot part of £1**, proceed as for the aliquot part and multiply the result by the number corresponding to the multiple.

Ex. 340 articles at 7s. 6d. each.

7s. 6d. = 2s. 6d. × 3 = $\frac{3}{8}$ of £1

340 ÷ 8 = £42$\frac{1}{2}$ price at $\frac{1}{8}$

£42$\frac{1}{2}$ × 3 = £127, 10s. **Ans.**

Conversely, to find the price of one article.

(1) Given price of a dozen—call shillings in price, pence.

(2) ,, ,, 2 ,, ,, florins ,, ,, pence.

(3) ,, ,, 4 ,, ,, shillings ,, ,, farthings.

(4) ,, ,, 1 gross ,, shillings ,, ,, pence and divide by 12.

(5) ,, ,, a score ,, pounds in price, shillings.

(6) ,, ,, 100 reduce price to pence and mark off two places.

(7) ,, ,, 240 call pounds in price pence.

(8) ,, ,, 480 ,, pounds ,, halfpence.

(9) ,, ,, 960 ,, pounds ,, farthings.

(10) ,, ,, 252 ,, guineas ,, ,, pence.

(11) ,, ,, any power of 10 reduce price to pence and mark off as many places as there are 0's in the power.

(12) ,, ,, 1 lb. divide pence in the price by 4, the result is the price of 1 oz. in farthings.

EXERCISE 64.

Calculate *mentally, as far as possible*, the price of—

(1) 1 dozen articles at 5d.; 10$\frac{1}{4}$d.; 1s. 3d.; 2s. 8d. each.

(2) 2 ,, ,, 9d.; 7$\frac{3}{4}$d.; 2s. 4$\frac{1}{2}$d.; 5s. 8$\frac{3}{4}$d. ,,

(3) 4 ,, ,, 4d.; 3$\frac{3}{4}$d.; 10$\frac{1}{2}$d.; 1s. 10$\frac{3}{4}$d. ,,

(4) 1 gross articles at $\frac{3}{4}$d.; $1\frac{1}{2}$d.; $8\frac{1}{4}$d.; 1s. 3d. each.
(5) 1 score ,, 7s.; 5s. 3d.; $7\frac{1}{2}$d.; 2s. $4\frac{1}{2}$d. ,,
(6) 100 ,, 7d.; $9\frac{1}{4}$d.; 2s. 3d.; 19s. 11d. ,,
(7) 240 ,, 8d.; $7\frac{3}{4}$d.; 1s. $11\frac{1}{4}$d.; 5s. $7\frac{1}{4}$d. ,,
(8) 480 ,, $1\frac{1}{2}$d.; 6d.; 1s. $3\frac{1}{2}$d.; 1s. $6\frac{3}{4}$d. ,,
(9) 960 ,, $\frac{3}{4}$d.; 3d.; $9\frac{1}{2}$d.; 1s. $1\frac{1}{4}$d. ,,
(10) 252 ,, 3d.; $9\frac{1}{2}$d.; 1s. 3d.; 1s. $5\frac{1}{4}$d. ,,
(11) 1000 ,, £12, 10s.; £3, 7s. 6d.; £4, 2s. 9d. each.
(12) 99 ,, $\frac{1}{4}$d.; $1\frac{1}{2}$d.; $2\frac{3}{4}$d. each.
(13) 1 cwt. at 3d.; 11d.; $2\frac{1}{2}$d.; 1s. $2\frac{1}{4}$d. a lb.
(14) 1 lb. at $1\frac{1}{2}$d.; $3\frac{3}{4}$d.; $5\frac{1}{2}$d.; 1s. $1\frac{1}{4}$d. an oz.
(15) Find the yearly salary at 4s.; 6s.; 7s. 6d. a day.
(16) Find the year's earnings at 1s. 3d.; 2s. 6d.; 3s.; 4s. 6d. each working day.
(17) Find the price of 8 dozen at $2\frac{1}{4}$d.; $1\frac{1}{2}$d.; $5\frac{3}{4}$d.; 1s. $6\frac{3}{4}$d. each.
(18) ,, ,, 5 cwts. at 3s.; 5s.; 10s. 6d. a ton.
(19) ,, ,, 432 articles at 1s. 3d.; 1s. 8d.; 2s. 6d.; 3s. 4d. each.
(20) ,, ,, 54 articles at 5s.; 6s. 8d.; 8s. each.
(21) ,, ,, 1 article at 2s. 3d.; 5s. 9d.; 27s. 6d.; £2, 4s. 3d. a dozen.
(22) ,, ,, 1 cwt. at £2; £2, 15s. a ton.
(23) ,, ,, 1 article at £5; £14, 15s. for 240.
(24) ,, ,, 1 article at £1, 4s.; 18s.; 15s. a gross.

PRACTICE.

An **aliquot part** of any quantity is a **smaller quantity** which **is contained in the larger an exact number of times**.

Thus 1s. ÷ 3d. = 12d. ÷ 3d. = 4, therefore 3d. (= $\frac{1}{4}$ of 1s.) is an **aliquot part** of 1s.

So 5s. (= $\frac{1}{4}$ of £1); 6s. 8d. (= $\frac{1}{3}$ of £1), &c., are **aliquot parts** of £1 which contains them exactly.

Practice is compound multiplication carried through by means of aliquot parts.

It is called **simple** or **compound** according as the multiplier in the exercise is of **one denomination** or of **several denominations**.

Thus, "Find the weight of 316 trucks weighing 8 tons 13 cwt. 2 qr. each," is an exercise in **simple practice**, while "What will 8 tons 13 cwt. 2 qr. cost at £1, 3s. 6d. a ton?" is an exercise in **compound practice**.

Simple practice is finding the cost (or weight, length, &c.) **of a certain number of articles at a given price** (or weight, length, &c.) **for each.**

The method, as will be seen from the example below, is to take some unit and divide the price into convenient aliquot parts (there may be a multiple to begin with) of that unit and of each succeeding aliquot part. The price of the given number of articles at each of these partial prices is found by multiplying the price at the unit, or other price in which this particular one is contained, by the fraction representing the aliquot part.

Ex. Find the cost of 274 articles at £3, 17s. 6d. each.

Taking £1 as the unit—£274, 0s. 0d. = cost of 274 articles at £1 each

£3 = 3 times £1	× 3	£822, 0s. 0d. = cost of 274 arts. at £3,	0s. 0d. each
10s. = $\frac{1}{2}$ of £1	÷ 2*	£137, 0s. 0d. = ,, ,, ,,	£0, 10s. 0d. ,,
5s. = $\frac{1}{2}$ of 10s.	÷ 2	£68, 10s. 0d. = ,, ,, ,,	£0, 5s. 0d. ,,
2s. 6d. = $\frac{1}{2}$ of 5s.	÷ 2	£34, 5s. 0d. = ,, ,, ,,	£0, 2s. 6d. ,,

Summing all partial prices—

£1061, 15s. 0d. = cost of **274** articles at **£3, 17s. 6d.** each

The above example might be worked more shortly by considering £3, 17s. 6d. = £4, *minus* 2s. 6d.

£274, 0s. 0d. = cost of 274 articles at £1, 0s. 0d. each

£4 = 4 times £1	× 4	£1096, 0s. 0d. = cost of 274 arts. at £4,	0s. 0d. each
2s. 6d. = $\frac{1}{8}$ of £1	÷ 8*	£34, 5s. 0d. = ,, ,, ,,	£0, 2s. 6d. ,,

By complementary addition—

£1061, 15s. 0d. = cost of **274** arts. at **£3, 17s. 6d.** each

Aliquot Parts. In order to be able to choose suitable instalments for easy calculation, the learner should make for himself **tables of aliquot parts** like the following for £1, 10s., 6s. 8d., 5s., 4s., 2s. 6d., 1s., and become quite familiar with their contents.

s.	d.				s.	d.			
10	0	=	$\frac{1}{2}$	of £1.	2	6	=	$\frac{1}{8}$	of £1.
6	8	=	$\frac{1}{3}$	,,	2	0	=	$\frac{1}{10}$	,,
5	0	=	$\frac{1}{4}$	,,	1	8	=	$\frac{1}{12}$	,,
4	0	=	$\frac{1}{5}$	,,	1	4	=	$\frac{1}{15}$	,,
3	4	=	$\frac{1}{6}$	,,	1	3	=	$\frac{1}{16}$	,,

* Care should be taken to divide price at the sum of which the money being dealt with is the aliquot.

s.	d.				s.	d.			
1	0	=	$\frac{1}{20}$	of £1.	1	8	=	$\frac{1}{6}$	of 10s.
0	10	=	$\frac{1}{24}$	,,	1	3	=	$\frac{1}{8}$	,,
0	8	=	$\frac{1}{30}$	,,	1	0	=	$\frac{1}{10}$	,,
0	6	=	$\frac{1}{40}$	,,	0	10	=	$\frac{1}{12}$	,,
0	4	=	$\frac{1}{60}$	,,	0	8	=	$\frac{1}{15}$	,,
0	3	=	$\frac{1}{80}$	,,	0	6	=	$\frac{1}{20}$	,,
0	2	=	$\frac{1}{120}$	,,	0	4	=	$\frac{1}{30}$	,,
0	1	=	$\frac{1}{240}$	,,	0	3	=	$\frac{1}{40}$	,,

5	0	=	$\frac{1}{2}$	of 10s.	3	4	=	$\frac{1}{2}$	of 6s. 8d.
3	4	=	$\frac{1}{3}$	,,	1	8	=	$\frac{1}{4}$	,,
2	6	=	$\frac{1}{4}$	,,	0	10	=	$\frac{1}{8}$	,,
2	0	=	$\frac{1}{5}$	,,				&c.	

Ex. 1. *Find the value of 365 articles at 1s. 10½d. each.*

		365s.	= cost of 365 articles at 1s. each
6d. = ½ of 1s.	÷ 2	182s. 6d. =	,, ,, ,, 6d. ,,
3d. = ½ ,, 6d.	÷ 2	91s. 3d. =	,, ,, ,, 3d. ,,
1½d. = ½ ,, 3d.	÷ 2	45s. 7½d. =	,, ,, ,, 1½d. ,,

£34, 4s. 4½d. = cost of 365 arts. at 1s. 10½d. each

A line is not drawn below the cost at 1s., for that amount is to be taken as part of the total cost.

Ex. 2. Find the weight of 126 bales, each weighing 13 cwt. 3 qr. 19 lb.

	Cwt.	qr.	lb.		Cwt.	qr.	lb.
Taking 1 cwt. as the unit of weight	126	0	0	= weight of 126 bales of 1	0	0	ea.

13 cwt. = 13 times 1 cwt.	× 13	1638	0	0	= weight of 126 bales of 13	0	0	ea.
2 qr. = ½ of 1 cwt.	÷ 2	63	0	0	= ,, ,, ,,	0	2	0 ,,
1 qr. = ½ of 2 qr.	÷ 2	31	2	0	= ,, ,, ,,	0	1	0 ,,
14 lb. = ½ of 1 qr.	÷ 2	15	3	0	= ,, ,, ,,	0	0	14 ,,
4 lb. = ¼ of 1 qr.	÷ 7	4	2	0	= ,, ,, ,,	0	0	4 ,,
1 lb. = ¼ of 4 lb.	÷ 4	1	0	14	= ,, ,, ,,	0	0	1 ,,

Ans. 1753 3 14 = weight of 126 bales of 13 3 19 ea.

When there is a **vulgar fraction** in the quantity to be valued, we either proceed as above shown and add in the

value of the given vulgar fraction at the given price, or **take this fraction of the unit** and work thus :—

Ex. Find the price of $749\frac{6}{11}$ cwt. at 11s. 8d. per cwt.

Taking £1 as the unit, $\frac{6}{11}$ of £1 = 10s. 10·90d.

Decimals should always be used in working beyond pence, two places being sufficient.

	£	s.	d.		£	s.	d.	
	749	10	10·90	= price of $749\frac{6}{11}$ cwt. at	1	0	0	each
10s. = $\frac{1}{2}$ of £1 ÷ 2	374	15	5·45	= price of $749\frac{6}{11}$ cwt. at	0	10	0	each
1s. 8d. = $\frac{1}{6}$ of 10s. ÷ 6	62	9	2·91	= ,, ,, ,,	0	1	8	,,
	437	4	8·36	= price of $749\frac{6}{11}$ cwt. at	0	11	8	each

Ans. (correct to nearest farthing), **£437, 4s. 8¼d.**

Simple Practice by Decimals. The above example could have been worked even more simply by expressing $£\frac{6}{11}$ as a decimal.

Thus $£\frac{6}{11} = £\cdot\dot{5}\dot{4}$.

	£	s.	d.	
£749·5454 = price of $749\frac{6}{11}$ cwt. at	1	0	0	each
10s. = $\frac{1}{2}$ of £1 ÷ 2	374·7727	= price of $749\frac{6}{11}$ cwt. at	0	10	0	,,
1s. 8d. = $\frac{1}{6}$ of 10s. ÷ 6	62·4621	= ,, ,, ,,	0	1	8	,,
	437·2348	= price of $749\frac{6}{11}$ cwt. at	0	11	8	,,

Ans. (correct to nearest farthing) **£437, 4s. 8¼d.**

EXERCISE 65.

(Answers, to the nearest farthing.)

Work, by the shortest *practice* method, the following :—

(1) Find the cost of 795 articles at £0, 3s. 9d. each
(2) ,, ,, 478 ,, £0, 11s. 8d. ,,
(3) ,, ,, 947 ,, £0, 13s. 4d. ,,
(4) ,, ,, 576 ,, £0, 11s. 3d. ,,
(5) ,, ,, 749 ,, £0, 5s. 10½d. ,,
(6) ,, ,, 654 ,, £0, 11s. 7¾d. ,,
(7) ,, ,, 586 ,, £2, 17s. 4d. ,,
(8) ,, ,, 496 ,, £1, 14s. 8d. ,,

(9) Find the weight of 216 articles weighing 6 oz. 17 dwt. 7 gr. each
(10) ,, ,, 64 ,, ,, 40 oz. 5 dwt. 16 gr. ,,
(11) ,, ,, 40 ,, ,, 3 tons 14 cwt. 3 qr. ,,
(12) ,, ,, 832 ,, ,, 7 cwt. 3 qr. 27 lb. ,,
(13) ,, ,, 37 ,, ,, 23 lb. 6 oz. 8 dr. ,,
(14) ,, ,, 313 ,, ,, 62 lb. 7 oz. 4 dr. ,,
(15) ,, length of 189 wires measuring 7 yd. 1 ft. 8 in. ,,
(16) ,, area of 84 plots ,, 5 ac. 3 ro. 11 po. ,,
(17) Find the cost of $794\frac{9}{10}$ articles at £2, 5s. 10d. each
(18) ,, ,, $823\frac{7}{16}$,, £1, 9s. 4d. ,,
(19) ,, ,, $273\frac{7}{16}$,, £1, 3s. 9d. ,,
(20) ,, ,, $423\frac{5}{8}$,, £0, 17s. $11\frac{1}{4}$d. ,,
(21) ,, ,, $342\frac{3}{10}$,, £0, 5s. $3\frac{1}{4}$d. ,,
(22) ,, ,, $827\frac{3}{5}$,, £0, 19s. $2\frac{1}{2}$d. ,,
(23) ,, ,, $286\frac{4}{5}$,, £0, 12s. $9\frac{1}{2}$d. ,,
(24) ,, ,, $889\frac{5}{8}$,, £2, 14s. $7\frac{3}{4}$d. ,,

Compound Practice. In compound practice the **quantity**, the price of which has to be found, is Compound, that is, **consists of more than one denomination.**

The quantity, of which the price is given, is taken as the unit, and the quantity, whose value is to be found, is broken up into aliquot parts.

Ex. Find cost of 7 tons 13 cwt. 3 qr. at 18s. 4d. a ton.

		£ s. d.		Tons cwt. qr.
Taking 1 ton as the unit - -		0 18 4	= price of 1	0 0
7 tons = 7 times 1 ton -	× 7	6 8 4	= price of 7	0 0
10 cwt. = $\frac{1}{2}$ of 1 ton -	÷ 2	0 9 2	= ,,	0 10 0
2 cwt. = $\frac{1}{5}$ of 10 cwt. -	÷ 5	0 1 10	= ,,	0 2 0
1 cwt. = $\frac{1}{2}$ of 2 cwt. -	÷ 2	0 0 11	= ,,	0 1 0
2 qr. = $\frac{1}{2}$ of 1 cwt. -	÷ 2	0 0 5·5	= ,,	0 0 2
1 qr. = $\frac{1}{2}$ of 2 qr. - -	÷ 2	0 0 2·75	= ,,	0 0 1
		£7 0 11·25	= price of 7	13 3

Ans. £7, 0s. $11\frac{1}{4}$d.

Questions in compound practice, like the above, can also be worked as simple practice by **expressing the compound quantity as a decimal of the unit, and taking aliquots of the price.**

When this method is taken the working can often be shortened, for any even number of shillings can be taken at once as so many tenths of £1 instead of having to be broken up into aliquots.

Ex. Find the cost of 3 cwt. 2 qr. 21 lb. at £4, 17s. 8d. per cwt.

3 cwt. 2 qr. 21 lb. = 3·6875 cwt.

				£	s.	d.
		£3·6875	- price at	1	0	0
£4 = 4 times £1 -	× 4	14·75	- - price at	4	0	0
16s. = $\frac{8}{10}$ of £1 -	× 8 and set digits one place farther to the right.	2·95	- - - ,,	0	16	0
1s. 8d. = $\frac{1}{12}$ of £1	÷ 12	·3073 (nearly)	,,	0	1	8
		£18·0073	- price at	4	17	8

Ans. £18, 0s. 1¾d. (correct to nearest farthing).

Decimalisation of weights and measures can often be done by inspection or by such short methods as the following :—

Ex. Express 32 cwt. 2 qr. 21 lb. as the decimal of 1 cwt.

Cwt.	qr.	lb.		Cwt.		Cwt.
32	0	0	=	32	=	32·
0	2	0	=	$\frac{1}{2}$	=	·5
0	0	14	=	$\frac{1}{8}$	=	·125
0	0	7	=	$\frac{1}{16}$	=	·0625
32	2	21				32·6875

or

28 | 7 21· lb.
4 | 3·
4 | 2·75 qr.
32·6875 cwt.

In any particular business where the same kind of quantities have to be frequently dealt with, tables should be constructed to save time and trouble.

Shipping agents, for instance, charge shippers either by weight (so much per cwt.), or by bulk (so much per cub. ft.), and for their calculations the tables here given are useful.

The method of construction is merely to express 1 unit of the lower denomination as a decimal of 1 unit of the higher. Thus 1 lb. = $\frac{1}{112}$ cwt. = ·0089285714 ; and 1 cub. in. = $\frac{1}{1728}$ cub. ft. = ·000578703. When this has been done the others are got by multiplying these decimals by 1, 2, 3, &c.

Tables for Expressing

(1) lbs. as decimal of 1 cwt.	(2) Cub. in. as decimal of cub. ft.
$1 = \cdot 0089\dot{2}85714\dot{2}$	$1 = \cdot 000578\dot{7}0\dot{3}$
$2 = \cdot 0178\dot{5}71428\dot{5}$	$2 = \cdot 00115\dot{7}4\dot{0}$
$3 = \cdot 0267\dot{8}57142\dot{8}$	$3 = \cdot 0017361\dot{1}$
$4 = \cdot 0357\dot{1}42857\dot{1}$	$4 = \cdot 0023\dot{1}4\dot{8}$
$5 = \cdot 0446\dot{4}28571\dot{4}$	$5 = \cdot 002893\dot{5}1\dot{8}$
$6 = \cdot 0535\dot{7}14285\dot{7}$	$6 = \cdot 003472\dot{2}$
$7 = \cdot 0625$	$7 = \cdot 004050\dot{9}2\dot{5}$
$8 = \cdot 0714\dot{2}85714\dot{2}$	$8 = \cdot 004629\dot{9}$
$9 = \cdot 0803\dot{5}71428\dot{5}$	$9 = \cdot 0052083\dot{3}$

Note.—The recurring figures have been indicated in the above tables, but without these the decimals have been carried far enough to make the results of calculations based on them sufficiently correct for most purposes.

In constructing such tables as the above it is unnecessary to go beyond 9, for the same digits which represent 3 also stand for 30, 300, 3000, &c., the point being moved one place farther to the right for each of these in turn.

Thus 7 lb. = ·0625 cwt.	2 cub. in. = ·00115 + cub. ft.
700 lb. = 6·25 cwt.	2000 cub. in. = 1·15 + cub. ft.
&c.	&c.

Ex. Express 16 cwt. 1 qr. 23 lb. as the decimal of 1 cwt.

16 cwt. 1 qr. 23 lb. = 16 cwt. 51 lb.
= $16 \cdot 4464\dot{2}85714\dot{2}$
$\cdot 0089\dot{2}85714\dot{2}$

$16 \cdot 4553\dot{5}71428\dot{5}$

Both the compound quantity and the price might be decimalised in the ways shown, and the exercises would then be worked as compound multiplication by decimals.

EXERCISE 66.

Work the following by the shortest *practice* method, and give answers **correct to the nearest farthing**.

(1) Find the cost of 18 cwt. 2 qr. 16 lb. at £3, 18s. 6d. per cwt.
(2) ,, ,, 21 cwt. 1 qr. 24 lb. at £4, 17s. 8d. per cwt.
(3) ,, ,, 9 cwt. 2 qr. 22 lb. at £2, 13s. 6d. per cwt.

(4) Find the cost of 2 qr. 27 lb. at £5, 15s. 9d. per cwt.
(5) ,, ,, 11 cwt. 13 lb. at £6, 18s. per cwt.
(6) ,, ,, 20 cwt. 3 qr. at £4, 17s. 8d. per cwt.
(7) ,, ,, 18 cwt. 3 qr. 4 lb. at £3, 18s. 6d. per cwt.
(8) ,, ,, 15 cwt. 3 qr. 18 lb. at £4, 11s. 9d. per cwt.
(9) ,, ,, 6 cwt. 2 qr. 7 lb. at £2, 19s. 3d. per cwt.
(10) ,, ,, 7 cwt. 3 qr. 18 lb. at 17s. 6½d. per cwt.
(11) ,, ,, 16 cwt. 2 qr. at £2, 6s. 11d. per cwt.
(12) ,, ,, 9 lb. 1 oz. 4 dwt. 18 gr. at £3, 12s. 6d. per lb.
(13) ,, ,, 4 lb. 6 oz. 15 dwt. at £4, 7s. 6d. per lb.
(14) ,, ,, 2 lb. 1 oz. 4 dwt. 18 gr. at £3, 12s. 6d. per lb.
(15) ,, ,, 18 lb. 1 oz. 16 dwt. 5 gr. at £4, 4s. 6d. per lb.
(16) ,, ,, 12 ac. 3 ro. 20 po. at 18s. 11d. an acre.
(17) ,, ,, 15 ac. 2 ro. 18 po. at 19s. 6d. an acre.
(18) ,, ,, 9 qr. 3 bush. 3 pk. 1 gal. at £4, 3s. 8d. per qr.
(19) ,, ,, 13 cwt. 2 qr. 14 lb. at £1, 17s. 4d. per cwt.
(20) ,, ,, 9 oz. 15 dwt. 23 gr. at £46, 14s. 6d. per lb.
(21) ,, ,, 17 ac. 2 ro. 20 po. at £6, 10s. per ac.
(22) ,, ,, 7 qr. 4 bush. 2 pk. at £2, 8s. per qr.
(23) ,, ,, 11 qr. 7 bush. 2 pk. at £2, 16s. 8d. per qr.
(24) ,, ,, 11 yd. 2½ qr. at 15s. 4d. per yd.

EXERCISE 67.

Problems to be worked by Practice Methods.

(*Answers to be correct to nearest farthing.*)

1. A bankrupt whose debts are £2016 offers a composition of 14s. 3¾d. in the £. Find his assets.
2. If the town taxes amount to 2s. 7½d. per pound of rent; what must a householder pay whose rent is £35?
3. A bankrupt's debts are £3753, 7s. 6d. What is the value of his estate if he can pay his creditors 14s. 6d. in the £?
4. How much did a man gain who bought 1350 articles at 3s. 7½d. each and sold them all at 5s. 1¼d. each?
5. The total number of depositors in a savings bank is 39033, and the value of the average deposit £7, 15s. 10½d. Find the total sum deposited.
6. An estate is planted with 429864 young trees at a cost of £1, 3s. 8d. per dozen; find the whole expense.

7. Find the total produce of 540 acres of wheat at 5 qr. 3 bush. 1 pk. per acre.

8. If one ton of goods cost £91, 1s. 9d., what is the value of a cargo of $3871\frac{5}{1\frac{1}{2}}$ tons?

9. Find the cost of $7215\frac{9}{1\frac{1}{0}}$ cwts. when the price per cwt. is £5, 12s. $11\frac{3}{4}$d.

10. What is the value of a nugget of gold weighing 1 oz. 13 dwt. 16 gr. when gold is worth £3, 17s. $10\frac{1}{2}$d. an ounce?

11. What is the value of the contents of a full jar containing 3 gall. 2 qt. 1 pt., if the cost is 18s. 6d. a gallon?

12. What should a shipper pay whose goods occupy 3 cub. yd. 1 cub. ft. 216 cub. in. of space, for which the charge is 5s. 6d. per cubic foot?

13. What should a man pay to buy outright a site whose area is 1 ro. 24 poles at £70, 11s. 8d. per acre?

14. Find the freight charge for 6 tons 1 cwt. 2 qr. 24 lb. of goods at £3, 16s. 9d. a cwt.

15. Find what should be paid for 15 lb. 6 oz. 12 drs. of Pekoe at 2s. 8d. a lb.

16. How much should a silversmith get for a silver epergne weighing 7 lb. 3 oz. 10 dwt. sold secondhand at 5s. per ounce?

17. What will a chemical manufacturer charge for 5 tons $16\frac{1}{2}$ cwt. of sulphate of ammonia at £19, 10s. a ton?

18. Find the rental of a farm of 375 ac. 2 ro. 30 po. at £3, 12s. 6d. an acre?

19. If the weight of the iron in the girders of a bridge is 3575 tons; what was the cost of the material at £3, 11s. $10\frac{1}{2}$d. a ton?

20. What is the weight of the cargo on a barge loaded with 562 sacks of flour, each weighing 1 cwt. 3 qr. 17 lb.?

21. If I am charged every year £2, 11s. 9d. per acre for my farm of 137·4375 acres; what is my yearly rent?

22. What is the weight of 200 bushels of oats, if the average weight of a bushel is 1 qr. 26 lb.?

23. If, in one year, 590767 ounces of gold coin were exported from the United Kingdom, find its value at £3, 17s. $10\frac{1}{2}$d. per ounce.

24. In the Edinburgh grain market 42915 quarters of wheat were sold at the average price of 73s. 7d. What was the value of the grain sold?

RATIO AND PROPORTION.

The relation which one number bears to another in respect of how often the one contains the other is called their **Ratio.**

The ratio of 9 to 4 is written 9 : 4 or $\frac{9}{4}$; and 9 in this relation is called the **antecedent** of the ratio, and 4 the **consequent.**

The mark : is used above as an alternative for the horizontal line separating numerator from denominator of a vulgar fraction. It is also therefore a **sign of division** when used to separate two numbers.

Quantities, as well as numbers, bear certain ratios to one another, but in order **to express the ratio in any particular case the quantities must be brought to the same denomination.**

Proportion. When four numbers are such that the ratio of the first to the second is equal to the ratio of the third to the fourth, they are said to be **in proportion** or to be **proportional.**

Since $\frac{9}{12}$ and $\frac{15}{20}$ both $= \frac{3}{4}$; the numbers 9, 12, 15, and 20 are said to be proportional, and this is expressed thus :—

$$9 : 12 = 15 : 20, \text{ or } 9 : 12 :: 15 : 20$$

These numbers are called the **terms** of the proportion; the first and the last terms are called the **extremes,** and the second and third (counting from left) the **means.**

When four numbers are proportional *the product of the means is equal to the product of the extremes.*

In what is commonly termed **Simple Proportion** a number or quantity has to be found to which a given number or quantity will bear a certain given ratio.

Ex. If 20 cwt. of rice cost £12, what will 35 cwt. cost?

The greater the quantity of the same material the greater will be the price, so whatever ratio 20 cwt. bears to 35 cwt. the same ratio will £12 bear to the number of pounds to be paid.

In **Compound Proportion** the ratio which the given number or quantity bears to the required number or quantity depends not on one but **several** given ratios.

Ex. If 8 labourers earn £14, 8s. in 12 days by working 10 hours a day, how much will 17 labourers earn by working 12 hours a day for 5 days?

The ratio which £14, 8s. bears to the sum to be earned, depends on the ratios of the different numbers of men, of the different numbers of days on which work was done, and on the varying number of hours worked a day.

Both simple and compound proportion problems can be solved by what is called the **Unitary Method**, by which the answer is reached by a series of elementary exercises.

Ex. If 13 bushels of oats serve 3 horses for 11 days, how many bushels will serve 7 horses for 12 days?

3 horses eat	13	bushels in	11 days
1 horse eats	$\frac{13}{3}$	,,	11 ,,
7 horses eat	$\frac{13 \times 7}{3}$	,,	11 ,,
7 ,, eat	$\frac{13 \times 7}{3 \times 11}$	,,	1 day
7 ,, eat	$\frac{13 \times 7 \times 12}{3 \times 11}$	,,	12 days

Ans. $\frac{13 \times 7 \times \overset{4}{\cancel{12}}}{\underset{1}{\cancel{3}} \times 11} = \frac{364}{11} = 33\frac{1}{11}$ **bushels.**

Cancelling should be resorted to whenever opportunity occurs, much time and labour being saved thereby.

The Unitary method of solution above exemplified is perhaps the simplest, but in some questions involving compound proportion it is cumbrous. It is strongly recommended that the modified form shown below should be used, the various steps in the work being done mentally.

Ex. If 15 mowers in 18 days of 10 hours can reap 15 fields, in how many days of 12 hours can 54 mowers cut 20 fields?

$$\frac{\overset{1}{\cancel{18}} \times \overset{1}{\cancel{15}} \times 10 \times \overset{5}{\cancel{20}}}{\underset{1}{\cancel{54}} \times \underset{3}{\cancel{12}} \times \underset{3}{\cancel{15}}} = \frac{50}{9} = 5\frac{5}{9} \text{ days}$$

The explanation will be more easily followed by the following vertical arrangement—the numerator being on the right-hand side of the vertical line.

Required term (always denominator)			No. to be put in numerator.
		18 days	
Mowers	54	15 *Less* time with 54 than 15	The *less*.
Hours	12	10 *Fewer* days of 12 hrs. than of 10	The *less*.
Fields	15	20 *More* time to reap 20 fields than 15	The *greater*.

A considerable number and variety of problems in Proportion are given in the following pages, but they are given as exercises in *thinking* and *cancelling*, and not because such will ever occur in business calculations. Expertness in cancelling will be found most serviceable in nearly every form of calculation.

EXERCISE 68.

Problems in Proportion to be solved by method shown.

1. An express train runs 40 ml. in 64 min.; how far will it run in 24 min.?
2. If 110 ac. of a West Indian plantation can produce 200 hhd. of sugar, find the produce of 176 ac.
3. If 48 reapers cut 20 ac. in a week; how many acres will 156 reapers cut in the same time?
4. If 20 reapers can cut a field in 6 days; in what time will 30 reapers do it?
5. If 42 men can do a work in 165 days; how many men will do it in 45 days?
6. If a steamer from Liverpool to Portland can make the passage of 2750 ml. in 11 days 6 hr.; in what time would the passage of 2980 ml. from Liverpool to New York be made?
7. If 3 qr. 5 lb. of tea cost £6, 6s. 1d.; find the price of 2 cwt. 1 qr. 20 lb. of the same kind.
8. If a traveller can drive between two towns 13 ml. apart in 1 hr. 25 min.; in what time can he drive 9 ml.?
9. In what time will an express train, which runs at the rate of 40 ml. an hour, traverse a distance which a parliamentary train, at 24 ml. an hour, runs in 3 hr. 15 min.?
10. If 1 cwt. 1 qr. 25 lb. of Mocha coffee may be had for £7, 4s. 4½d.; for what may 2 cwt. 3 qr. 11 lb. be obtained?
11. Find the sum on which a payment at the rate of 1s. 4d. per £ amounts to £19, 4s.
12. A landlord, after paying a tax of £2, 2s. 1d. on rent, had £98, 17s. 11d. over; what was the rate per £ of the tax?
13. If the shadow of a staff 3 ft. 7 in. high measures 4 ft. 9 in.; find the height of a steeple whose shadow at the same time is 158 ft. 4 in.
14. A farmer used stone weights of 26 lb. 8 oz. each instead of 28 lb.; what would 2 tons 13 cwt. of grain appear by these weights to be
15. A merchant used weights of 27 lb. 12 oz. instead of 28 lb.; find the true weight, which would appear 1 cwt. *more* by the false weights.

16. If $39\frac{3}{5\,6}$ cwt. of rice cost £$18\frac{9}{4\,0}$; how much rice can be bought with £$3\frac{1\,1}{4\,8}$?

17. If $93\frac{3}{4}$ yd. of damask cost $\frac{1}{4}$ of £$45\frac{2\,7}{8\,0}$; what will $113\frac{7}{1\,6}$ yd. of the same cost?

18. If 54 men can complete a work in $29\frac{5}{8}$ days; how many men will do it in $35\frac{1\,7}{2\,0}$ days?

19. For every $5\frac{1}{4}$ ml. that A walks, B goes $4\frac{5}{8}$ ml.; how long will B take to go a journey which takes A $6\frac{1}{2}$ hr.?

20. A train going at the rate of $25\frac{3}{8}$ ml. an hour runs a certain distance in $3\frac{1}{2}$ hr.; how long will another train whose rate is $24\frac{1}{2}$ ml. an hour take to run the same distance?

21. If 4·06 cwt. of rice cost £3·480; how much rice may be bought for £7·625?

22. A wall, whose height is 9·1875 ft., casts a shadow of 10·5 ft.; find the length of the shadow of a steeple 93·8 ft. high.

23. Suppose the ratio of the diameter to the circumference of a circle to be as 112 : 352; find the circumference of a fly-wheel 10 ft. in diameter.

24. A cistern can be filled by a pipe running $3\frac{7}{8}$ gal. per min. in 54 min.; in what time can it be filled by another running $4\frac{1}{2}$ gal. per min.?

25. If 300 labourers can make an embankment in 48 days; in how many *more* days will 60 *fewer* do it?

26. 77 tailors can execute an order of regimental clothing in 30 days; how many *more* must be engaged to fulfil the order 8 days *sooner?*

27. If 33 masons can build a wall in 47 days, and if, after working 11 days, 15 leave; in how many days *after* the 15 leave will it be finished?

    33 masons can *finish* the wall in 47 − 11 or 36 days. Since 15 masons *have left*, 18 *remain.*

    masons. days.

    Hence, 18 : 33 :: 36 : x = the number of days after the 15 have left.

28. If 17 men can do a work in 89 days, and if, after working 33 days, 3 men leave; in how many days in *all* will the work be done?

29. If 64 men can perform a work in 57 days, and if, after working for 12 days, notice is sent to finish the work 9 days *before* the stipulated time; how many *additional* men must be engaged?

30. If 3 men can do as much as 4 youths, and if 13 men can do a work in 9 days; in what time can 12 men *and* 8 youths do it?

    youths. men.
    $4 : 8 :: 3 : x = 6$

    $6 + 12 = 18$ men.

    men. days.
    $18 : 13 :: 9 : x$

31. If 3 families of 6 persons each consume 28 loaves in a week; how many will 9 families of 5 persons each consume in the same time?

32. A housekeeper, having used 6 pots of jelly with 14 loaves each 12 slices, wishes to know how many will be used with 8 loaves each 7 slices?

33. If 13 bush. of oats serve 3 horses for 11 days; how many bush. will serve 7 horses for 12 days?

34. If 6 boys are boarded for 10 months for £270; for what ought 13 boys to be boarded for 7 months?

35. If 8 labourers earn £14, 8s. in 12 days; what will 17 labourers earn in 5 days?

36. If 7 compositors set up 15 sheets in 6 days;
    (1) In how many days will 21 compositors set up 30 sheets?
    (2) How many sheets will 27 compositors set up in 14 days?
    (3) How many compositors will set up 25 sheets in 7 days?

37. If 36 labourers clear 513 yd. for a railway in 6 days;
    (1) How many will clear 3800 yd. in 10 days?
    (2) How many yards will be cleared by 156 labourers in 18 days?
    (3) In how many days will 16 labourers clear 190 yd.?

38. If 4 masons build 27 yd. of a wall in 5 days, working 9 hr. a day; in how many days will 32 masons build 81 yd. of a similar wall, working 10 hr. a day?

39. If 12 boys are boarded 10 months for £498; find the board of 18 boys for 9 months, supposing that the cost of boarding 4 of the former = that of 3 of the latter.

40. If £5 is sufficient to maintain 8 labourers for a fortnight when corn is at 28s. per qr.; how much will be required to maintain 6 labourers 29 days when corn is at 32s. per qr.?

41. If 6 bars of metal, 2 ft. long, 6 in. broad, and $3\frac{1}{4}$ in. thick, weigh 126 lb.; find the weight of 7 bars 3 ft. long, $4\frac{1}{3}$ in. broad, and 3 in. thick.

42. The weight of 35 cub. in. of gold, of which the sp. gr. is 19·258, is 355·270 oz. troy; find the weight of 49 cub. in. of silver, of which the sp. gr. is 10·474.

43. A slab of granite containing $3\frac{3}{10}$ cub. ft. weighs 541 lb.; find the weight of a piece of pumice stone containing $1\frac{3}{4}$ cub. ft., the sp. gr. of the former being to that of the latter as 175 to 61.

44. A contractor having engaged to lay 10 ml. of railway in 150 days, finds that 90 men have finished 3 ml. in 80 days; how many additional men must be engaged to finish it within the time?

45. A stabler lays in 80 bush. of oats to feed 15 horses for 16 days; at the end of 4 days he receives other 5 horses; how many additional bush. will be required for the given time?

46. If 236 men eat 160 qr. of wheat in 108 days; how many quarters will 76 men eat in a year and 67 days?

47. If a man travel 360 ml. in 12 days of 8 hr. each; how many miles will he travel in 60 days of 6 hr. each?

48. If 24 men perform a piece of work in 36 days of 12 hr. each; in what time will 30 men accomplish it, when the days are only 8 hr. long?

49. If 30 cwt. are carried 15 ml. for £5, 8s. 9d.; how many miles ought 90 cwt. to be carried for £29?

50. If the 9d. loaf weighs 3 lb. when wheat is at 10s. per bush.; what is the price per bush. when the penny loaf weighs 9 oz.?

51. If 18 men eat 16s. worth of bread in 3 days, when wheat is at 54s. per qr.; what value of bread will 45 men eat in 27 days, when wheat is at 45s. per qr.?

52. 8 men accomplished 30 yd. of ditching in 12 days, working 8 hr. per day; in what time will 12 men finish a ditch, supposing its whole length 60 yd., when they work only 6 hr. per day?

53. If 12 men build a wall 60 ft. long, 4 thick, and 20 in height in 24 days, working 12 hr. per day; what length of wall, 3 ft. thick and 12 high, will 18 men build in 18 days, working 8 hr. per day?

54. If $27\frac{1}{2}$ yd. of cloth, $\frac{1\,3}{1\,2}$ yd. wide, cost £10, 13s. 4d.; what will $7\frac{3}{4}$ yd. cost, when only $\frac{1\,1}{1\,2}$ yd. wide?

55. If the $2\frac{1}{2}$d. loaf weighs $8\frac{2}{5}$ oz., when wheat is at 5s.; what should the 6d. loaf weigh, when wheat is at 8s. 3d.?

56. If 63 cwt. be carried 42 ml. for £3, 10s., when the rate of carriage is $\frac{1}{2}$d. per mile per cwt.; what distance should 142 cwt. be carried for £8, 17s. 6d. when the rate is 1d. per mile per cwt.?

57. If 40 masons build a wall 56 yd. long in 10 days of $10\frac{1}{2}$ hr. each; how many hours a day must 60 masons work to build a wall 120 yd. long in 20 days?

58. If 120 men can dig a trench 150 yd. long, 4 yd. wide, and 2 deep, in $7\frac{1}{2}$ days of 10 hr. each; what length of a trench, 5 yd. wide and 3 deep, will 200 men dig in 15 days of 12 hr. each?

59. If 14 horses plough 112 acres in 40 days; how many horses would plough 64 acres in 16 days?

60. If 30 men earn £80, 14s. in 15 days; how many men will earn £107, 12s. in 12 days?

61. If 135000 bricks, 8 in. long, $3\frac{1}{2}$ in. broad, and $2\frac{3}{4}$ in. thick, be required to build the walls of a magazine; how many bricks, 14 in. long, 4 in. broad, and 3 in. thick, would be sufficient for the same?

62. If 7 compositors set up a volume of 12 sheets in 21 days of 12 hr. each; how many would be required to set up 3 volumes of 10 sheets in 35 days of 9 hr. each?

63. If 35 masons build 48 yd. of a wall which is to be 192 yd. long in 12 days of 12 hr. each; how many additional masons will be required to finish the wall in 18 days of 10 hr. each?

64. If 15 men build a wall, 80 ft. long, $3\frac{1}{2}$ ft. thick, and 9 ft. high, in 27 days of 10 hr. each; in how many days of 12 hr. each will 25 men build a wall 100 ft. long, $2\frac{1}{3}$ ft. thick, and 8 ft. high?

65. A garrison of 4050 men, with provisions for 5 months at the rate of 32 oz. a day for each, is reinforced by 750 men, and cannot be relieved for 8 months; how many ounces a day must each man be allowed that the provisions may last that time?

66. If 2 examiners working 7 hr. a day look over 420 papers in 12 days; how long, at the same rate of progress, will it take 10 examiners working 11 hr. a day to look over 880 papers?

67. If 18 men do a piece of work in 16 days working 10 hr. a day; how many men will do half as much again in 45 days working 8 hr. a day?

68. A locomotive engine making 162 strokes per minute, travels 90 ml. in 2 hr.; how many strokes per minute must the same engine make to do 200 ml. in $4\frac{1}{2}$ hr.?

69. If 15 men working $8\frac{1}{2}$ hr. a day can in $4\frac{3}{4}$ days excavate 114 yd., the substance being of 5 deg. of hardness; how many hours a day must 25 men be employed for $4\frac{1}{4}$ days to excavate 85 yd. where the material is of 11 deg. of hardness?

70. If 40 yd. of cloth, 70 in. wide, can be made from 60 lb. of wool; what will be the width of a piece of similar cloth 35 yd. long which is made from 90 lb. of wool?

71. The travelling expenses of 7 tourists for 5 weeks amounted to £75, 5s.; a second party of 18 made the same tour in 6 weeks, their average weekly expenditure per man being $\frac{4}{9}$ of that of the first party. What were the total expenses of the second party?

72. If 18 tons of coal are carried 250 ml. by rail for £7, 10s.; how much will it cost to send 280 tons a distance of 360 ml. by a ship whose charges are $\frac{3}{10}$ of the charges by rail?

PERCENTAGES.

By "*per cent.*" (L. *centum* = a hundred) is meant "*for every hundred*," so that 5 per cent. means 5 for every 100. Five per cent. is usually written thus—**5 %**. A **percentage** of any number or quantity therefore means **a certain number of hundredths** of that number or quantity; and the **number of hundredths** is called the **rate per cent.**

Thus, 5 per cent. of £40 means $\frac{5}{100}$ of £40, and 5 is said to be the rate per cent.

A percentage or rate per cent. then is **a ratio whose consequent is 100,** so that any ratio may be expressed as a percentage by converting it into a vulgar fraction with denominator 100.

Ex. 1. Express $\frac{2}{5}$ as a percentage.

$$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\ \%.$$

Ex. 2. Express $\frac{7}{12}$ as a percentage.

$$\frac{7}{12} = \frac{7 \times 100}{12 \times 100} = \frac{\frac{7 \times 100}{12}}{100} = \frac{7 \times 100}{12} \text{ per cent.}$$

$$= \frac{700}{12}\ \% = 58\tfrac{1}{3}\ \%.$$

Ex. 3. Express $6\frac{1}{3}$ as a percentage.

$$6\tfrac{1}{3} = \frac{6\frac{1}{3} \times 100}{100} = (6\tfrac{1}{3} \times 100) \text{ per cent.}$$

$$= (\tfrac{19}{3} \times 100) \quad ,,$$

$$= 633\tfrac{1}{3}\ \%$$

Conversely any percentage may be expressed as a fraction of the whole by making it the numerator of a fraction and writing 100 as the denominator.

Ex. What fraction of the whole is represented by $33\frac{1}{3}$ %?

$$33\tfrac{1}{3}\ \% = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{\frac{1}{3}}{1} = \frac{1}{3}$$

Problems in percentages are often a matter of simple ratio.

Ex. A man's income this year is £650, which is an increase of 4 % on his last year's salary. Find (*as independent exercises*)—(1) his salary last year; (2) the total increase.

(1) £650 = last year's salary + 4 % of last year's salary

= 104 % of last year's salary

For 1 (representing the total) as a percentage is $\frac{1 \times 100}{100} = 100\ \%$

$$\frac{100}{104} \times \text{£}650 = \text{last year's salary}$$

$$= \underline{\text{£}625} \text{ Ans.}$$

(2) Again, the total increase is 4 % of last year's salary ; but every £100 of last year's salary has been increased to £104 this year.

$$\text{Therefore } \frac{4}{104} \text{ of } £650 = \text{the increase}$$

$$\frac{4}{104} \times £650 = \underline{£25} \text{ Ans.}$$

This could, of course, have been found after finding last year's salary by taking 4 % of £625 ; or by subtracting £625 from £650 ; and similarly the first could be solved after finding the second by subtracting £25 from £650, or by finding $\frac{100}{4}$ (ratio of salary to increase) of £25.

For rapid calculations, a table of the more common rates per cent. expressed as fractions should be constructed and committed to memory as far as possible.

Table of Percentages as Fractions.

$2\ \% = \frac{1}{50}$	$8\frac{1}{3}\ \% = \frac{1}{12}$	$33\frac{1}{3}\ \% = \frac{1}{3}$
$2\frac{1}{2}\ \% = \frac{1}{40}$	$10\ \% = \frac{1}{10}$	$40\ \% = \frac{2}{5}$
$3\frac{1}{3}\ \% = \frac{1}{30}$	$12\frac{1}{2}\ \% = \frac{1}{8}$	$50\ \% = \frac{1}{2}$
$4\ \% = \frac{1}{25}$	$16\frac{2}{3}\ \% = \frac{1}{6}$	$60\ \% = \frac{3}{5}$
$5\ \% = \frac{1}{20}$	$20\ \% = \frac{1}{5}$	$66\frac{2}{3}\ \% = \frac{2}{3}$
$6\frac{1}{4}\ \% = \frac{1}{16}$	$25\ \% = \frac{1}{4}$	$75\ \% = \frac{3}{4}$
$80\ \% = \frac{4}{5}$	$90\ \% = \frac{9}{10}$	
$85\ \% = \frac{17}{20}$	$95\ \% = \frac{19}{20}$	

For many everyday transactions it is convenient to know and remember the equivalent **per £1, per shilling,** &c., of some of these percentages ; 5 %, 10 %, &c., being allowances often deducted from accounts on each sovereign, or half-sovereign, or shilling, &c., of the amount.

Thus, $1\frac{1}{4}$ % = 3d. in the £1 ;	$1\frac{1}{2}$d.	in 10s.
$2\frac{1}{2}$ % = 6d. ,, ;	3d.	,,
5 % = 1s. ,, ;	6d.	,,
$8\frac{1}{3}$ % = 1s. 8d. ,, ;	10d.	,,
10 % = 2s. ,, ;	1s.	,,
$12\frac{1}{2}$ % = 2s. 6d. ,, ;	1s. 3d.	,,
20 % = 4s. ,, ;	2s.	,,
25 % = 5s. ,, ;	2s. 6d.	,,
$33\frac{1}{3}$ % = 6s. 8d. ,, ;	3s. 4d.	,,
&c.	&c.	

$8\frac{1}{3}$ °/₀ = 1d. in the 1s. ; ½d. in 6d.
$12\frac{1}{2}$ °/₀ = 1½. ,, ; ¾d. ,,
$16\frac{2}{3}$ °/₀ = 2d. ,, ; 1d. ,,
25 °/₀ = 3d. ,, ; 1½d. ,,
$33\frac{1}{3}$ °/₀ = 4d. ,, ; 2d. ,,

EXERCISE 69.

Do the first ten sums mentally.

1. What is 5 °/₀ of £80; of £200; of £800?
2. What is the value of $12\frac{1}{2}$ °/₀ of £64; of £240; of £20; of £42?
3. Find the worth of $3\frac{1}{3}$ °/₀ of £90; of £270; of £585; of £750?
4. What is the value of 40 °/₀ of £25; of £75; of £110; of £160?
5. What percentage of £120 is £10; is £12; is £80; is £150?
6. Express as a percentage of £150 the following:—£3; £5; £25; £90.
7. What percentage of £1000 is £40; is £75; is £33, 6s. 8d.; is £12, 10s.?
8. At $12\frac{1}{2}$ °/₀ what allowance would be made on £2; on £3, 4s.; on £8, 9s.; on £23?
9. At 25 °/₀ less than marked price what will a bookseller charge me for books marked 1s. 6d.; 3s. 9d.; 12s. 6d.; 2 guineas?
10. A draper allows $8\frac{1}{3}$ °/₀ off for ready money; what should he charge for cash purchases of 18s.; of £1, 5s.; of £3, 7s. 6d.?
11. If 1 lb. troy of sterling gold contains 1 oz. of alloy, find the percentage of alloy in sterling gold.
12. If out of 335966 emigrants, 257372 are Irish, how much per cent. is this?
13. If in a city of 561300 inhabitants, 4947 are old men over seventy years of age, what percentage is this?
14. If the numbers of poor relieved in this country during three consecutive years were respectively 100961, 106434, and 101454, find the increase per cent. from the first to the second, and the decrease per cent. from the second to the third year.
15. The number of letters delivered one year was 82470596, and in the next 168768344; find the increase per cent.
16. The population of a country was 8175124, and at the end of ten years it is now 6552385; find the decrease per cent.
17. Find what is the salary of a gentleman who earns 44 °/₀ of what is earned by his brother whose salary is £1250.
18. If a mixture is made of 73 lb. of Ceylon tea and 56 lb. of China tea, what is the percentage of each in the mixture?

19. An increase of 5 per cent. represents a rise of 10d. in the price of an article; find the former price.

20. An article formerly sold at 4s. 2d. is now sold at 4s. 4½d.; what percentage of increase is this?

21. A man's income has been increased by £25, which represents a rise of 10 per cent.; what was his former income?

22. What percentage of increase is represented by an increase of £40 which raises a man's salary to £350?

23. The number of first-class passengers in a train is 120, and this represents 24 per cent. of the total; how many passengers, other than first-class, are in the train?

24. Of a farm of 600 acres 10 per cent. is grass land, $\frac{1}{8}$ is taken up with turnips and potatoes, and the remainder with grain; what percentage of the whole farm is under grain?

25. From a barrel of 30 gallons of oil 26 quarts leaked away; what percentage of the total remained?

26. Two pints of water are added to 5 gallons of milk; what percentage of the mixture is milk, and what water?

27. A man's present salary of £550 is an increase of 10 per cent. on his former salary; find the total increase and the former salary.

28. The number of attendances made by a class of 36 pupils in a week (10 attendances per pupil) is 352; what is the daily percentage of absentees?

29. Teas of three qualities are mixed in the proportion of 2, 7, and 6; what is the percentage of each kind in the mixture?

30. The proportion of spirits in a blend of three qualities is $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$; find the percentage of each.

Percentages in Business. The use of percentages obtains in almost every kind of business transaction, and special applications of it will now be considered.

For this purpose percentages may be divided into :—

(1) **Percentages not involving the element of time.**

(2) **Percentages in which time is an element.**

The first set includes **Brokerage, Commission,** and **Discounts for Cash** payments; **Insurance**; **Rates and Taxes, Bankruptcies** and the payment of **Dividends**; **Profit and Loss**; and questions of **Mixtures.**

Partnership calculations fall under the first or the second set according as the *duration of the partnership is equal or unequal* in the case of the individual members of the firm.

Percentage calculations *involving time* include all **Interest** and **Bill Discounting** questions.

These various applications of percentage will now be considered in the order given above.

COMMISSION, BROKERAGE, CASH DISCOUNT.

An agent employed to assist in buying or selling goods, or in the transferring of property rights, is called a **broker.**

In all business centres brokers manage the sale of produce sent from a distance or imported, and are paid by **an allowance of so much per cent.** on the money paid or received.

The percentage is called their **Commission.** A house "factor" or agent who represents the landlord in dealings with the tenants is also paid a commission.

The agent whose business it is to carry through the transfer of the property rights of stocks and shares is called a **Stockbroker,** and one who negotiates for merchants in the matter of ship cargoes, a **Shipbroker.** These agents are paid in the same way as the produce broker, but the percentage allowance in these cases is known as **Brokerage.**

Cash Discount is a percentage deduction made from the amount of an account for prompt payment. Some firms allow a larger or smaller percentage deduction according as payment is made on delivery, within a week, &c., but that is merely a matter of arrangement, and does not bring the *time* consideration into any calculations to be made.

In all calculations regarding any of the above **the principal sum should be decimalised** (*see page 72*). If the percentage can be readily expressed as a short terminating decimal the point is moved 2 places to the left in the principal sum, and the calculation made by contracted multiplication.

Ex. Find the commission on £540, 12s. 6d. at $2\frac{1}{2}$ °/₀.

£540, 12s. 6d. ÷ 100 = £5·40625

$2\frac{1}{2}$ = 2·5 — 52

10·812

2·703

13·515

Ans. (correct to nearest farthing) **£13, 10s. 3½d.**

If the percentage cannot be expressed as a terminating decimal, the numerator may be broken up into aliquots of the denominator and the results of these added to get the answer.

Ex. Find brokerage on £2117, 1s. 8d. at $\frac{9}{64}$ °/₀.

£2117, 1s. 8d. ÷ 100 = £21·17083̇ = Brokerage at 1 °/₀

Break up $\frac{9}{64}$ into $\frac{8}{64}$ and $\frac{1}{64}$

$\left(\frac{8}{64} = \frac{1}{8} \text{ of } £1\right)$ 2·64635 = ,, at $\frac{8}{64}$ °/₀

$\left(\frac{1}{64} = \frac{1}{8} \text{ of } \frac{8}{64}\right)$ ·33079 = ,, at $\frac{1}{64}$ °/₀

2·9771

Ans. (correct to nearest farthing) **£2, 19s. 6½d.**

EXERCISE 70.

Find the Commission or Brokerage on—

1. £978, at £$2\frac{1}{8}$ per cent.
2. £436, 13s., at £$2\frac{3}{4}$ per cent.
3. £853, at £$1\frac{7}{12}$ per cent.
4. £1242, at 2s. 6d. per cent.
5. £957, at 3s. 4d. per cent.
6. £937, at 7s. 6d. per cent.
7. £197, 13s. 4d., at £$1\frac{5}{8}$ per cent.
8. £673, 17s., at £$\frac{1}{8}$ per cent.
9. £754, 14s. 4d., at 4s. per cent.
10. £598, at 4s. 6d. per cent.
11. £675, at 2s. 9d. per cent.
12. £820, at £$\frac{1}{2}$ per cent.
13. £764, at £$3\frac{1}{2}$ per cent.
14. £829, at £$1\frac{7}{8}$ per cent.
15. £689, 17s., at £$12\frac{1}{2}$ per cent.
16. £678, 19s. 8d., at 5 per cent.
17. I purchase through a broker 250 £10 shares; what brokerage must I pay at £$\frac{5}{16}$ per cent.?
18. An agent gets £45, 12s. 6d. for selling goods; if his commission is £$2\frac{1}{2}$ per cent., what value of goods did he sell?
19. What is the commission on £672, 13s. 4d. at £2, 12s. 6d. per cent.?
20. An agent sells a consignment of goods for £540 and remits £515, 14s. to the consigner; what is his rate of commission?

21. What cash discount at the rate of $3\frac{1}{2}$ per cent. will be allowed on the furnishings of a mansion if they cost £5000?

22. A merchant gives a discount of $1\frac{7}{8}$ per cent. on accounts paid within 3 months; what will he allow on a bill for £712, 18s. 10d. settled within that time?

23. On an account of £856, 15s. $7\frac{1}{2}$d. what discount will be allowed at $2\frac{3}{4}$ per cent.?

24. A broker who received $\frac{1}{2}$ per cent. for selling some securities, paid £2666, 12s. to the owner after deducting his own brokerage; for what sum did he sell the securities?

25. What brokerage at 2s. 11d. per cent. will be paid on the sale of goods which bring £350, 12s. 6d.?

26. What will the commission on property sold at £1040, 17s. 9d. amount to at $\frac{3}{16}$ per cent.?

27. Brokerage, calculated at the rate of 3s. 8d. per cent., is paid to the amount of £55; find the value of the property involved.

28. A produce broker sells goods to the value of £5960, 12s. 8d.; what will he remit to the seller after he deducts his commission of $2\frac{3}{4}$ per cent.?

29. To what value must a broker sell goods so that his commission at $3\frac{5}{8}$ per cent. may amount in all to £54, 16s. 6d.?

30. If, after the broker's commission of 4 per cent. has been deducted, there remains £3000, 10s. 6d.; what value of goods must have been sold?

TRADE INVOICES AND ACCOUNTS.

When a quantity of goods is bought, it is the business of the seller to see that a list of the goods, showing the quantities and prices of each separate kind as well as the total amount of the purchase, is made out and forwarded along with the goods to the purchaser, that he may see that the goods delivered are as stated in the list. Such a list is called an **Invoice.**

If the purchaser is himself a business man he will enter this invoice in what is called his **Invoice Book**, just as the seller enters a copy of it when he despatches the goods in his **Sales Book.**

From these books the various records of purchases and sales, or Goods bought and Goods sold, are transferred to the various accounts in the **Ledger,** which is the merchant's record

of his relation to his various clients or customers as **Creditor** or Debtor; that is of whether he owes money to any or has money due to him by others.

Periodically (once a month or 3 months as the custom may be) he sends a reminder to those indebted to him, which is usually merely a statement of the amount of the various invoices with their dates and total. This is called an **Account.**

Tweeddale Court,
Edinburgh, 21*st April* 1903.

Mr. James Campbell,

Bought of OLIVER & BOYD,
Publishers.

	£	s.	d.
4 doz. Le Harivel's French Grammars, at 2s. 6d. each	6	0	0
1 gross Tweeddale Geographies, at 6d. each - -	3	12	0
516 Dick's English Grammars, at 6s. per doz. - -	12	18	0
12 doz. Tweeddale Arithmetics, Book I., at 3s. per dozen	1	16	0
	£24	6	0

Paid
Pro Oliver & Boyd.
A. M.
21/4/03

14 English Street,
Carlisle, *June* 1903.

J. Armstrong, Esq.

Bought of BROWN & ROBINSON,
General Drapers.

		£	s.	d.
April 21	To Goods as per Invoice - - - - -	6	13	3
May 13	,, ,, - - - - -	3	9	11
June 20	,, ,, - - - - -	8	5	3
		£18	8	5

The first of the above specimens is an example of an **Invoice.** The goods, however, have been paid when purchased,

and the invoice is shown *receipted*. That means, the seller or one of his representatives writes the words "**Paid**" or "Received Payment" and **adds his signature.** Where, as in the example, the amount is £2 or more, **a penny stamp must be affixed** and the **signature** of the one receiving the money **written across it.** When goods are paid when purchased it is usual to allow *Discount*.

The second is an example of **an Account.** Where merely the dates of the various purchases and their amounts, with the sum of the whole, is given, it is called simply an **Account**; but, were the various items again entered as in the several invoices, it would be called a **Detailed Account.**

In calculations use should be made of all "Short methods" applicable; and the cost found to the nearest farthing in the case of each separate item.

EXERCISE 71.

Write out in proper form:—

1. 2½ lb. tea at 1s. 10d. a lb.; ½ lb. tea at 1s. 8d. a lb.; 1½ lb. coffee at 1s. 4d. a lb.; 6 lb. loaf sugar at 2½d. a lb.; 4 lb. brown sugar at 1½d. a lb.

2. 8 lb. best salt butter at 1s. 1d. a lb.; 2 lb. cooking butter at 10d. a lb.; 2½ lb. Cheddar at 10d. a lb.; 3 doz. cooking eggs at 10d. a doz.; 2 doz. farm eggs at 1s. 2d. a doz.; 7¼ lb. bacon at 9½d. a lb.

3. ½ stone potatoes at 8d. a stone; ¼ lb. tea at 1s. 8d. a lb.; ¼ lb. coffee at 1s. 6d. per lb.; 1½ lb. salt butter at 1s. 1d. per lb.; ½ lb. fresh butter at 1s. 3d. a lb.; threepence worth of firewood.

4. 2 boxes Sunlight soap at 7½d. a box; 1 box Lifebuoy soap at 8d. a box; 1 doz. squares of blue at 2 for 1½d.; 12 packets washing powder at 5½d. per package of 6; 2 quarts household ammonia at 9d. a quart; 3½ lb. Robin starch at 6d. a lb.; 3 bottles furniture polish at 9d. a bottle; ½ doz. tins enameline at 2d. a tin; 3 bars soap at 3½d. a bar.

5. 3 oz. black pepper at 1s. 4d. a lb.; 6 oz. white pepper at 1s. 8d. a lb.; 2 pints ketchup at 5½d. a pint; 1½ lb. mustard at 1s. 8d. a lb.; 2 bottles sauce at 9½d. a bottle; 2 jars mixed pickles at 11½d. a jar.

6. 2 pairs cotton sheets at 6s. 11d. a pair; 3 yd. muslin at 5½d. a yd.; ½ doz. linen towels at 5s. 6d. a doz.; ½ doz. diaper towels at 3s. 6d. a doz.; ½ doz. dusters at 3s. 1d. a doz.; 2 lengths towelling at 1s. 7d. each.

7. 3 tablecloths at 11s. 6d. each; 2 do. at 13s. 11d. each; 2 teacloths at 4s. 11½d. each; 1½ doz. table napkins at 5s. 11d. the half dozen; 2 pairs twill linen sheets at 12s. 6d. a pair.

8. $2\frac{1}{2}$ pecks prepared flour at 1s. 3d. a peck; $1\frac{1}{2}$ pecks whole meal at 1s. 1d. a peck; 2 pecks oatmeal at $11\frac{3}{4}$d. a peck; $\frac{1}{2}$ peck flour at 1s. a peck; 7 lb. rice at $2\frac{1}{2}$d. a lb.; 3 lb. green peas at 2d. a lb.; 5 lb. split peas at $1\frac{1}{2}$d. a lb.

9. $\frac{1}{2}$ stone tapioca at 3d. a lb.; 3 lb. sago at $2\frac{1}{2}$d. a lb.; 5 lb. ground rice at 3d. a lb.; 7 lb. whole rice at $2\frac{1}{2}$d. a lb.; $7\frac{1}{2}$ lb. corn flour at 7d. per lb.; $6\frac{1}{2}$ lb. haricot beans at 3d. a lb.; 1 stone barley at 2d. a lb.

10. $9\frac{1}{2}$ lb. mutton at 10d. a lb.; 5 lb. steak at $11\frac{1}{2}$d. a lb.; $7\frac{1}{2}$ lb. beef at 11d. a lb.; $8\frac{1}{2}$ lb. pork at $7\frac{1}{2}$d. a lb.; $2\frac{1}{4}$ lb. suet at 8d. a lb.

11. 12 lb. currants at $5\frac{3}{4}$d. a lb.; 7 lb. raisins at $6\frac{1}{2}$d. a lb.; 4 lb. figs at $8\frac{1}{2}$d. a lb.; 6 lb. apples at $4\frac{1}{2}$d. per lb.; $3\frac{1}{2}$ lb. nuts at 5d. per lb.; 7 doz. oranges at 16 for 1s.

12. $\frac{1}{2}$ doz. champagne at 63s. per doz.; $\frac{1}{2}$ doz. dinner sherry at 38s. a doz.; 3 bottles brandy at 70s. a doz.; 1 gal. whisky at a guinea a gal.; $\frac{1}{2}$ gal. liqueur whisky at 28s. a gal.; 1 doz. syphons aerated water at $5\frac{1}{2}$d. each.

13. 1 turkey, $13\frac{1}{2}$ lb., at 1s. a lb.; $7\frac{1}{2}$ lb. sirloin roast at 11d. per lb.; 3 lb. sausages at 8d. a lb.; $\frac{1}{2}$ lb. mutton suet at $8\frac{1}{2}$d. a lb.; 1 goose, $11\frac{1}{2}$ lb., at 10d. a lb.

14. $8\frac{1}{4}$ yd. Brussels carpet at 4s. 3d. a yd.; $14\frac{1}{2}$ yd. border at 4s. a yd.; making up above, 9s. 9d.; 9 yd. underfelt at 1s. 6d. a yd.; 2 door mats at 5s. 11d. each; $17\frac{1}{2}$ sq. yd. linoleum at 2s. 11d. per sq. yd.; upholsterer's time fitting and laying linoleum, 7s. 8d.

15. $34\frac{1}{3}$ yd. Brussels carpet at 3s. 11d. a yd.; 14 yd. tape at 2d. a yd.; making up carpet, 8s. 8d.; $27\frac{1}{3}$ yd. carpet at 3s. 3d. a yd.; 13 yd. tape at 2d.; making up above, 6s. 10d.; $20\frac{2}{3}$ yd. carpet at 3s. 3d. a yd.; making up carpet and running ends, 5s. 2d.

16. $26\frac{1}{2}$ lb. sugar at $1\frac{1}{2}$d. per lb.; $56\frac{1}{2}$ lb. cheese at $8\frac{1}{2}$d. per lb.; $39\frac{3}{4}$ lb. cocoa at 1s. 10d. per lb.; $7\frac{3}{4}$ doz. articles at 2s. $3\frac{1}{2}$d. each.

17. 1000 eggs at 1s. per doz.; 4 doz. loaves at 2d. each; 100 pens at $\frac{1}{2}$d. each; 8 stones sugar at 2d. per lb.

18. 150 articles at 3 for $\frac{1}{2}$d.; 19 articles at 10d. each; 25 articles at 6s. 3d. each; 6 doz. articles at 3 for 2d.

19. $1\frac{3}{4}$ cwt. sugar at 32s. 8d. per cwt.; $\frac{1}{2}$ lb. soap at $3\frac{1}{2}$d. per lb.; 2 lb. 12 oz. tea at 2s. 8d. per lb.; $5\frac{1}{2}$ lb. coffee at 1s. 8d. per lb.

20. 5 pieces merino, each 46 yd., at 4s. 3d. a yd.; 8 pieces cotton, each 80 yd. at $4\frac{1}{2}$d. per yd.; $3\frac{1}{2}$ pieces linen, each 54 yd., at 2s. 9d. a yard. Deduct 25 % discount for cash payment.

21. $37\frac{1}{2}$ lb. at 2s. $10\frac{1}{2}$d. per lb.; $9\frac{1}{2}$ yd. at 1s. 10d. per yd.; $58\frac{1}{2}$ lb. at $5\frac{1}{2}$d. per lb.; 25 articles at 1s. 8d. per score.

22. 1000 pencils at $\frac{3}{4}$d. each; $6\frac{1}{2}$ doz. copies at 1s. 8d. per doz.; 100 slates at 6s. per doz.; $7\frac{1}{2}$ doz. books at 1s. 5d. each. Deduct 5 % discount.

23. 16 pieces of cloth, each 28 yd., at 2s. $3\frac{1}{4}$d. per yd.; 19 yd. lace at 10d. per yd.; 6 score articles at 7s. 9d. per doz., 34 pairs gloves at 3s. 8d. per pair. Deduct $12\frac{1}{2}$ % discount.

24. 36 pairs gloves at 3s. 11d. per pair ; 1158 needles at 3½d. per doz. ; 18½ yd. tweed at 3s. 2d. per yd. ; 9 pieces sheeting, each 22 yd., at 1s. 3d. per yd.

25. 4 gal. 1 pt. at 12s. per gal. ; 16⅝ yd. at 8s. 6d. per yd. ; 247 articles at 9d. per doz. ; 16 pieces, each 28 yd., at 2s. 3¼d. per yd.

26. 14 lb. mutton at 11½d. per lb. ; 29 lb. beef at 8½d. per lb. ; 26 chickens at 1s. 8d. each ; 25 rabbits at 11s. per doz.

27. 7 yd. of lawn at 2s. 8d. a yd. ; 8 yd. silk at 5s. 7d. a yd. ; 9 yd. cambric at 2s. 9d. a yd. ; 3 yd. merino at 3s. 1d. a yd. ; 12 yd. satin at 8s. 4d. a yd. ; 28 yd. lace at 2s. 9d. a yd.

28. 8 yd. silk at 13s. 8d. a yd. ; 7 yd. satin at 12s. 10d. a yd. ; 11 yd. brocade at 18s. 10½d. a yd. ; 12 yd. silk at 15s. 11d. a yd. ; 9 yd. sarcenet at 4s. 3d. a yd. ; 16 yd. velvet at 25s. 10d. a yard.

29. 9 pairs stockings at 4s. 9d. a pair ; 7 pairs thread stockings at 3s. 5d. a pair ; 8 pairs silk stockings at 13s. 10d. a pair ; 12 pairs cotton stockings at 5s. 8d. a pair ; 18 yd. fine flannel at 2s. 3d. a yd. ; 27 yd. flannel at 1s. 9d. a yd.

30. 5 chests Congou, each 2 qr. 11 lb., at 3s. 8d. a lb. ; 3 hhd. sugar, each 13 cwt. 2 qr., at 39s. 4d. a cwt. ; 3 cwt. 1 qr. 14 lb. coffee at 49s. 6d. a cwt. ; 14 cwt. 2 qr. 3 lb. cheese at 5½d. a lb.

31. 286 loaves at 7½d. ; 140 loaves at 6½d. ; 89 fancy loaves at 8d. ; 147 doz. biscuits at 3d. per doz. ; 176 lb. flour at 2½d.

32. 648 silk mantles at 14s. 10½d. ; 420 richly trimmed mantles at 45s. ; 600 yd. satin at 8s. 10½d. ; 252 silk velvet mantles at 71s. 9d. ; 140 Paisley shawls at 47s. 6d. ; 246 foreign shawls at 66s. 8d. Deduct 33⅓ °/₀ discount.

33. 900 yd. moleskin at 1s. 2½d. ; 500 yd. plaiding at 1s. 4d. ; 250 yd. flannel at 1s. 5d. ; 600 yd. gingham at 4⅜d. ; 1800 yd. unbleached cotton at 3½d. ; 200 yd. twilled linen at 1s. 5d. ; 80 yd. pilot cloth at 6s. 5½d. ; 200 yd. pack sheeting at 5½d. Allow 8⅓ °/₀ discount.

34. 348 squares of soap at 5½d. per square ; 440 doz. squares of honey soap at 10s. 6d. per doz. ; 200 bottles marrow oil at 11¾d. ; 288½ pints castor oil at 1s. 2d. ; 350 pots polishing paste at 5¾d. ; 1 cwt. 2 qr. 7 lb. starch at 6½d. per lb. Deduct 25 °/₀ for cash payment.

35. 740¾ lb. coffee, No. I., at 1s. ; 370½ lb. coffee, No. II., at 1s. 2d. ; 561¼ lb. coffee, No. III., at 1s. 4d. ; 311 lb. coffee, No. IV., at 1s. 8d. Allow 12½ °/₀ discount.

36. 496⅞ qr. wheat at 41s. 4d. ; 236½ qr. barley at 39s. 2d. ; 483¾ qr. oats at 26s. 1d. ; 146¼ qr. beans at 39s. 5d.

37. 14 pieces, each 37½ yd., at 10s. 5d. per yd. ; 11 pieces, each 53½ yd., at 12s. 4d. per yd. ; 20 pieces, each 44¾ yd., at 13s. 8½d. per yd. ; 24 pieces, each 59¼ yd., at 16s. 7¼d. per yd. Deduct 25 °/₀ discount for cash payment.

38. 124 qr. 6 bush. wheat at 55s. 5d. per qr. ; 88 qr. 4 bush. barley at 45s. 3d. per qr. ; 138 qr. 3 bush. oats at 23s. 8d. per qr. ; 181 qr. 5 bush. beans at 40s. 8d. per qr.

39. 6 chests Congou, each 2 qr. 17 lb., at 3s. 9d. per lb. ; 13 hhd. brown sugar, each 13 cwt. 1 qr. 18 lb., at 23s. 4d. per cwt. ; 4 casks molasses, each 7 cwt. 2 qr. 14 lb., at 13s. 9d. per cwt.

40. 14 cwt. 2 qr. 14 lb. Cheshire cheese at 50s. per cwt. ; 17 cwt. 3 qr. 14 lb. Wiltshire at 40s. per cwt. ; 23 cwt. 1 qr. 18 lb. Gouda at 28s. per cwt. ; 15 cwt. 2 qr. 13 lb. American at 35s. per cwt. ; 27 cwt. 3 qr. 16 lb. Carlow butter at 77s. per cwt. ; 39 cwt. 1 qr. 14 lb. Waterford at 72s. ; 47 cwt. 2 qr. 20 lb. Dutch at 84s. per cwt. ; 23 cwt. 2 qr. 7 lb. Limerick at 66s. 8d. per cwt.

INSURANCE.

Insurance is a contract between parties by which, in return for certain fixed payments by the one party, the other party engages to pay a certain sum on the occurrence of a specified contingency.

In **Life Insurance** the contingency is the death of a named individual or his attainment of a specified age ; in **Fire Insurance** it is the loss of property by fire ; in **Marine Insurance** it is the loss of ships or cargo at sea.

The deed of contract is called the Insurance **Policy**; the sum, of which payment is guaranteed, the **Amount** of the policy ; and the fixed payment, by the insured party to the party taking the risk, the **Premium.** In Life and Fire Insurance the Premium is paid periodically ; in Marine Insurance it is paid for a certain voyage or sometimes for a specified period.

In the case of Life Insurance and Fire Insurance the contracting parties are usually the individual whose life or property is insured, and one of the many **Insurance Companies.**

In Marine Insurance the risk is usually shared by a number of individuals called **Underwriters.**

There are also companies of underwriters, the best known being "**Lloyd's.**"

In Marine Insurance a small percentage is paid—Policy Duty—on the sum insured for, by the owner. When the sum insured for is not a multiple of £100, the duty is charged on the next greater multiple.

Calculations in Insurance are made as in Brokerage; Policy Duty, where mentioned, being found separately and added on.

Ex. Find the expense of insuring a cargo of hemp from Manilla to London for £3433, 6s. 8d. at 3 °/₀; allowing 10 °/₀ discount on the premium, reduced 5 °/₀ for brokerage; duty 4d. °/₀.

Premium on £3433, 6s. 8d. at 3 °/₀		= £103 0 0	
	£103 0 0		
Brokerage on £103 at 5 °/₀ =	5 3 0		
Discount at 10 °/₀ on	£97 17 0	= £9 15 8	(nearest penny)
		£93 4 4	
Duty on £3500 (next higher multiple of £100) at 4d. °/₀ =		11 8	
Total Expense		£93 16 0	

To insure so as to cover premium as well as loss of property, the value of the property must be increased in the ratio of (100 − rate °/₀) : 100 and this sum insured for.

Ex. What sum must be insured for, to cover £3880 and premium at 3 °/₀?

$$£3880 \times \frac{100}{100 - 3} = \frac{3880 \times 100}{97} = \text{Ans. } £4000.$$

EXERCISE 72.

1. Find the annual expense of insuring household property for £1200 at 1s. 6d. °/₀.
2. Find the annual premium on a theatre insured for £4500 at 3 °/₀.
3. Find the increase of annual premium on the insurance of household property for £700 raised on account of extra risk from 1s. 6d. to 5s. °/₀.
4. Find the annual premium paid for insuring furniture for £500 at 1s. 6d. °/₀ and scientific instruments for £150 at 3s. °/₀.
5. A gentleman insures his life for £2700. Find the annual premium paid at £2, 14s. 11d. °/₀.
6. What was paid for insuring a cargo for £1370 at $2\frac{1}{4}$ °/₀; duty 3d. °/₀?
7. A ship worth £5500 had a cargo worth £2670. All the expenses connected with insuring the ship and cargo to their full value amounted to £4, 1s. 8d. °/₀. How much was paid?

8. Find what was paid for insuring a cargo of tar from Archangel to Leith for £1170 at 30s. %; allowing 10 % of discount on the premium, reduced 5 % for brokerage; duty 3d. %.

9. Find the amount paid for insuring a cargo of tea from Hong Kong to London for £6300 at 50s. %; allowing 10 % of discount on the premium, reduced 5 % for brokerage; duty 3d. %.

10. What sum must be insured to cover £1530, the expense of insuring being £4, 7s. 6d. %?

11. How much must be insured to cover £7831, premium being 42s. and duty 3d. %?

12. How much must be insured to cover £5005, premium being 74s. 6d. and duty 6d. %?

13. A cargo is worth £2442, and the expense of insuring it is £2, 17s. 6d. %. What must be insured to cover the value?

14. What annual premium is payable on an insurance policy of £525 at £2, 15s. 5d. %?

15. A warehouse contains goods worth £17,230 which are insured for only 86·3 % of their value. What sum would be lost by the owner in case of their destruction by fire?

16. For how much should a ship worth £5122 be insured at $2\frac{1}{2}$ % so that in case of loss the value of the ship and the insurance premium may be repaid?

17. A merchant insures a cargo on the system of covered insurance for a premium of £140, the rate being $2\frac{1}{2}$ %. What is the value of the cargo?

18. Find the cost of insuring property to the amount of £375 at 2s. 6d. %.

19. An agent insures £3750 at 2 gu. %, and charges commission $\frac{1}{2}$ %; find the whole cost?

20. Insured £4735 on a ship at 50s. %; what was the expense of insuring?

21. Find the cost of insuring £65, 13s. 4d. worth of goods on a voyage, at 12s. 6d. %.

22. What should be paid for insuring goods to the amount of £275, 16s. 8d. on a voyage at $1\frac{1}{2}$ guinea %?

23. Insured £2650 on a ship at $3\frac{1}{2}$ gu. %, commission $\frac{1}{2}$ %; she sustained damage to the amount of £575: how much will be recovered, allowing 2 % discount upon the loss?

24. A man pays for insurance on his life 15 % of his gross income; and after paying the premium has £527 left. What is his gross income?

RATES AND TAXES, BANKRUPTCIES AND DIVIDENDS.

The only point of difference, between calculations of the above and calculations of brokerage, &c., already dealt with, is that in the present case the **payments are calculated on the basis of so much per £1**, instead of a given percentage. The two are the same thing expressed in different ways; for example, 3s. 4d. in the £1 = $\frac{1}{6}$ of the principal sum = $16\frac{2}{3}$ %.

Rates are certain payments which the various local authorities—Borough, Town, County, Parish Councils, &c.—are empowered to levy for the defraying of the expenses of all matters under their management.

Taxes are payments made to the Imperial Authorities for the general purposes of government.

Both rates and taxes are calculated at so much per £1 on what is called the **Assessment** or **Rateable Value** of a person's property.

Property consisting of houses and lands (*fixed* property) is called **Real Estate**, while *moveable* property (such as money, furniture, ships, &c.) is called **Personal Estate**.

The Imperial Authorities levy what is called **Income Tax** on all incomes above £130 a year, but on incomes not exceeding £700 certain abatements are allowed.

"On Incomes . . . . . the following reliefs are granted:—

"Not exceeding £130, Total Exemption.

"Not exceeding £400, Abatement of Duty on £120.

"Not exceeding £600, Abatement of Duty on £100.

"Not exceeding £700, Abatement of Duty on £70."

Certain other **Duties** (Excise, Customs, &c.) are levied by Government and calculated variously on value, on weight or measure, but these call for no special treatment.

Dividends are payments which a **Bankrupt** is able to make to his **Creditors.** They are calculated at so much per £1 of his indebtedness. The total sum of a bankrupt's debts

is given the name **Liabilities**, while the value of his possessions together with all *moneys due to him by his* **Debtors** is his **Assets.**

The problems which come up for solution in Rates, Taxes, and Bankruptcies are all similar in kind.

(1) Given the assessment (in Bankruptcy this is called *Liability*) and the amount of the levy (in Bankruptcy, *Assets*) to find the payment per £1.

(2) Given the Rate per £1 (in Bankruptcy, *Dividend*) and Assessment (in Bankruptcy, *Liability*) to find the total payment.

(3) Given the payment per £1 and the amount of the levy (in Bankruptcy, *Assets*) to find the Assessment (in Bankruptcy, *Liability*).

The following examples show the method of calculation.

Ex. (1) A District Council needs £416, 13s. 4d. for public purposes ; if the assessment is £5000, how much in the £1 will the rate be?

$$\text{Rate} = \frac{\text{£416, 13s. 4d.}}{\text{£5000}} = \text{£}416\cdot\dot{6} \div 5000$$
$$= \text{£}\cdot 41\dot{6} \div 5$$
$$= \text{£}\cdot 08\dot{3} = \textbf{Ans. 1s. 8d.}$$

Ex. (2) Find the inhabited house duty on an assessed rental of **£75** at 9d. in the £1.

Duty = 75 × 9d. = 675d. = **£2, 16s. 3d.**

or 9d. = £·0375 ∴ Rate = ·0375

57

·187

2·625

2·812 = **£2, 16s. 3d.**

Ex. (3) A bankrupt pays 12s. 8d. in the £1, and his assets amount to £950. Find his liabilities.

$$\text{Liabilities} = \frac{\text{£1}}{\text{12s. 8d.}} \times \text{£950}$$

$$= \text{£}\frac{\overset{30}{\cancel{240}} \times \overset{50}{\cancel{950}}}{\underset{19}{\underset{1}{\cancel{152}}}} = \textbf{£1500 Ans.}$$

Decimalising would be used where the reduction necessary is long.

EXERCISE 73.

1. A man pays £116, 2s. 9d. income-tax; if the rate be 1s. in the pound, what is the man's income?
2. Find the total rental of a town if a tax of 9d. in the pound on rents brings in £2163, 14s. 3d.
3. In a certain parish a rate of 1s. 4½d. in the pound brings in £2268, 15s.; what rate in the pound will bring in £3025?
4. A bankrupt makes a composition of 12s. 7½d. in the pound; what does he pay to a creditor to whom he owes £359, 10s.?
5. Find the dividend on £4287, 17s. 6d. at 14s. 4d. in the pound.
6. A rate of £1497, 17s. is to be raised from property of which the rental is £49,584; how much is this in the pound?
7. A creditor receives from a bankrupt's estate the sum of £174, 13s. 6d. in discharge of a claim for £493, 4s.; how much does he lose in the pound?
8. A bankrupt's debts amount to £5729, 10s., and his assets to £893; find, correct to the nearest penny, how much will be received by a creditor to whom he owes £476, 15s.
9. A bankrupt's debts amount to £10,659, 5s., and his effects, when realised, amount to £888, 5s. 5d. Find (1) what rate of dividend per pound he can pay; (2) how much a creditor for £150 will lose.
10. A bankrupt possessed £5850, one-eighth of which went in legal expenses, and he paid his creditors 7s. 7d. in the pound; what was the total amount of his debts?
11. A man has a total income of £1260; he pays 8d. in the pound income-tax, $\frac{1}{5}$ of his total income as house rent, and rates amounting to 4s. 6d. in the pound on his house rent; how much has he left?
12. If the rateable value of a certain district is £4736, 5s.; what amount will be raised by levying a rate of 2s. 8d. in the pound?
13. A man's income is £950; he pays £57, 10s. for life insurance on which he claims exemption from income-tax; how much income-tax does he pay at 11d. in the pound?
14. The assessed rental of a house occupied by the owner is £45; as occupier he pays a rate of 1s. 10d. per pound and as owner 10½d. in the pound; find the amount of rate he pays.
15. In respect of property assessed at £48 (less 10 °/₀ deducted therefrom) the proprietor and occupier is charged for Relief of the Poor $6\frac{4}{6}\frac{1}{4}$d. in the pound, and for Education $11\frac{3}{4}$d. in the pound; how much does he pay towards both purposes?
16. A bankrupt's estate pays a dividend of 13s. 4d. in the pound, 25 per cent. of the assets being spent in costs, which amounted to £1216; what was the amount of the debts and assets respectively?
17. At 10d. in the pound a person pays £78, 14s. 2d. income-tax; what will his net income be if the tax be raised 1½d. in the pound?

18. The rateable value of a town is £653,784; an assessment has to be made for a charge of £27,846; what will be the necessary rate in the pound, expressed in pence and hundredths of a penny: how much will be collected in excess of the requirement if all the assessment is paid in?

19. My income-tax last year at 1s. 3d. in the pound amounted to £151, 5s.; this year, the rate being 11d. per pound, it amounts to £122, 0s. 2d.; supposing my income to again increase in the same ratio, what would be the amount of my next year's income?

20. A person has a net income of £1816, 8s. 1½d. after paying an income-tax of 7½d. in the pound; what was his gross income?

21. A man has to pay taxes amounting to a rate of 4s. 4½d. per pound on his rental; the rent and taxes together amount to £170, 12s. 6d.; find the rent.

22. The income-tax was raised from 7d. to 8d. in the pound; at the same time a man's gross income was raised 2½ per cent.; find by what percentage his net income was altered.

23. A man can afford £125 for rent, rates, and house tax; if the house tax is 9d. per pound upon the whole rent, and the rates are 3s. 10d. per pound upon ⅔ of the rent, find, to the nearest shilling, how much rent he can pay.

24. In 1893 a man paid income-tax at the rate of 7d. in the pound on the portion of his income exceeding £120; in 1894 he paid at the rate of 8d. in the pound on the portion of his income exceeding £160; if his income was the same in each year and he paid 5s. less in 1894 than in 1893, what was his income?

PROFIT AND LOSS.

Profit and Loss is the name given to **the difference** between what a merchant **pays for an article** and what he **sells it for.**

The price the dealer pays is called the **Prime Cost or Buying Price,** and the price he gets by selling it the **Sale Price or Selling Price.**

When the *selling price exceeds* the buying price a **Profit** is made; when the *buying price is greater* than the selling price a **Loss** is sustained.

For the **same article or quantity** of articles therefore—

$$\textit{Selling}\ \text{Price} = \begin{cases} \text{Cost Price} + \text{Gain}, \\ \text{or} \\ \text{Cost Price} - \text{Loss}. \end{cases}$$

$$\textit{Buying}\ \text{Price} = \begin{cases} \text{Selling Price} - \text{Gain}, \\ \text{or} \\ \text{Selling Price} + \text{Loss}. \end{cases}$$

The Profit or Loss on a certain article is found by subtracting. Thus, on the sale of an article which cost £30 and realised by its sale £35, a profit of £5 is made; and the same profit is made on another bought for £100 and sold for £105.

Comparison between different gains or losses, however, can only be completely made when not only the difference of price but also **the amount of money involved**—called the **Outlay**—in the business transaction is considered.

Comparative Profit and Loss is therefore expressed by *percentages*, and unless otherwise specified, by Percentages of the **Buying Price**.

The problems which arise are :—

(1) Given the buying price and the selling price to find profit or loss per cent.

(2) Given the buying price and gain or loss per cent. to find the selling price.

Ex. 1. Find the profit or loss per cent. when coal is bought at 14s. 7d. for 21 cwt. and sold at 16s. 8d. a ton.

Selling price is greater, therefore a profit is made.

$$\text{Profit per cent.} = \frac{\frac{21}{20}\text{ of 16s. 8d.} - \text{14s. 7d.}}{\text{14s. 7d.}} \times 100$$

$$= \frac{\text{17s. 6d.} - \text{14s. 7d.}}{\text{14s. 7d.}} \times 100 = \frac{\text{35d.}}{\text{175d.}} \times 100 = 20\ \%\ \textbf{Ans.}$$

Ex. 2. An article which cost £18, 15s. is sold at a profit of 12 % on the cost price. Find the selling price.

Selling Price = Cost Price + Percentage increase on the Cost Price
= [1 (expressed as a percentage) + given percentage] of the Cost Price
= (100 °/₀ + 12 °/₀) of the Cost Price
$= \frac{112}{100} \times$ Cost Price $\left(= \frac{100 + \text{gain } °/_0}{100} \times \text{Cost Price} \right)$
$= \frac{112}{100} \times £18\frac{3}{4}$
$= \frac{\overset{7}{\cancel{112}}}{\underset{1}{\cancel{100}}} \times \frac{\overset{3}{\cancel{75}}}{\underset{1}{\cancel{4}}} =$ **Ans. £21, 0s. 0d.**

In connection with the **Pricing of Goods for sale** the custom of giving discount for prompt payment must be taken into consideration.

Ex. What price should be put on an article which cost £2, 6s. 8d. so that a profit of 10 °/₀ may be made after allowing discount of $12\frac{1}{2}$ °/₀ for cash payment.

$$\text{Marked Price} = \text{Buying Price} \times \frac{100 + \text{gain } °/_0}{100 - \text{discount } °/_0}$$
$$= £2\frac{1}{3} \times \frac{110}{87\frac{1}{2}} = \quad \textbf{Ans. £2, 13s. 4d.}$$

The method will be better understood if the working is broken up into two parts.

(1) Find selling price, were no discount mentioned.

$$\text{Selling Price} = \text{Buying Price} \times \frac{100 + \text{gain } °/_0}{100}$$
$$= \left(£2\tfrac{1}{3} \times \frac{110}{100} \right) = \textbf{Price after discount is taken off.}$$

(2) As this sum remains after the stated percentage of discount has been deducted, the marked price will be proportionally greater in the ratio of 100 : 100 − percentage of discount.

$$\text{Marked Price} = \text{Price after deduction} \times \frac{100}{100 - \text{disct. } °/_0 \ (12\frac{1}{2})}$$
$$= \text{from (1) } £2\tfrac{1}{3} \times \frac{110}{\underset{1}{\cancel{100}}} \times \frac{\overset{1}{\cancel{100}}}{87\frac{1}{2}}$$
$$= £2\tfrac{1}{3} \times \frac{110}{87\frac{1}{2}}$$

$$\textbf{Marked Price} = \textbf{Buying Price} \times \frac{100 + \text{gain } °/_0}{100 - \text{disct. } °/_0}$$

To price goods so as to secure a greater or less gain merely requires the former price to be multiplied by

$$\frac{100 + \text{new profit per cent.}}{100 + \text{old profit per cent.}}$$

for, $\text{Former Selling Price} = \frac{100 + \text{old profit per cent.}}{100} \times \text{Cost Price}$

and $\text{New Selling Price} = \frac{100 + \text{new profit per cent.}}{100} \times \text{Cost Price}$

and therefore the ratio of the new selling price to the former one is a ratio (that given) compounded of these two.

Converse problems are worked by the same method.

Though it does not occur in business calculations, the following form of problem should be noticed :—

Ex. Find the cost price when $12\frac{1}{2}$ per cent. profit is made by selling an article at £3, 3s.

Ratio of buying price to selling price is 100 : 100 + profit per cent., therefore

$$\text{Cost Price} = \frac{100}{112\frac{1}{2}} \times 63\text{s.} = \underline{\text{£2, 16s.}}$$

EXERCISE 74.

1. Bought 428 yd. of cloth at 14s. 8d., and sold it at 16s. 3d.; what did I gain?
2. Bought 57 cwt. of cheese at £4, 3s. 6d. per cwt., and sold it at $9\frac{3}{4}$d. per lb.; what was gained?
3. Bought 136 yd. of muslin at 3s. 8d.; how must it be sold per yd. to gain £12 on the whole?
4. Sold 257 yd. of linen at 3s. 9d., and lost £9; what was a yard bought at?
5. Sold 13 doz. pairs of stockings at 3s. 7d. per pair, and gained £11, 10s.; what were they bought at?
6. Bought cloth at 17s. 6d. a yard; how much of it must be sold at 19s. a yard to gain £43, 13s. 6d.?
7. By selling tobacco, which had been bought for £14, 10s. per cwt., at 3s. 6d. per lb., I gained £130; how much did I sell?
8. By selling biscuits, which had been bought for 4 guineas per cwt., at $8\frac{1}{4}$d. per lb., I lost £85; what quantity did I sell?
9. Bought 236 ft. of wood, at 3s. 10d., and sold it at 3s. 5d. per ft.; what did I lose on it?
10. Bought 234 cwt. of iron at 4s. 8d. per st. of 14 lbs., but I am willing to lose £14, 12s. in selling it; what should it be sold at per lb.?

11. What is gained by selling 367 yd. of cloth at 7s. 9d. a yd. which was bought at 6s. 5d. a yd. ?

12. How much is gained by selling 3 cwt. 1 qr. of cheese at 6½d. per lb., bought at the rate of £2, 6s. 8d. per cwt. ?

13. How much is gained per cwt. by selling sugar at 5½d. per lb., bought at £2, 4s. 4d. per cwt. ?

14. Find the loss on 364 qr. of wheat, bought at 65s. 6d. per qr., and sold at 7s. 11½d. per bushel ?

15. What is gained by selling 10 doz. of pears at 2 for 1½d., bought at the rate of 5 a penny ?

16. What did a publisher gain by buying the remainder of an edition consisting of 420 copies for £57, 10s. 6d., and selling 300 copies at 3s. 6d., and the remaining number at 3s., each ?

17. Bought 3 cwt. 1 qr. 9 lb. of soap at £2, 11s. 4d. per cwt., and sold it at 6d. per lb., but found that the soap had inlaked 27 lb. ; what was gained or lost by the transaction ?

18. Bought 2 cwt. 27 lb. sugar at 58s. 4d. per cwt., and sold 1 cwt. 3 qr. at 7½d. per lb., but by a fall of the market was obliged to sell the remainder at 5d. per lb. ; what was gained or lost by the transaction ?

19. How must 288 yd. of cloth, bought at 4s. 5½d. per yd., be sold per yd. to gain 12 guineas by the transaction ?

20. How must 3 pieces of cloth, each 89 yd., bought for £73, 8s. 6d., be sold per yd. to gain £2, 4s. 6d. per piece ?

21. Find the prime cost of 6 chests of tea, each containing 2 qr. 27 lb., sold at 4s. 8d. per lb. with a total gain of £15, 2s. 6d.

22. At what rate per cwt. must a merchant purchase a lot of hams, so as to retail them at 9d. per lb. with a gain of 1¾d. per lb. ?

23. What was paid for 4 cwt. 3 qr. 16 lb. of Cheshire cheese, sold at 6¾d. per lb. with a gain of 4s. 8d. per cwt. ?

24. At what rate must soap be retailed per lb. so as to gain 1¼d. per lb. on 3 cwt. 2 qr. 14 lb., purchased in all for £8, 9s. 2d. ?

EXERCISE 75.

Do the first 10 mentally.

1. I buy goods at £100 and sell at £113 ; what is the profit per cent. ?

2. If I sell for £85 what costs £100, what is the loss per cent. ?

3. I bought a bicycle for £8, and sold it for £10 ; what was my profit per cent. ?

4. A motor cycle costing £70 was sold for £56 ; what was the loss per cent. ?

5. Goods bought for £30 are sold at 30 per cent. profit ; find the selling price.

5. I sell at a loss of 5 per cent. goods which cost me £18; what do I get for them?

7. By selling an article at 30s. I gain 25 per cent.; what did it cost me?

8. I lose $12\frac{1}{2}$ per cent. by selling an article for £21; what was the cost price?

9. I sell an article at 5s. and gain 6d.; what is the profit per cent.?

10. By selling goods at £40 I lose £8; what is the loss per cent.?

11. Bought cloth at 3s. 8d., and sold it at 4d. per yd. profit; what was the gain per cent.?

12. Sold cloth worth 15s. per yd. at a loss of 1s. 6d. per yd.; what was the loss per cent.?

13. Bought cloth at 9s. 6d. per yd., and sold it at 12s.; what was the gain per cent.?

14. Bought goods at 5s. 6d. per lb., but as they got damaged, I was obliged to sell at 4s. 9d.; what was the loss per cent.?

15. Bought 7 cwt. 3 qr. of cheese at $5\frac{3}{4}$d. per lb., and sold it at 9d.: what did I gain per cent., and in all?

16. How much profit per cent. is $2\frac{1}{2}$d. per shilling?

17. Bought a house for £315, paid for repairs £20, and sold it for £400; what was the gain per cent.?

18. Gained $9\frac{1}{17}$ per cent. by selling cloth which I bought at 3s. 8d.; what did I sell it at?

19. Lost 10 per cent. by selling cloth which I bought at 15s.; what did I sell it at?

20. Bought cloth at 9s. 6d.; at what must I sell it to gain $26\frac{6}{19}$ per cent.?

21. On goods bought at 5s. 6d. per lb., and which got damaged, I am obliged to lose $13\frac{7}{17}$ per cent.; at what must I sell per lb. to lose so much?

22. Bought cheese at $5\frac{3}{4}$d. per lb.; what must I sell at per lb. to gain $56\frac{1}{3}\frac{2}{3}$ per cent.?

23. Bought goods at 4s. per lb.; at what price must I sell per lb. to gain $20\frac{5}{8}$ per cent.?

24. What charge per gallon must a merchant make for wine which cost him 15s. a gallon so that he may allow the purchaser 6 per cent. for cash payment and still make a profit of $17\frac{1}{2}$ per cent.?

25. A jeweller allows 5 per cent. discount on cash sales and clears $18\frac{3}{4}$ per cent. profit. How must he have priced an article for which he paid £9, 10s.?

26. A manufacturer sells goods, which cost £4162, 10s. to produce, to a retailer to whom he allows a discount of $7\frac{1}{2}$ per cent. How must he have priced them so as to still have 5 per cent. profit?

27. How must cloth bought at 5s. 6d. a yd. be priced so as to allow 4 per cent. discount for cash and yet bring in a return of $9\frac{1}{11}$ per cent. ?

28. Lost $13\frac{7}{11}$ per cent. by selling goods at 4s. 9d. ; what was the prime cost ?

29. Gained $56\frac{1}{3}\frac{2}{3}$ per cent. by selling goods at 9d. ; what was the prime cost ?

30. Sold a quantity of cloth at 4s. 10d. per yd., by which I cleared $20\frac{5}{6}$ per cent. ; what did I buy it for ?

31. Sold cloth at 12s., on which I gained $26\frac{6}{19}$ per cent. ; what was the prime cost ?

32. By selling goods at 5s. 3d., I gained 16 per cent. ; the same articles were afterwards sold at 4s. 6d. ; what was lost or gained per cent. by the latter price ?

33. By selling goods at 5 per cent. profit, I gained £44, 16s. ; what did I pay for them ?

34. I lost £34, 18s. on a quantity of cloth at 4 per cent. ; what was it bought and sold for ?

35. Sold 342 cwt. sugar at 3 per cent. profit, and gained £53, 14s. ; what was it bought and sold at per cwt. ?

36. Bought muslin at 5s. 8d., and by selling it at $4\frac{1}{2}$ per cent. profit, I gained £29, 19s. 3d. ; what quantity did I sell ?

AVERAGES AND MIXTURES.

If the four numbers 3, 6, 8, and 11 be added and the sum divided by 4 (the number of separate numbers) we get the quotient 7. The number 7 is called the *mean* of the four given numbers, for if it were substituted for each of them the sum would still be the same.

$$3 + 6 + 8 + 11 = 28$$
$$7 + 7 + 7 + 7 = 28$$

To the mean of several unequal numbers or quantities the name **Average** is given.

Ex. 1. Find the average of 8·9, 14·8, and 16·7.

$$\frac{8{\cdot}9 + 14{\cdot}8 + 16{\cdot}7}{3} = \frac{40{\cdot}4}{3} = 13{\cdot}4\dot{6}$$

Ex. 2. The average weight of a crew of four rowers and a coxswain is 11 st. 3 lb. If the rowers weigh respectively 11 st. ; 11 st. 6 lb. ; 12 st. 10 lb. ; and 12 st. 8 lb. ; find the weight of the coxswain.

5×11 st. 3 lb. = 56 st. 1 lb.
11 st. + 11 st. 6 lb. + 12 st. 10 lb. + 12 st. 8 lb. = 47 st. 10 lb.

Weight of coxswain = 8 st. 5 lb.

To find the average price of a *mixture* composed of various quantities at different prices—**Multiply each quantity by its price and divide the total of all the prices so found by the total quantity.**

Ex. A merchant mixes 45 gal. of spirits at 14s. 8d. ; 20 at 13s. ; 84 at 13s. 4d. ; and 21 gal. of pure water. Find the average price per gallon of the blend.

45 × 14s. 8d.	=	660s.
20 × 13s.	=	260s.
84 × 13s. 4d.	=	1120s.
21 gallons of pure water costs		0s.
170 gallons of blend cost		2040s.

$\frac{2040\text{s.}}{170}$ = **12s. = price per gallon.**

The converse of the above is another form of a mixtures problem which constantly occurs in some businesses.

The solution depends upon the principle of proportion, and the method used is known as **Alligation**, because the prices are *linked* or bound together (*ad—ligo* = to bind) in pairs in the working of the problem.

Ex. In what proportion must teas at 2s. 6d. a lb. and 1s. 8d. a lb. be mixed so as to produce a mixture worth 2s. a lb. ?

$$\begin{matrix} 30 \\ 24 \\ 20 \end{matrix}\Bigg\rangle \begin{matrix} \text{2s. 6d.} \\ \\ \text{1s. 8d.} \end{matrix}$$

Ratio—**2 lbs. at 2s. 6d. to 3 lbs. at 1s. 8d.**

Bring all prices to one denomination ; set the given prices in a column, arranged in order of magnitude, and leave a space between those greater and those less than the required price. To the left of this space set the required price. Link each number above the space with each number below, and set opposite each, the difference between the one to which it is linked and the required price. These give the quantities to be taken at the price opposite to which they have been set. If they can be cancelled as in above example this should be done.

The number of possible *linkages* of a number above with a number below the space indicates the number of possible solutions, but the case where the *number of prices* given is *odd* should be noticed.

Ex. How should teas at 2s. 6d., 2s. 3d., and 1s. 6d. a lb. be mixed so as to be retailed at 1s. 10d. a lb.?

$$\begin{array}{rcl} 30 & 4 & \\ 27 & 4 & \\ 22 & & \\ 18 & 8+5 & \end{array}$$

Ans. 4 lb. at 2s. 6d.; 4 lb. at 2s. 3d.; 13 lb. at 1s. 6d.

Proof.

4 lb. at 2s. 6d.	= 10s.
4 lb. at 2s. 3d.	= 9s.
13 lb. at 1s. 6d.	= 19s. 6d.
21	38s. 6d.
	1s. 10d.

The 18 is linked with both 27 and 30, and therefore (30 – 22) and (27 – 22) are both set opposite 18.

The dealer may wish to find how to produce a mixture which he can sell at a certain price and **by so doing gain a stated percentage of profit.**

In this case the **cost price** must be found, and with that as the *required price* the problem is solved by Alligation.

Ex. How must I mix teas at 1s. 6d. and 2s. 2d. a lb. so as to gain 10 % by selling the mixture at 1s. 10d. a lb.?

$$\text{Cost price of mixture} = \frac{100}{100+10} \times \text{1s. 10d.}$$

$$= \frac{100}{110} \times \text{22d.} = \text{20d.}$$

$$\begin{array}{rcl} 18 & 6 & 3 \\ 20 & & \\ 26 & 2 & 1 \end{array}$$

Ratio—3 lb. at 1s. 6d. to 1 lb. at 2s. 2d.

Where any particular quantity is specified at one price, or where the total quantity of the mixture is specified, after finding *ratio*, as above, by alligation, the required *quantities* will be found by proportion.

EXERCISE 76.

1. Find the average of 25, 36, and 74.
2. What is the average of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$?
3. Determine the average of 1·03, 1·02, 1·04, 1·01, and 1·025.
4. A man walks 160 yards in one minute, 150 in the next, 140 in the third, and 135 in the fourth; find his average distance per minute.
5. A body falls 16·1 feet the first second, 48·3 the second, 80·5 the third, 112·7 the fourth; find the average velocity for the first four seconds.

6. The average attendance of a school for five days is 723. For the first four days it was 731. How many were present on the fifth day?

7. Find the average price of 4 gal. at 5s., 12 gal. at 3s. 5d., and 8 gal. at 2s. 6d.

8. Find the average price of 100 lb. of rice at 1d. per lb., 300 lb. at 2d., 400 lb. at $1\frac{1}{4}$d., and 100 lb. at 4d.

9. Find the price per gal. of a mixture of 50 gal. at 4s. 6d., 40 at 4s. 2d., 45 at 4s. 4d.

10. Find the average price of 23 qr. wheat at 40s., 32 at 48s., 12 at 69s., 24 at 38s., and 17 at 50s.

11. Find the average price of a consignment of wheat, including 8 qr. at 96s., 6 at 80s. 8d., 21 at 78s., 13 at 76s., and 1 at 75s.

12. Of a herd of 60 cows, $\frac{1}{5}$ is sold at £10 each cow, $\frac{1}{4}$ at £12, $\frac{1}{3}$ at £15, and $\frac{1}{6}$ at £18, and the rest at £20. Find the average.

13. Bought five horses for 500 guineas, and sold them for £110, £120, £80, £70, and £120. Find the average loss per head.

14. A baker sold 143 loaves on Monday, 118 on Tuesday, 201 on Wednesday, 139 on Friday, and 217 on Saturday. The daily average for the week was 167. How many were sold on Thursday?

15. The average of six fractions is $\frac{2}{3}$. Five of them are $\frac{5}{9}$, $\frac{3}{4}$, $\frac{16}{21}$, $\frac{4}{7}\frac{8}{2}$, and $\frac{7}{12}$. Find the sixth.

16. Find the price per lb. of a mixture of 17 lb. coffee at 10$\frac{1}{2}$d. a lb. with 13 lb. at 1s. 3$\frac{1}{2}$d. a lb.

17. A mixture is made of 16 lb. of tea at 3s., 11 lb. at 4s. 1d., 5 lb. at 3s. 7d., and 12 lb. at 2s. 4$\frac{1}{2}$d. per lb. At what price can the mixture be sold?

18. A dealer mixes 16 lb. of coffee at 1s. 1$\frac{1}{2}$d., 6 lb. at 1s. 8d., and 20 lb. at 1s. 9d. a lb. What price must he charge per lb. of the mixture to gain 10s. 6d. on the sale of the whole?

19. Find the proportional quantities of sugar at 2$\frac{1}{2}$d. and 4d. that must be sold to make the average price 3$\frac{1}{2}$d. per lb.

20. What proportional quantities of potatoes at 2s., 3s., and 3s. 6d. per bushel must be sold to make the average price 2s. 9d. per bushel?

21. Mix tea at 4s. 6d., 4s. 2d., 3s. 4d., and 3s. 9d. per lb., so that the compound may be worth 3s. 11d. per lb.

22. What proportional quantities of wine at 15s., 12s., 18s., 19s., and 21s. per gal. must be sold to make the average price 16s. per gal.?

23. How much spirits at 18s. per gal. and water should be mixed to form a mixture worth 15s. per gal.?

24. The men employed at a manufactory receive 2s. 6d. each per day, the women 2s., the boys 1s. 3d., and the girls 9d. ; the average of the whole is to be 1s. 10d. each per day ; what proportion of each may be employed ?

25. How many lb. of tea at 2s. and how many at 2s. 4d. should be mixed with 15 lb. at 3s. to give a mixture to sell at 2s. 8d. a lb. ?

26. How much wheat at 42s. and 56s. must be sold with 13 qr. of wheat at 60s. to make the average price 50s. per qr. ?

27. How much sugar at 10d. and 11d. must be mixed with 9 lb. of 7d. sugar to make the whole worth $8\frac{1}{4}$d. ?

28. How many gallons of water must be mixed with 63 gal. of spirits at 18s. so that the prime cost may be 15s. 9d. per gal. ?

29. How many gallons of water must be mixed with $47\frac{1}{4}$ gal. of spirits at 16s. 8d. to make the prime cost 12s. 6d. per gal. ?

30. How many gallons of wine at 15s. 3d., 16s. 4d., 17s. 2d., and 18s. 1d. must be sold to make the average price of 154 gal. 17s. per gal. ?

31. The specific gravity of an alloy of gold and copper is 16·65, while that of gold is 19·2, and that of copper 9. Find the weight of gold and of copper in 144 oz. of the alloy.

32. A crown 150 oz. in weight, and made of gold and silver, displaces 13·824 cub. in. of water. Had it been all gold it would have displaced 12·96 cub. in., and had it been all silver 23·04 cub. in. Find the weight of gold and silver in the crown.

33. A grocer mixes equal quantities of sugar worth 25s., 50s., and 62s. a cwt. In what proportion must these be mixed with a fourth kind at 70s. a cwt. to get a sugar worth 58s. a cwt. ?

34. Find two proportions in either of which a mixture may be made, worth 18s. a gallon, of spirits at 12s., 15s., and 20s. a gallon respectively.

35. A mixture of currants at $4\frac{1}{2}$d., 7d., $11\frac{1}{2}$d., and 1s. 2d. a lb. is to be made so as to have 560 lb. worth 10d. a lb. ; find the quantities.

36. Three qualities of spirits are mixed in the proportion 1, 1, and 2 ; if the prices of the first two qualities are 12s. and 15s., and the price of the mixture 17s. per gallon, what is the price of the third quality ?

37. How should I mix coffee costing 2s. a lb. with chicory at 7d. a lb. so as to get a profit of $16\frac{2}{3}$ % by selling the mixture at 1s. 9d. a lb. ?

38. In what proportion should spirits at 15s. a gal. be mixed with a better quality at 21s. a gal. so that the sale of the blend at 18s. a gal. may yield a profit of $12\frac{1}{2}$ % ?

39. How should tea at 3s. a lb. be mixed with teas at 1s. 5d. and 1s. 6d. a lb. so as to give a return of 25 % when the mixture is retailed at 2s. 1d. a lb. ?

40. When teas at 1s. 4d., 1s. 6d., 2s. 6d., and 3s. 6d. are mixed in a certain ratio and the mixture retailed at 2s., a profit of 2d. a lb. is made. Find the ratio.

PROPORTIONAL PARTS AND PARTNERSHIP.

By Proportional Parts is meant the dividing of a quantity into parts which bear a given ratio to each other.

A quantity is said to be divided into parts proportional to the various terms of a ratio when each part in order divided by each of the terms in order gives the same quotient.

To divide a quantity into parts proportional to certain given numbers, **multiply the quantity to be divided by the ratio of one of the given numbers to the total of these,** taking each number in turn as the antecedent of the ratio.

Ex. Divide £33 among A, B, and C in the proportion of 2 : 3 : 6.

$$\text{A gets } £33 \times \frac{2}{2+3+6} = £33 \times \frac{2}{11} = £6$$

$$\text{B gets } £33 \times \frac{3}{2+3+6} = £33 \times \frac{3}{11} = £9$$

$$\text{C gets } £33 \times \frac{6}{2+3+6} = £33 \times \frac{6}{11} = £18$$

$$\text{Total } £33$$

Partnership. When two or more individuals each subscribe a certain amount of money towards the carrying out of some business enterprise they are said to have formed a **Partnership,** and each subscriber is spoken of as a **Partner of the Firm.** The money subscribed is called the **Capital,** and any profits made or losses sustained are divided among the partners *in proportion to the Capital each subscribed.*

Calculations connected with any Partnership into which the element of time does not enter—called **Simple Partnership**—are therefore simply questions of *dividing into proportional parts,* and are solved in the same way.

Ex. Towards an undertaking which realised a profit of £4617, 12s., A furnished £1875, B £1500, and C £1250 of capital; what share of the profit should each receive?

$$\text{A gets } £4617{\cdot}6 \times \frac{1875}{1875+1500+1250} = £4617{\cdot}6 \times \frac{1875}{4625}$$

$$= £4617{\cdot}6 \times \frac{15}{37} = £1872$$

$$\text{B gets } £4617{\cdot}6 \times \frac{1500}{4625} = £1497,\ 12s.$$

$$\text{C gets } £4617{\cdot}6 \times \frac{1250}{4625} = £1248$$

Total £4617, 12s.

Compound Partnership. When the partners give the use of their money towards the conducting of the business *for varying periods*, a new element of *time* is introduced, and calculations to be made depend on both the ratio of share to total capital; and duration of partnership to total period under consideration. The proportional parts are therefore in a ratio *compounded* of two ratios, hence the name.

Ex. A begins business with a capital of £400 and is joined after 3 months by B with £600. Six months from the start of the business C becomes a partner subscribing £500. How should the total profit for the year, £374, be divided among them?

A unit of time is taken common to all the different periods—a month in this case—and the capital of each man for the given time is expressed as an equivalent amount for the unit; thus £500 in use for 6 months = £3000 in use for 1 month. With these equivalents as each man's contribution to the business the calculation is made as in Simple Partnership.

£400 × 12 = £4800
£600 × 9 = £5400
£500 × 6 = £3000

£13200

$$\text{A gets } £374 \times \frac{4800}{13200} = £374 \times \frac{4}{11} = £136$$

$$\text{B gets } £374 \times \frac{5400}{13200} = £374 \times \frac{9}{22} = £153$$

$$\text{C gets } £374 \times \frac{3000}{13200} = £374 \times \frac{5}{22} = £85$$

£374

EXERCISE 77.

Give answers correct to nearest farthing.

1. Divide 84 into parts having the mutual ratios of 2, 3, 7.
2. Divide 1200 into parts having the mutual ratios of 11, 12, 13, 14.

3. Divide a line 4 ft. long into parts having the ratios of the first four odd numbers.

4. Divide 100 into parts having the ratios of the cubes of the first three numbers.

5. Divide 390 into parts having the ratios of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

6. Divide 1331 into parts having the ratios of the *reciprocals* of the first three even numbers.

7. Apportion a house tax of £6, 18s. 8d. among 3 joint proprietors, who pay in the proportion of the annual values of their properties, which are £30, £40, and £60 respectively.

8. A vessel is divided into 64 equal shares, of which A, B, C, D have 6 shares each; E, 12; F, 16; G, 4; and H the remainder. Find their respective shares in sustaining a joint loss of £158, 10s. 1d.

9. Divide a profit of £689 among 3 partners, of whom the first owns $\frac{2}{13}$ of the joint stock and the second $\frac{5}{13}$.

10. A, B, C, D invest £450, £230, £190, and £110 respectively in a speculation. Find their respective liabilities in a joint loss of £313, 12s.

11. Three partners respectively claim $\frac{1}{3}$, $\frac{1}{13}$, and $\frac{7}{13}$ of the gain of an adventure amounting to £1260. Give to each a proportionate share.

12. Divide 5 guineas among George, James, and Henry, who respectively claim $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{1}{2}$, so that they may have proportionate shares.

13. An analysis of a manure gives the following results for every 100 parts:—Water, 13·97; organic matter, 15·71; soluble phosphates, 21·63; insoluble phosphates, 11·43; sulphate of lime, 15·83; sulphuric acid, 15·63; alkaline salts, 1·1; silica, the remainder. Find the weight of each in a ton.

14. Oil of vitriol contains by weight, 1 of hydrogen, 32 of oxygen, and 32 of sulphur. Find the weight of each in a gallon, which weighs $18\frac{1}{2}$ lb.

15. In a copartnery, A's capital of £400 has continued for 9 months; B's of £350 for 8 months; C's of £600 for 2 months. Divide £570 of gain among them.

16. Three cattle-dealers rent a field of 9 acres at £5 per acre: A puts in 6 cows for 2 months; B, 9 cows for 1 month; C, 12 cows for 3 months. How much does each pay?

17. At the end of 12 months, D, E, F, having a joint capital of £6000, find that they have lost £625. D's capital of £2500 has been in trade for 12 months, E's of £1500 for 8 months, and F's for 4 months. What is the loss of each?

18. A and B enter into partnership, the former with £1800, the latter with £900: in 8 months B adds £300 to his capital. Divide a profit of £840 between them at the end of 12 months.

19. A has £300 in trade for 7 months, when B joins him with £400. At the end of the next 3 months C joins them with £300. Divide £549 of gain among them after 18 months' trade.

20. A, B, and C enter into partnership on Jan. 1, 1884, with a capital of £1000 *each*. On April 30, B withdraws £400, and C makes up the sum. On Aug. 28, A withdraws £200, and C makes up the sum. On balancing their books for the year 1884, they find they have a gain of £365. What is the share of each?

21. Three graziers rent a field from May 11 to October 19, 1885, for £43. A agrees to pay £13 for grazing 12 oxen; B, £18 for 18 oxen; and C the remainder for 20 oxen. To how many days is each grazier entitled; and if the oxen go into the field in the order A, B, C, on what days do B's and C's severally enter?

22. 3 men and 4 boys are loading carts with sand. A man takes 7 shovelfuls for a boy's 6, and 4 shovelfuls of a man's = 5 of a boy's. Divide £3, 7s. proportionally among them.

23. A, B, and C formed a company for 9 months; A's capital of £975 continued during the whole time, B's of £1275 for 5 months, and C's of £1550 for 6 months. They gained £1365. If A is allowed £142, 10s. of the profit for managing the business, how much should each receive at the end of the time?

24. A, B, and C, each with a capital of £5000, become partners in an existing firm on April 4, May 7, and June 10 respectively. The net profit is declared (Dec. 31) at 12½ per cent. per annum; find what interest each receives.

INTEREST.

Interest (I.) is a payment or **rent for the use of money for a time**, paid by the borrower to the lender.

The payment is usually expressed as a percentage called the *Rate per cent. per annum*, or simply the **Rate** (R.), while the money for the use of which interest is paid is called the **Principal** (P.). When the Interest due is added to the Principal the sum is called the **Amount** (A.).

$$A = P + I.$$

Simple Interest is interest **reckoned on the principal alone** for the stated time at the given rate per cent. per annum.

To calculate the Simple Interest on a sum of money lent for a given time, at a given rate per cent. per annum, this rule will be found useful—*Multiply the*

ratio of the principal to £100, *by the product of the rate* (in pounds) *and the time* (in years); or, in the form of a formula :—

$$I = \frac{P \times R \times T}{100}$$

An example, worked by the unitary method, will show how that rule is arrived at.

Ex. 1. Find the simple interest on £600 for 3 years at 5 per cent. per annum.

Interest on £100 for 1 year at 5 °/₀ = £5

.. .. £1 .. 1 year at 5 °/₀ = £$\frac{5}{100}$

.. .. £1 .. 3 years at 5 °/₀ = £$\frac{5}{100} \times 3$

.. .. £600 .. 3 years at 5 °/₀ = £$\frac{5}{100} \times 3 \times 600 = \frac{600 \times 5 \times 3}{100}$

$$= \frac{P \times R \times T}{100}$$

Ex. 2. Find the simple interest on £8635, 7s. 6d. for 9 months at 6 per cent. per annum.

Interest on £100 for 1 year at 6 °/₀ = £6

,, ,, £1 ,, 1 year at 6 °/₀ = £$\frac{6}{100}$

,, ,, £1 ,, $\frac{3}{4}$ year (9 months) at 6 °/₀ = £$\frac{6}{100} \times \frac{3}{4}$

,, .. £8635·375 for 9 months at 6 °/₀ = £$\frac{6}{100} \times \frac{3}{4} \times 8635{\cdot}375$

$$= \frac{£8635{\cdot}375}{100} \times 6 \times \frac{3}{4}$$

$$= \frac{P}{£100} \times R \times T \text{ or } \frac{P \times R \times T}{100}$$

MENTAL OR SHORT METHODS FOR CALCULATING INTEREST.

1. To find the interest on any principal for **1 year at 5 °/₀—consider the number expressing the principal in pounds as shillings**, because 5 °/₀ = $\frac{1}{20}$, and therefore 5 °/₀ of £1 = $\frac{1}{20}$ of 20s. = 1s.

For 1 month the interest will be exactly $\frac{1}{12}$ of the interest for a year, therefore

2. To find the interest on any principal for **1 month at 5 °/₀—consider the principal expressed in pounds as pence.**

As $2\frac{1}{2}$ °/₀ is just the half of 5 °/₀ the amount reckoned for any period will be half of that at 5 °/₀ for the same period.

3. To find interest on any principal for **1 year** and for **1 month at $2\frac{1}{2}$ °/₀—reckon half the principal in pounds as shillings** and **as pence** respectively.

4. To find interest on any principal for **1 year** and for **1 month at 10 °/₀—reckon twice the principal in pounds as shillings** and **as pence** respectively.

5. When the rate is :—

6 °/₀ —proceed as for 5 °/₀ and add one-fifth.
4 °/₀ ,, ,, 5 °/₀ ,, deduct ,,
$5\frac{1}{2}$ °/₀ ,, ,, 5 °/₀ ,, add one-tenth.
$4\frac{1}{2}$ °/₀ ,, ,, 5 °/₀ ,, deduct ,,
3 °/₀ ,, ,, $2\frac{1}{2}$ °/₀ ,, add one-fifth.
2 °/₀ ,, ,, $2\frac{1}{2}$ °/₀ ,, deduct ,,
$7\frac{1}{2}$ °/₀ ,, ,, 5 °/₀ and for $2\frac{1}{2}$ °/₀ and add results; and so from 10 °/₀ we can get interest at 11 °/₀, 12 °/₀, 8 °/₀, 9 °/₀, &c., by addition or deduction of a fraction of the result.

6. To convert any rate into a **5 °/₀** rate—**Multiply the principal by double the rate and divide by 10.**

Ex. Find interest for 1 year and for 1 month on £30 at 7 per cent.

$\frac{30 \times 14}{10}$ s. = 42s. = £2, 2s. interest for 1 year.

$\frac{30 \times 14}{10}$ d. = 42d. = 3s. 6d. interest for 1 month.

7. For **any number of years or of months** at a given per cent.—proceed as in (1) to (6), and multiply the result by the given number of years or months.

Ex. What is the interest on £35, 10s. for 5 months at 5 °/₀?

P £35·5 gives interest for 1 month 35·5d.

35·5d. × 5 = 177·5d. = 14s. $9\frac{1}{2}$d.

EXERCISE 78.

Find mentally or by short methods the simple interest on—

1. £400 for 1 year at 5 per cent.
2. £650 for 1 year at 5 per cent.
3. £700 for 2 years at 5 per cent.
4. £520 for 4 years at 5 per cent.
5. £365 for 3 years at 5 per cent.
6. £260 for 5 years at 1 per cent.
7. £60 for 2 years at $2\frac{1}{2}$ per cent.
8. £485 for 2 years at $2\frac{1}{2}$ per cent.
9. £250 for 4 years at $2\frac{1}{2}$ per cent.
10. £440 for 5 years at $2\frac{1}{2}$ per cent.
11. £450 for 1 year at 3 per cent.
12. £1200 for 4 years at 3 per cent.
13. £750 for 2 years at $4\frac{1}{2}$ per cent.
14. £150 for 7 years at 3 per cent.
15. £500 for $3\frac{1}{2}$ years at 2 per cent.
16. £400 for 4 years at $3\frac{1}{2}$ per cent.
17. £600 for 3 years at $4\frac{1}{2}$ per cent.
18. £420 for 2 years at 4 per cent.
19. £325 for 4 years at 4 per cent.
20. £720 for 3 years at $8\frac{1}{3}$ per cent.
21. £420 for $8\frac{1}{3}$ years at 2 per cent.
22. £3000 for 1 year at $6\frac{2}{3}$ per cent.
23. £960 for 5 years at $6\frac{2}{3}$ per cent.
24. £240 for 2 years at $6\frac{1}{4}$ per cent.
25. £560 for 12 years at $6\frac{1}{4}$ per cent.
26. £400 for 1 month at 5 per cent.
27. £576 for 1 month at 5 per cent.
28. £600 for 1 month at 5 per cent.
29. £360 for 1 month at 6 per cent.
30. £720 for 1 month at $5\frac{1}{4}$ per cent.
31. £480 for 1 month at $4\frac{1}{2}$ per cent.
32. £540, 6s. 9d. for 10 months at 5 per cent.
33. £875, 10s. for 4 months at $3\frac{1}{4}$ per cent.
34. £8635, 7s. 6d. for 6 months at 2 per cent.
35. £1020, 17s. 6d. for 3 months at 7 per cent.
36. £144, 16s. for 4 months at $12\frac{1}{2}$ per cent.

From the formula established for the calculation of Simple Interest,

$$I = \frac{P \times R \times T}{100}$$

or by solving exercises by the unitary method, other similar expressions of methods of working are arrived at.

$$\text{Thus, since } I = \frac{P \times R \times T}{100}$$

$$100 \times I = P \times R \times T$$

$$\text{Therefore (1) } \frac{100 \times I}{P \times R} = T$$

(Formula for solutions in which *Time* is to be found.)

(2) $\frac{100 \times I}{P \times T} = R$ (Formula for solutions when *Rate* is to be found.)

(3) $\frac{100 \times I}{R \times T} = P$ (Formula for solutions when *Principal* is to be found.)

From the definition of Amount the formula was got $A = P + I$.

Therefore (4) $A = P + I$.

$= P + \frac{P \times R \times T}{100}$ (Formula for solutions to find *Amount.*)

To find the Principal which, in a given time at a given rate, **will produce a certain Amount.**

Such problems depend for solution on the principles of Proportion. The required Principal will be to £100 (or any chosen sum) in the same ratio as the given Amount bears to the amount of £100 (sum chosen in first ratio) for the given time at the given rate.

$$\frac{\text{Required P}}{\text{£100}} = \frac{\text{Amount}}{\text{Amount of £100 in given Time at given Rate}}$$

Ex. What Principal will in 8 months at 4½ °/₀ amount to £175, 2s. ?

$$\text{£}\frac{P}{100} = \frac{\text{£175·1}}{\text{£100 + Int. on £100 for 8 months at 4½ °/₀}}$$

$$\text{£}\frac{P}{100} = \frac{175{\cdot}1}{103}, \text{ therefore } P = \text{£}\frac{175{\cdot}1 \times 100}{103} = \text{£170 Ans.}$$

Formula for solving—$\mathbf{P = \frac{A \times 100}{100 + (R \times T)}}$

EXERCISE 79.

(*Answers to be correct to the nearest farthing.*)

Find the interest for one year on—

(1) £320, at 3 per cent. per annum.
(2) £647, 15s. 6d., at 4 per cent. per annum.
(3) £802, 11s. 6d., at 3½ per cent. per annum.
(4) £772, 16s. 9d., at 4¼ per cent. per annum.

Find the interest on—

(5) £750 for 7 years, at 5 per cent. per annum.
(6) £216, 4s. 6d. for 5 years, at 3 per cent. per annum.
(7) £802, 17s. 6d. for 4 years, at 2½ per cent. per annum.

(8) £564, 13s. 4d. for 3 years, at $2\frac{1}{3}$ per cent. per annum.
(9) £361, 14s. 6d. for 5 years, at $3\frac{1}{2}$ per cent. per annum.
(10) £874, 18s. 8d. for 8 years, at $2\frac{1}{4}$ per cent. per annum.
(11) £374, 17s. 3d. for 5 months, at $3\frac{1}{2}$ per cent. per annum.
(12) £769, 13s. 3d. for 8 months, at $3\frac{3}{4}$ per cent. per annum.
(13) £467, 2s. $4\frac{1}{2}$d. for 5 months, at $4\frac{1}{4}$ per cent. per annum.
(14) £876, 14s. $6\frac{1}{4}$d. for 2 years 3 months, at $2\frac{1}{2}$ per cent. per annum.
(15) £723, 16s. $3\frac{3}{4}$d. for 3 years 11 months, at $3\frac{1}{4}$ per cent. per annum.
(16) £846, 12s. 6d. for 2 years 7 months, at 5 per cent. per annum.
(17) £754, 14s. $8\frac{3}{4}$d. for a year, at 4 per cent.
(18) £1256, 14s. $8\frac{1}{4}$d. for 7 years, at $3\frac{1}{2}$ per cent.
(19) £938, 18s. for $8\frac{3}{4}$ years, at $4\frac{5}{8}$ per cent.
(20) £2346 for 9 years 7 months, at $4\frac{7}{12}$ per cent.
(21) £127, 15s. $7\frac{1}{2}$d. for 19 months, at 4 guineas per cent.
(22) £364, 17s. 5d. for 5 months, at $3\frac{1}{4}$ per cent.
(23) £36, 17s. 7d. for 9 months, at $4\frac{1}{3}$ per cent.
(24) £560 for 7 weeks, at 4 per cent.

Find the amount at simple interest of—

(25) £375 for 4 years, at $2\frac{1}{4}$ per cent.
(26) £1251, 13s. 4d. for $5\frac{1}{2}$ years, at $6\frac{1}{2}$ per cent.
(27) £565 for $3\frac{1}{2}$ years, at 4 per cent.
(28) £360 for 2 years, at $2\frac{1}{2}$ per cent.
(29) £3733, 6s. 8d. for $5\frac{1}{2}$ years, at 3 per cent.
(30) £500 for 84 days, at $6\frac{1}{12}$ per cent.
(31) What principal will produce £384 of interest in 6 years, at 4 per cent. ?
(32) What principal will produce £153 of interest in $4\frac{1}{4}$ years, at $4\frac{1}{2}$ per cent. ?
(33) Find the principal of which the interest for 50 days at 4 per cent. is £14, 12s.
(34) Find the principal which in $4\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent., will amount to £962.
(35) What principal will amount to £1017, 15s. $0\frac{1}{2}$d. in $6\frac{1}{2}$ years, at 3 per cent. ?
(36) What principal lent for 73 days, at 5 per cent., will amount to £712, 9s. 5d. ?
(37) At what rate must £424 be lent for $2\frac{1}{2}$ years to produce £26, 10s. of interest ?
(38) At what rate must £255, 10s. be lent for 80 days to produce £2, 16s. of interest ?
(39) How long must £670 be lent to produce £134 at 5 per cent. ?
(40) How long must £91, 5s. be lent to produce £2 of interest at 5 per cent. ?

Interest for a given number of Days. The methods shown above should be used for days when the number is a fraction of 365, which can be expressed in simple terms, but in many cases this cannot be done.

For such cases the rule is—**Divide double the continued product of Principal, Rate, and Number of Days by 73000**, which is a special application of—

$$I = \frac{P \times R \times T\,(\text{years})}{100} = \frac{P \times R \times T\,(\text{days})}{100 \times 365}$$

$$= \frac{2 \times P \times R \times T\,(\text{days})}{2 \times 100 \times 365} = \frac{2 \times P \times R \times T\,(\text{days})}{73000}$$

The division by 73000 is usually carried through by what is called the **Third, Tenth, and Tenth Rule**, the working of which an example will best show.

Ex. Find the interest on £372, 8s. 4½d. for 56 days at 3½ per cent.

$$\frac{2 \times 56 \times 3\frac{1}{2}}{73000} = \frac{350}{73000}$$

$$P \times \frac{350}{73000} = \frac{P \times 350 \times 100}{100000 \times 73} = \left(\frac{P}{100000} \times 350\right) \times \frac{100}{73}$$

but $\quad \frac{100}{73} = 1{\cdot}3698 +$

and $\quad 1{\cdot}3698 = 1 + {\cdot}\dot{3} + {\cdot}0\dot{3} + {\cdot}00\dot{3} - {\cdot}0001$

$$= 1 + \tfrac{1}{3} + (\tfrac{1}{10} \text{ of } \tfrac{1}{3}) + (\tfrac{1}{10} \text{ of } \tfrac{1}{10} \text{ of } \tfrac{1}{3}) - \tfrac{1}{10000}$$

Therefore to the product of $\frac{P}{100000} \times 350$ we must add $\frac{1}{3}$ of of the product, $\frac{1}{10}$ of that result, and $\frac{1}{10}$ of the last; and then deduct $\frac{1}{10000}$ of the total.

This gives a result correct to 4 decimal places.

Decimalise P. and move point 5 places to the left.	·00372417708$\dot{3}$ = $\frac{P}{100000}$
Multiply by numerator got as shown above.	053 × 350
	1·1172531
	1862088
	1·3034619
Add ⅓ of product	·4344873
,, $\frac{1}{10}$ of last	·0434487
,, $\frac{1}{10}$,,	·0043448
	1·7857427
Deduct $\frac{1}{10000}$ to make answer correct to nearest farthing	·00018
Ans.	1·78556 = £1, 15s. 8½d.

To calculate the number of days from one date to another, subtract from the days of that month the figures of the first date, to this add the number of days in any intervening months and the figures of the second date. The sum is the number wanted.

Ex. How many days from May 3 to September 27?

(No. of days in May =) 31 − 3 (figure of 1st date) = 28

Intervening months: June 30 + July 31 + Aug. 31 + 27 (figure of 2nd date) = 119

No. of days = 147

EXERCISE 80.

1. Find the interest on £608, 6s. 8d. for 58 days at 4 per cent.
2. What do I pay as interest on £7000 for 85 days at 4 per cent.?
3. Find the simple interest on £1368, 15s. for 106 days at $4\frac{3}{4}$ per cent.
4. Calculate the amount of £19314, 11s. 8d. in 19 days at the rate of 4 per cent. per annum.
5. What is the simple interest on £500 for 84 days at $6\frac{1}{2}$ per cent.?
6. What amount of interest is due on a loan of £518, 19s. 2d. for 170 days at $9\frac{1}{8}$ per cent. per annum?
7. A sum of £1250 was borrowed on the 1st of April and repaid on the 25th of August of the same year, with interest at $3\frac{3}{4}$ per cent. per annum; what sum had to be paid?
8. Find the simple interest on £3591, 5s. at $3\frac{1}{4}$ per cent. from 2nd January 1896 to 3rd May 1896.
9. A customer deposits £100 in a bank on 31st January, £150 on 30th April, and £125 on 31st July; what interest is due to him on 31st December, the rate being $2\frac{1}{2}$ per cent. per annum, $\frac{1}{12}$ of a year's interest being allowed for each calendar month?
10. The interest on a certain sum for $\frac{5}{6}$ year at 3 per cent. is £9, 7s. 6d.; what will be the interest on the same sum for $\frac{3}{5}$ year at $4\frac{1}{2}$ per cent.?
11. On the 31st December 1895, a sum of £2962, 10s. is borrowed from the bank, and it is repaid on 13th March 1896; at 4 per cent. how much simple interest is due?
12. If the interest charged for a loan of £555 for 130 days be exactly what is charged on £520 lent at $2\frac{3}{4}$ per cent. for 37 days, what rate is charged in the first instance?

TABLE OF DAYS.

For use when it is required to find the number of days between two given dates.

Ex. How many days from Feb. 23 to July 1?

July 1	182
Feb. 23	54
	128 days in the period.

Jan.	1.	1	Feb.	12.	43	Mar.	26.	85	May	7.	127
,,	2.	2	,,	13.	44	,,	27.	86	,,	8.	128
,,	3.	3	,,	14.	45	,,	28.	87	,,	9.	129
,,	4.	4	,,	15.	46	,,	29.	88	,,	10.	130
,,	5.	5	,,	16.	47	,,	30.	89	,,	11.	131
,,	6.	6	,,	17.	48	,,	31.	90	,,	12.	132
,,	7.	7	,,	18.	49	April	1.	91	,,	13.	133
,,	8.	8	,,	19.	50	,,	2.	92	,,	14.	134
,,	9.	9	,,	20.	51	,,	3.	93	,,	15.	135
,,	10.	10	,,	21.	52	,,	4.	94	,,	16.	136
,,	11.	11	,,	22.	53	,,	5.	95	,,	17.	137
,,	12.	12	,,	23.	54	,,	6.	96	,,	18.	138
,,	13.	13	,,	24.	55	,,	7.	97	,,	19.	139
,,	14.	14	,,	25.	56	,,	8.	98	,,	20.	140
,,	15.	15	,,	26.	57	,,	9.	99	,,	21.	141
,,	16.	16	,,	27.	58	,,	10.	100	,,	22.	142
,,	17.	17	,,	28.	59	,,	11.	101	,,	23.	143
,,	18.	18	Mar.	1.	60	,,	12.	102	,,	24.	144
,,	19.	19	,,	2.	61	,,	13.	103	,,	25.	145
,,	20.	20	,,	3.	62	,,	14.	104	,,	26.	146
,,	21.	21	,,	4.	63	,,	15.	105	,,	27.	147
,,	22.	22	,,	5.	64	,,	16.	106	,,	28.	148
,,	23.	23	,,	6.	65	,,	17.	107	,,	29.	149
,,	24.	24	,,	7.	66	,,	18.	108	,,	30.	150
,,	25.	25	,,	8.	67	,,	19.	109	,,	31.	151
,,	26.	26	,,	9.	68	,,	20.	110	June	1.	152
,,	27.	27	,,	10.	69	,,	21.	111	,,	2.	153
,,	28.	28	,,	11.	70	,,	22.	112	,,	3.	154
,,	29.	29	,,	12.	71	,,	23.	113	,,	4.	155
,,	30.	30	,,	13.	72	,,	24.	114	,,	5.	156
,,	31.	31	,,	14.	73	,,	25.	115	,,	6.	157
Feb.	1.	32	,,	15.	74	,,	26.	116	,,	7.	158
,,	2.	33	,,	16.	75	,,	27.	117	,,	8.	159
,,	3.	34	,,	17.	76	,,	28.	118	,,	9.	160
,,	4.	35	,,	18.	77	,,	29.	119	,,	10.	161
,,	5.	36	,,	19.	78	,,	30.	120	,,	11.	162
,,	6.	37	,,	20.	79	May	1.	121	,,	12.	163
,,	7.	38	,,	21.	80	,,	2.	122	,,	13.	164
,,	8.	39	,,	22.	81	,,	3.	123	,,	14.	165
,,	9.	40	,,	23.	82	,,	4.	124	,,	15.	166
,,	10.	41	,,	24.	83	,,	5.	125	,,	16.	167
,,	11.	42	,,	25.	84	,,	6.	126	,,	17.	168

June	18.	169	Aug.	8.	220	Sept.	28.	271	Nov.	18.	322
,,	19.	170	,,	9.	221	,,	29.	272	,,	19.	323
,,	20.	171	,,	10.	222	,,	30.	273	,,	20.	324
,,	21.	172	,,	11.	223	Oct.	1.	274	,,	21.	325
,,	22.	173	,,	12.	224	,,	2.	275	,,	22.	326
,,	23.	174	,,	13.	225	,,	3.	276	,,	23.	327
,,	24.	175	,,	14.	226	,,	4.	277	,,	24.	328
,,	25.	176	,,	15.	227	,,	5.	278	,,	25.	329
,,	26.	177	,,	16.	228	,,	6.	279	,,	26.	330
,,	27.	178	,,	17.	229	,,	7.	280	,,	27.	331
,,	28.	179	,,	18.	230	,,	8.	281	,,	28.	332
,,	29.	180	,,	19.	231	,,	9.	282	,,	29.	333
,,	30.	181	,,	20.	232	,,	10.	283	,,	30.	334
July	1.	182	,,	21.	233	,,	11.	284	Dec.	1.	335
,,	2.	183	,,	22.	234	,,	12.	285	,,	2.	336
,,	3.	184	,,	23.	235	,,	13.	286	,,	3.	337
,,	4.	185	,,	24.	236	,,	14.	287	,,	4.	338
,,	5.	186	,,	25.	237	,,	15.	288	,,	5.	339
,,	6.	187	,,	26.	238	,,	16.	289	,,	6.	340
,,	7.	188	,,	27.	239	,,	17.	290	,,	7.	341
,,	8.	189	,,	28.	240	,,	18.	291	,,	8.	342
,,	9.	190	,,	29.	241	,,	19.	292	,,	9.	343
,,	10.	191	,,	30.	242	,,	20.	293	,,	10.	344
,,	11.	192	,,	31.	243	,,	21.	294	,,	11.	345
,,	12.	193	Sept.	1.	244	,,	22.	295	,,	12.	346
,,	13.	194	,,	2.	245	,,	23.	296	,,	13.	347
,,	14.	195	,,	3.	246	,,	24.	297	,,	14.	348
,,	15.	196	,,	4.	247	,,	25.	298	,,	15.	349
,,	16.	197	,,	5.	248	,,	26.	299	,,	16.	350
,,	17.	198	,,	6.	249	,,	27.	300	,,	17.	351
,,	18.	199	,,	7.	250	,,	28.	301	,,	18.	352
,,	19.	200	,,	8.	251	,,	29.	302	,,	19.	353
,,	20.	201	,,	9.	252	,,	30.	303	,,	20.	354
,,	21.	202	,,	10.	253	,,	31.	304	,,	21.	355
,,	22.	203	,,	11.	254	Nov.	1.	305	,,	22.	356
,,	23.	204	,,	12.	255	,,	2.	306	,,	23.	357
,,	24.	205	,,	13.	256	,,	3.	307	,,	24.	358
,,	25.	206	,,	14.	257	,,	4.	308	,,	25.	359
,,	26.	207	,,	15.	258	,,	5.	309	,,	26.	360
,,	27.	208	,,	16.	259	,,	6.	310	,,	27.	361
,,	28.	209	,,	17.	260	,,	7.	311	,,	28.	362
,,	29.	210	,,	18.	261	,,	8.	312	,,	29.	363
,,	30.	211	,,	19.	262	,,	9.	313	,,	30.	364
,,	31.	212	,,	20.	263	,,	10.	314	,,	31.	365
Aug.	1.	213	,,	21.	264	,,	11.	315			
,,	2.	214	,,	22.	265	,,	12.	316			
,,	3.	215	,,	23.	266	,,	13.	317			
,,	4.	216	,,	24.	267	,,	14.	318			
,,	5.	217	,,	25.	268	,,	15.	319			
,,	6.	218	,,	26.	269	,,	16.	320			
,,	7.	219	,,	27.	270	,,	17.	321			

In the case of Leap Years the number for Mar. 1 and all succeeding dates would be *one* more than number in Table.

BANK CALCULATION OF INTEREST.

The calculation of interest is an important part of the work in Banks, where, however, to save time and labour, much use is made of **Tables of Interest** calculated at various rates per cent. for all periods from 1 day to 365 days, and usually on £1, £2, £3, &c., by single pounds up to £100; thereafter by ten pounds to £1000; and so on.

Laurie's Tables, perhaps the ones most used, are calculated in the above way at $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5 per cent. King's Tables giving the interest for 1 day, and others dealing with special calculations, are also found useful in most Banks.

The example given below shows the interest calculated by what is called "Extending the Decimal," but which is really multiplying the balance by the number of days so as to reduce it to an equivalent amount for 1 day.

Ex. Borrowed £302, 17s. 6d on April 1; paid £100 on April 29; £50, 10s. on June 8; and the balance on September 11. Find the interest due at $3\frac{1}{2}$ per cent.

Date.	Dr.	Cr.	Balances.	Da.	Decimal.
April 1 -	£302·875		Dr. £302·875	28	8480500
,, 29 -		£100·000	202·875	40	8115000
June 8 -		50·500	152·375	95	14475625
Sept. 11 -		152·375			3107112557

73000)217497875

£2, 19s. 7d. +

At each date on which any change is made in the account, the balance for the previous period is multiplied by the number of days in that period, and *extended*. The total of these extensions gives the equivalent of all the various balances for the several periods, for 1 day.

The interest has been calculated by multiplying the sum by twice the rate per cent. (No. of days is 1, therefore no multiplying) and dividing by 73000.

The next example shows the calculation of interest by *conversion to 5 per cent. rate*, which is the method followed in many Banks.

Ex. Borrowed £3000 on Jan. 1; £500 on Feb. 1; £1200 on March 11; £300 on July 5. Paid the whole on Aug. 3. Find interest due at 4 per cent.

Date.	Dr.	Cr.	Balances.		Da.	Products.
Jan. 1 - -	£3000		Dr.	£3000	31	93000
Feb. 1 - -	500		Dr.	3500	38	133000
Mar. 11 - -	1200		Dr.	4700	116	545200
July 5 - -	300		Dr.	5000	29	145000
Aug. 3 - -		£5000				
						916,200

At 4 °/₀ one day's interest on £916200 = £100, 8s. 1d.

In dealing with the above total product, bankers would say:—"At 4 °/₀ (=80 per cent. of 5 °/₀) £916200 for 1 day = 80 days in 'Laurie' on £9162 at 5 °/₀."

= £100, 8s. 1d.

In provincial Banks where Interest is allowed on Cr. current accounts—(London Banks give no interest on such accounts, and, as a compensation, do not charge for cashing cheques)—a usual practice is to calculate the interest on the **Minimum Monthly Balance**. The lowest daily balance for each month is taken, and Interest calculated on that for the month. Where that practice is not followed with Interest-bearing Cr. Current Accounts, the same method is followed as in Dr. Current Accounts. (Examples above.)

Date.		Dr.			Cr.				Balance.			Da.	Decimal.
		£	s.	d.	£	s.	d.		£	s.	d.		
July 1	By cheque				1736	2	6	Cr.	1736	2	6	12	20832
,, 13	,, cash				112	15	0	Cr.	1848	17	6	6	11094
,, 19	To cheque	1000	0	0				Cr.	848	17	6	6	5094
,, 25	,, ,,	512	10	6				Cr.	336	7	0		

When the shillings are above 10 the decimal is found from the next higher number of pounds; when below 10 no account is taken.

The method of *forming a table of days* is found most suitable for the calculation of interest on Deposit Accounts, or, as they are often called, Deposit Receipts.

Interest on these varies with the Bank of England rate, and the practice followed is to proceed on the conversion or *equalising to 5 °/₀ principle.* For this purpose decimal tables for Deposit Receipt (D/R) Interest are kept up daily.

Thus, at present, when the D/R rate is $1\frac{1}{2}$ °/₀ = $\frac{3}{10}$ of 5 °/₀ Interest for each day at this rate on £1 is equal to the Interest on £1 for **·3 of a day** at 5 °/₀. The equivalent period at 5 °/₀ is therefore ·3, which is added on daily till the rate changes.

Date.	Equivalent No. of days at 5 °/₀.
July 1	·3
,, 2	·6
,, 3	·9
,, 4	1·2
,, 5	1·5
,, 6	1·8
,, 7	2·1
Interest from June 30 to July 7 at $1\frac{1}{2}$ °/₀	= Int. for 2·1 days at 5 °/₀

Ex. Find the Interest on a deposit of £200 from July 21 to July 30, supposing D/R rate $1\frac{1}{2}$ °/₀ till July 26 and 2 °/₀ (= ·4 of 5 °/₀) afterwards.

By ordinary method :—

$$\text{Int.} = \frac{200 \times (5 \text{ days} \times 3 [\text{twice } 1\frac{1}{2} \text{ °/₀}] + 4 \times 4 [\text{twice } 2 \text{ °/₀}])}{73000}$$

$$= \frac{200 \times 31}{73000} = \frac{200 \times 3{\cdot}1 \times 10}{73000}$$

= Int. on £200 at 5 °/₀ for 3·1 days as shown in decimal table.

July 22	-	·3
,, 23		·6
,, 24	-	·9
,, 25	-	1·2
,, 26	-	1·5
,, 27	-	1·9
,, 28	-	2·3
,, 29	-	2·7
,, 30	-	3·1

EXERCISE 81.

1. Borrowed £600 on June 1; paid £200, July 1; £300, August 1. Find Interest at 5 per cent. due on October 1.

2. Lent £950 on May 28; received £200, June 12; £300, July 4; Balance, August 2. Find Interest at $2\frac{1}{2}$ per cent.

3. Lent £500 on Feb. 2; received £300 on May 15; £100 on August 1. Find Interest at $4\frac{1}{4}$ per cent. due on November 11.

4. Borrowed £525 on March 25; paid £200, June 24; £150, September 29. Find Interest at $2\frac{3}{4}$ per cent. due on December 25.

5. Borrowed £500 on January 11; paid £200 on May 8; £125 on June 12; and the Balance on November 25. Find the Interest at $3\frac{1}{2}$ per cent.

6. Borrowed £300, 15s. on January 1; paid one-fifth on March 1, and one-fifth on the first day of each alternate month thereafter till all was paid. Find the Interest at 5 per cent.

7. A bank advanced £3000 on January 1, 1896, and received £500 on the first day of every quarter till the whole was paid. Find the Interest at 4 per cent.

8. Borrowed £506, 12s. 6d. on June 12, 1902; paid £200, 19s. on September 15; £190, 7s. 6d. on December 14; and £30, 10s. on January 5, 1903. Find the Interest at 3 per cent. due April 5, 1903.

9. A freshman at Cambridge borrows 30 guineas *nine* days after the beginning of Michaelmas Term, Oct. 1; 25 guineas at the beginning of Lent Term, Jan. 13; £30 at the beginning of Easter Term, April 22. Find Int. at 4 per cent. due *sixteen* days after the end of Easter Term, June 24.

10. The inventor of a patent machine borrows £200 on Jan. 13; £100 on April 3; £50 on May 6; £75 on July 13. Find Int. at 4 per cent. due on Dec. 31.

11. An Oxonian received 50 guineas in loan on the first days of Lent, Easter, Trinity, and Michaelmas Terms, when they fell respectively on Jan. 14, April 22, June 3, and Oct. 10. Find Int. at 5 per cent. due on Dec. 17.

12. Lent £509, 12s. 6d. on April 1; £392, 15s. 6d. on June 8; £96, 8s. 6d. on June 26; and £341, 17s. 6d. on Sept. 11. Find Int. at $4\frac{1}{2}$ per cent. due on Dec. 31.

13. Deposited £200 in the bank on April 10 when Interest was 3 per cent. On May 15 Interest fell to 2½ per cent. ; on June 30 to 2 per cent. ; and on October 8 it rose to 3 per cent. Find the Interest due on November 7.

Find Interest on the following Deposit Receipts :—

14. £300 from September 24 to September 30 at 2½ per cent. ; and to October 15 at 2 per cent.

15. £500 from August 1 to October 7 at 2½ per cent. ; to May 15 at 3½ per cent. ; and to July 10 at 4 per cent.

16. £400 from April 1 to May 15 at 3 per cent. ; to June 30 at 2½ per cent. ; and to July 16 at 2 per cent.

COMPOUND INTEREST.

In Simple Interest the interest is reckoned on principal alone; but in **Compound Interest** the **interest due** is, at stated intervals, **added to the principal** and **interest reckoned on this increased principal** for the next period, and so on, the principal being increased at each period by the amount of interest then due.

Compound Interest is a rent paid not only for the use of the money borrowed but also for the use of the interest after it becomes due.

To find **Compound Interest** and **Amount.**

Calculations in compound interest may be made as a series of simple interest calculations; thus—At the given rate, find the **amount** of the Principal for the first period; find the **amount of this amount** for the second period, &c. &c. **Interest = Final Amount − Principal.**

Should there be a number of periods *and a fraction of a period*, the Amount is found for the number of complete periods and the *Simple Interest* on this Amount found for the fraction of a period.

Calculations in compound interest are usually, in Banks, &c., made by means of Tables (see example appended) of the amount of £1 for any period at certain rates per cent.

Table showing the Amount, at Compound Interest, of £1 at various Rates per cent. from 1 to 10 years.

No. of Years.	1 %	1½ %	2 %	2½ %	3 %	3½ %	4 %	4½ %	5 %	6 %	10 %
1	1·01000	1·01500	1·02000	1·02500	1·03000	1·03500	1·04000	1·04500	1·05000	1·06000	1·10000
2	1·02010	1·03022	1·04040	1·05062	1·06090	1·07122	1·08160	1·09202	1·10250	1·12360	1·21000
3	1·03030	1·04567	1·06121	1·07688	1·09273	1·10871	1·12486	1·14116	1·15763	1·19101	1·33100
4	1·04060	1·06136	1·08243	1·10379	1·12551	1·14752	1·16986	1·19251	1·21551	1·26247	1·46410
5	1·05101	1·07728	1·10408	1·13140	1·15927	1·18768	1·21665	1·24618	1·27628	1·33822	1·61051
6	1·06152	1·09344	1·12616	1·15969	1·19405	1·22925	1·26532	1·30226	1·34010	1·41851	1·77156
7	1·07213	1·10984	1·14868	1·18868	1·22987	1·27227	1·31593	1·36086	1·40710	1·50363	1·94872
8	1·08285	1·12649	1·17166	1·21839	1·26677	1·31680	1·36857	1·42210	1·47746	1·59384	2·14359
9	1·09368	1·14339	1·19509	1·24885	1·30477	1·36299	1·42331	1·48609	1·55133	1·68947	2·35795
10	1·10462	1·16054	1·21899	1·28010	1·34392	1·41069	1·48024	1·55297	1·62889	1·79084	2·59374

When calculations have to be made without these, the same principle of working is made use of—The amount of £1 for 1 year is raised to the power indicated by the number of years and the result multiplied by the Principal.

(Amount of £1 for given time at given Rate) × P = Amount.

Amount − Principal = Compound Interest.

To find the **Principal producing a given Amount** at a given rate in a given number of periods is the reverse process. This is also expressed as finding the **Present Worth** or Present Value of the Amount due in the given time and at the given rate.

(Amount ÷ Amount of £1 for given time at given rate) = P.

When the Interest is payable half-yearly or quarterly take as many periods as there are half-years or quarters, and use *half* or *quarter* the given rate.

Ex. Find the amount of £500 for 7 years at 4 %.

Amount of £1 for 1 year at 4 % = 1·04

Amount of £1 for 7 years at 4 % = £1·04 raised to the 7th power.

= £1·3159

£1·3159 × 500 = £657·96

= **£657, 19s. 2d.**

EXERCISE 82.

Use Table for the working of this Exercise.

1. Required the amount of £400 for 3 years at 5 per cent.
2. Required the amount of £640 for 4 years at 3 per cent.
3. Required the amount of £800 for 5 years at 4 per cent.
4. Required the amount of £800 for 5 years at 4 per cent., the interest payable half-yearly, and also quarterly.
5. What is the compound interest of £500 for 6 years at 3 per cent., payable yearly?
6. Find the compound interest of £450 for 5 years at 6 per cent., the interest payable half-yearly.
7. What is the compound interest of £725 for 3 years at 4 per cent., the interest payable quarterly?
8. Required the compound interest of £840 for 2 years at 3 per cent., the interest payable three times a year.

9. What principal will amount to (what is the *Present Value* of) £840 in 8 years at 3 per cent. per annum compound interest?
10. Find the present value of £210 due 10 years hence at 5 per cent. compound interest.
11. What principal will amount to £350 in 10 years at 4 per cent. per annum compound interest?
12. What sum will amount to £300 in 7 years at $2\frac{1}{2}$ per cent. per annum compound interest?
13. What principal put out at compound interest for 5 years at 5 per cent. will amount to £1280, 10s. 6d.?
14. What principal will amount to £1260 in 10 years at $4\frac{1}{2}$ per cent. compound interest?
15. What principal will amount to £1000 in 4 years at 4 per cent. per annum, interest payable half-yearly?
16. What principal will amount to £850 in $2\frac{1}{2}$ years at 3 per cent. per annum, interest payable half-yearly?
17. Find the compound interest on £970, 5s. 6d. for 6 years at $2\frac{1}{2}$ per cent. per annum.

In the following calculate the interest for each period on the balance during that period, and deduct payment made for next period's balance.

18. Lent on bond, June 1, 1874, the sum of £750, at 5 per cent.; of which I received, Oct. 8, 1875, £180; Jan. 1, 1877, £360; and the balance Aug. 14, 1878; how much did I then receive, interest at 5 per cent.?
19. Borrowed on bond, March 18, 1877, £600; of which I paid, July 27, 1878, £250; Nov. 17, 1879, £225; Feb. 24, 1880, £175; the balance is to be paid May 30, 1881; how much will it amount to at $4\frac{1}{2}$ per cent.?
20. Lent to A. B. on bond, Jan. 1, 1875, £1000; of which I received, May 25, 1876, £400; Sept. 3, 1877, £350; and the balance, Feb. 24, 1879; how much did I then receive, interest at $4\frac{3}{4}$ per cent.?

EXERCISE 83.

To be worked without Tables, and answers found correct to 3 places of decimals.

Find the amount of the following sums:—

1.	£600	for 2	years	at 3	%	7.	£697	15	0	for 6	years	at $2\frac{1}{2}$ %
2.	300	,, 3	,,	5	%	8.	468	10	6	,, 4	,,	4 %
3.	800	,, 4	,,	3	%	9.	232	7	6	,, 8	,,	3 %
4.	400	,, 4	,,	4	%	10.	35	3	9	,, 3	,,	$3\frac{1}{2}$ %
5.	700	,, 4	,,	$2\frac{1}{2}$	%	11.	666	13	4	,, 5	,,	$2\frac{1}{2}$ %
6.	834	,, 5	,,	$3\frac{1}{2}$	%	12.	267	19	2	,, 7	,,	$4\frac{1}{2}$ %

13. Find the amount of £670 for 3 years at 6 °/₀, supposing the interest to become due half-yearly.

14. Find the amount of £684 for 3 years at 4 °/₀, supposing the interest to be due quarterly.

15. What is the compound interest on £764, 12s. 6d. for 4 years at 5 °/₀, due half-yearly?

16. Find the compound interest on £29, 15s. for 3½ years at 3½ °/₀, due quarterly.

17. Find the difference between the simple and the compound interest on £750 for 3 years at 4½ °/₀.

18. A sum of £300 is lent for one year at 4 °/₀; find the difference between the simple and the compound interest, due quarterly.

19. To what will a legacy of £500 left to a boy 11 years of age have accumulated at compound interest, on his attaining *majority* at 21 years of age, allowing interest at 5 °/₀?

20. A legacy of £2500 was left to a young lady in 1890 on condition that it should be improved at compound interest for a marriage-portion. To what will it have accumulated at her marriage in 1898, reckoning interest at 5 °/₀?

Find the present value of—

21. £900 due in 2 years at 4 °/₀

22. 700 ,, 4 ,, 5 °/₀

23. 1200 ,, 4 ,, 3 °/₀

24. 1405 11 6 due in 4 years at 4 °/₀

25. 105 11 3 ,, 3 ,, 3½ °/₀

26. 333 3 4 ,, 5 ,, 2½ °/₀

27. What sum in 3 years at 4 °/₀ will amount to £100 supposing the interest to be paid quarterly?

28. Find the sum which, with half-yearly payments of interest, will at 6 °/₀ amount in 4 years to £253·354.

29. A merchant who has increased each year's capital by *a tenth*, finds that at the end of 12 years he has £3985, 16s. 1½d. Find his original capital.

30. A sloop was bought by A, who sold it to B, by whom it was sold to C, who finally disposed of it. Each gained 30 °/₀ on his prime cost. C sold it for £659, 2s.; what did A pay for it?

DISCOUNT AND PRESENT WORTH.

When a borrower does not repay the sum due till after the specified date, the creditor exacts payment of the sum due + interest on this sum for the further period till payment is

made; and just so, should the repayment be made before the specified time, the borrower is entitled to *claim a deduction*, that is, to pay a sum less than the amount due. The deduction made—that is, the **difference between the amount due and the sum actually paid—is called Discount.** This calculation of discount differs from the Cash Discount dealt with earlier in that *Time enters into the calculation* here.

To correctly estimate the discount to be allowed, the Principal is found which will amount to the given sum in the stated time and at the given rate. This is called the **Present Worth** or Present Value, and

Amount − Present Worth = Discount.

Discount so calculated is called, for the sake of distinguishing it, Arithmetical or **True Discount.**

Ex. Find the present worth of and true discount on a debt of £609 paid three months before due at 6 %.

$$\text{By interest formula } P = \frac{100 \times A}{100 + (R \times T)}$$

$$= \frac{100 \times 609}{100 + (\frac{1}{4} \text{ yr.} \times 6)}$$

$$= \frac{100 \times 609 \times 2}{203} = £600$$

Ans. Present Worth £600, and **True Discount** £609 − £600 = **£9.**

The True Discount can also be found, without finding the Present Worth, *and vice versâ*, by the proportion principle.

Thus, in the above example—

(£100 + interest on £100 for 3 months at 6 %) : £100
= Amount : Present Worth.

$$\frac{101\frac{1}{2}}{100} = \frac{609}{\text{P.W.}}; \quad 101\tfrac{1}{2} \times \text{P.W.} = 609 \times 100; \quad \text{P.W.} = \frac{609 \times 100}{101\frac{1}{2}} = £600$$

and so also

(£100 + interest on £100 for 3 months at 6 %) : (interest on £100 for 3 months at 6 %) = Amount : True Discount.

$$\frac{101\frac{1}{2}}{1\frac{1}{2}} = \frac{609}{\text{T.D.}}; \quad 101\tfrac{1}{2} \times \text{T.D.} = 609 \times 1\tfrac{1}{2}; \quad \frac{609 \times 1\frac{1}{2}}{101\frac{1}{2}} = £9$$

In business transactions where large sums are dealt with payments are less frequently made in money than by other and more convenient means. Other forms of making payment are by **Cheques**, by **Promissory Notes**, and by **Bills of Exchange** or Drafts.

1. Form of Cheque.

No. X 137.	Bank of England.	No. X 137. *3rd Sept. 1903.*	(Embossed Penny Stamp)
3rd Sept. 1903.		*To the* **Bank of England.**	
Payable to		**Pay to** *Mr John Johnston*	
Mr J. Johnston		or Bearer, *Twenty Pounds Ten Shillings*	
or Bearer.		*and Sixpence Sterling.*	
£20 : 10 : 6		£20 : 10 : 6	*Thomas R. Smith.*

2. Form of Promissory Note.

£100. Edinburgh, *July* 22, 1903.

Two months from date I promise to pay to Mr George White, Glasgow, the sum of One Hundred Pounds : value received.

John Goodfellow.

3. Form of Bill of Exchange.

£250. Edinburgh, *July* 22, 1903.

Three months after date pay to me or my order the sum of Two Hundred and Fifty Pounds Sterling for value received.

Mr James Banks, Glasgow. John Brown.

The third of the above is the form of an Inland Bill of Exchange or Draft. It is said to be **drawn** by the one who signs it, and is sent by him to a customer to whom he has consigned goods. If acknowledged as a correct statement of his indebtedness, the customer **accepts** it by writing across it the word "Accepted" and his signature; and returns it to the **drawer**, who **holds** it now as legal warrant of his claim to be paid the sum mentioned, at the stated time.

He may, however, require money for some purpose before that date, and in such case he applies to a banker to **discount the Bill.** If satisfied with the security, the banker *advances a certain sum less than the amount of the bill*—the difference being the **Banker's Discount.** This discount, called also "*ordinary discount*," is the **Simple Interest on the**

amount of the bill for the period from the date of discounting till the date on which the bill is due, at the stated rate per cent.

In bill discounting the date of payment is reckoned 3 days later than the date stated in the bill. These are spoken of as "**Days of Grace.**"

The following should be noted:—

Banker's Discount – True Discount
= Interest on True Discount

for the specified time at the stated rate.

For:—Banker's Discount – True Discount

$$= \frac{A \times R \times T}{100} - \frac{A \times R \times T}{100 + (R \times T)}$$

$$= (A \times R \times T) \times \left(\frac{1}{100} - \frac{1}{100 + (R \times T)}\right)$$

$$= (A \times R \times T) \times \frac{100 + (R \times T) - 100}{100 \times [100 + (R \times T)]}$$

$$= \frac{A \times R \times T}{100 + (R \times T)} \times \frac{R \times T}{100} = \text{True Discount} \times \frac{R \times T}{100}$$

= Interest on True Discount.

When the period till the bill is due (called the **currency** of the bill) is a number of years, it is usual to make the calculation, *e.g.*, in questions of Annuity and Insurance, by Compound Interest; but for shorter periods simple interest is reckoned, in which case

$$\text{Present Worth} = \frac{A \times 100}{100 + (R \times T \text{ years})}$$

$$\text{True Discount} = \frac{A \times R \times T \text{ (years)}}{100 + (R \times T \text{ years})}$$

$$\text{Ordinary or Banker's Discount} = \frac{A \times R \times T}{100}$$

EXERCISE 84.

Use Table of Days. Answers correct to nearest farthing.

Find the *Banker's* and the *True* Discount on the following bills : –

		Drawn.	Discounted.
1.	£300	Mar. 25 for 3 months	April 16 at 4 per cent.
2.	£600	June 23 ,, 3 ,,	July 15 ,, 4 ,,
3.	£275	Aug. 4 ,, 2 ,,	Aug. 31 ,, 5 ,,
4.	£360	Mar. 19 ,, 2 ,,	April 10 ,, 3 ,,
5.	£275	Mar. 11 ,, 3 ,,	April 1 ,, 5 ,,
6.	£720	Oct. 19 ,, 2 ,,	Nov. 10 ,, 3 ,,
7.	£137, 10s.	Mar. 7 ,, 2 ,,	April 3 ,, 5 ,,
8.	£315, 10s.	July 10 ,, 4 ,,	Sept. 11 ,, 3½ ,,
9.	£480, 12s. 6d.	Jan. 1 ,, 6 ,,	Mar. 31 ,, 4 ,,
10.	£157, 15s.	Nov. 30 ,, 3 ,,	Dec. 30 ,, 3½ ,,
11.	£68, 15s.	Oct. 31 ,, 4 ,,	Jan. 25 ,, 5 ,,
12.	£240, 6s. 3d.	Oct. 31 ,, 4 ,,	Nov. 28 ,, 4 ,,

13. What sum will at the rate of 5 per cent. amount in a year to £75?

14. Find the present worth of £89 due in a year at 5 per cent.

15. The price of goods, allowing 6 months' credit at 5 per cent., is £4, 8s. 10d. Find the ready-money price.

16. What ready money is equivalent to 30s. 6d. with 4 months' credit at 5 per cent.?

17. The credit price of a newspaper per annum is £2, 4s. Find the ready money payable in advance, taking true discount at 10 per cent.

18. What sum due in one day will produce 1d. of true discount at 5 per cent.?

19. What sum due in one day will produce 1s. of ordinary discount at 5 per cent.?

20. Find the *ordinary* discount on a sum for 1 year at 5 per cent., of which the *true* discount for the same time and rate is 5s. 5d.

21. Find the *true* discount on a sum from April 1 to June 13, at 2 per cent., of which the *ordinary* discount for the same time and rate is £12, 11s.

22. Bought goods for £65, and 4 months' credit, and am offered a discount of £5 for present payment; at what rate per cent. is the offer made?

23. The present value of £393, 1s. 10d., due a certain time hence, is £365, 13s. 4d.; required the time, interest at 3 per cent.

24. Find the net proceeds of a bill of £191, 12s. 6d., dated March 8, at 3 months, and discounted April 12, at 5 per cent.

25. Find the discount upon a bill of £573, 16s. 8d., dated March 4, at 4 months, and discounted May 3 at 5 per cent.

26. Find the net proceeds of a bill of £478, 14s., dated July 6, at 2 months, and discounted July 24 at 4½ per cent.

27. What is the discount on a bill of £237, 12s., dated October 16, at 5 months, and discounted January 12 at 3¾ per cent. ?

28. What is the net proceeds of a bill of £348, dated January 25, at 6 months, which was discounted May 3 at 5 per cent., and commission ½ per cent. ?

29. Required the discount of a bill for £579, dated May 24, at 3 months, discounted June 6 at 5 per cent.

30. Required the net proceeds of the following bills, discounted on September 17: W. S. on I. M., £450, dated August 12, at 4 months; S. T. on A. B., £346, 10s., dated July 29, at 5 months; D. E.'s bill of £196, dated September 3, 3 months, and E. G. on H. L., £268, dated August 1, 3 months, deducting 5 per cent. and ½ per cent. commission.

EQUATION OF PAYMENTS.

A problem which often occurs in business, especially in Bill transactions, is to find on what date a fair discharge of a number of payments due on various dates may be made by a single payment of the full amount.

This date is called the **Equated Time**, and the calculation is spoken of as one in **Equation of Payments**.

The principle involved is one of average.

Ex. Find the equated time of payment of £300 due in 2 months, £250 due in 3 months, and £200 due in 4 months.

300 × 2	=	600
250 × 3	=	750
200 × 4	=	800
7'5'0		)2150(2·86 months, Ans.
		650
		50

When *the dates* on which the bills fall due are given, the method followed is this:—An *arbitrary date* is fixed on—either the first day of the month in which the first payment falls due, or more *usually the date of the first payment due.*

Each sum is multiplied by the number of days between that fixed date and its date of payment, and the total of the products divided by the total of the payments due.

Ex. Find the equated time of a bill for £100 due on Aug. 5, one for £60 due Sept. 1, and a third for £200 due Sept. 10.

(1) Taking Aug. 1 as the arbitrary date—

100 × 4	=	400
60 × 31	=	1860
200 × 40	=	8000
360		)10260(28 days
		720
		3060
		2880

28 days after Aug. 1 is **Aug. 29. Ans.**

(2) Taking date of first payment due, *i.e.*, Aug. 5—

100 × 0	=	0
60 × 27	=	1620
200 × 36	=	7200
360		)8820(24 days
		720
		1620
		1440

24 days after Aug. 5 is **Aug. 29. Ans.**

If one payment is *overdue*, count as from its date in all cases, and deduct the number of days overdue from the final result for the correct answer.

EXERCISE 85.

Find the equated time for paying the following sums due in the following number of days :—

1. £40 in 54 days, £80 in 36 days.
2. £30 in 58 days, £90 in 26 days.
3. £19 in 12 days, £22 in 24 days, £31 in 36 days.
4. £360 in 15 days, £140 in 20 days, £400 in 17 days.
5. $\frac{1}{3}$ of a debt in 6 mo., $\frac{5}{12}$ in 7 mo., $\frac{1}{6}$ in 8 mo., and the remainder in 9 mo.

6. $\frac{4}{7}$ of a debt in 3 mo., $\frac{2}{7}$ in 4 mo., and the remainder in $4\frac{1}{2}$ mo.
7. £190 payable *to-day*, £220 in 12 days, £310 in 24 days.
8. £95 payable 3 days *ago*, £110 in 9 days, £155 in 21 days.
9. $\frac{1}{8}$ of a debt payable *to-day*, $\frac{5}{8}$ in 48 days, and the remainder in 64 days.
10. Find the time for paying at once, £20 due in 3 mo., £30 due in 4 mo., £40 due in 6 mo., and £50 due in 7 mo.
11. I owe £60 in 40 days, £80 in 60 days, and £120 in 108 days; when ought the whole to be paid?
12. I owe £74 in 50 days, £108 in 250 days, and £100 in a year; required the equated time for paying the whole.
13. I owe £100 in 30 days, £270 in 118 days, and £365 in 1 year and 110 days; when ought the whole to be paid?
14. £240 is to be paid as follows: £60 in 60 days, £80 in 96 days, £40 in 250 days, and the rest in a year and 35 days; required the equated time for paying the whole.
15. Find the equated time for paying in one sum $\frac{3}{5}$ of a debt due on Christmas, $\frac{1}{5}$ on Lady-day (March 25), $\frac{1}{5}$ on Midsummer-day (June 24), and the rest on Michaelmas (Sept. 29).

Find the date on which the sum of the following debts can be adequately paid:—

16. £115 due on Mar. 2; £300 on Mar. 20; £600 on Mar. 21; £500 on Mar. 29.
17. £30 due on Apr. 1; £50 on Apr. 16; £30 on Apr. 26; £25 on May 1; and £15 on May 21.
18. £64 due on Apr. 1; £60 on Apr. 13; £50 on Apr. 18; £30 on Apr. 20; £28 on Apr. 24.

STOCKS AND SHARES.

In many commercial undertakings the necessary capital is so great that no individual is likely to be able to provide it, but a number of people subscribe the amount among them. These form together what is called a **Joint-Stock Company**, or shortly a **Company**; and the whole of the capital is spoken of as their **Stock**. Each member of the company "holds" or owns **Shares** of this stock in proportion to the part of the Capital he subscribed.

Government Stocks consist of all the various loans granted to the Government, and which together form the

National Debt. Those who lent the money originally, or those to whom they have transferred their claims, are said to have shares in the "**Funds**" or to have money invested in "**Consols**," both terms being abbreviations of *British Consolidated Fund*.

Holders of Government Stock receive quarterly payments of Interest at a certain fixed rate on the value of the stock which they hold. These payments are called **Dividends**. Holders of shares usually also receive Dividends but at variable rates.

A person who has invested money in Government or other stock is not entitled to ask a return of the money paid, but he may sell or otherwise transfer his stock to others. The buying and selling of Stocks and Shares is done in a special market known as the **Stock Exchange**, and the ordinary public can only buy or sell through the agency of a **Stock Broker** who charges a certain percentage (usually $\frac{1}{8}$ %) as Commission.

This charge *increases* the market price to a buyer, and *decreases* the market value to a seller.

Stocks are usually quoted at *so much per cent.*, that is to say, the quoted price is the market value of stock which, for calculation of dividend, is valued at £100.

The market prices vary according as the demand for a certain stock is great or small. When Stock valued at £100 sells for exactly £100 it is said to be selling **At Par**; when the market price rises above the valuation the stock is **At a Premium**, and when it falls below the valuation the stock is spoken of as **At a Discount**.

Shares have many *nominal* values—Five-pound shares; Twenty-pound shares, &c. By "nominal value" is meant the original value of the share; and however the market price may rise or fall, whether the buyer pays £5 or £15 for a Ten-pound share, the dividend will be calculated at the settled rate per cent. on the nominal value.

In Stock Exchange quotations the expressions "**Cum Div.**" and "**Ex. Div.**" sometimes occur. The meaning of "Cum Div." (*with dividend*) is, that the price quoted entitles the purchaser to draw the dividend on the shares for

the period ending at the date of purchase or very shortly thereafter. "Ex. Div." (*without dividend*) means that the seller retains the claim for dividend on the shares for the period preceding the sale. The "Cum Div." price of any particular stock is therefore always higher than the "Ex. Div." price of the same stock.

In both Stocks and Shares, interest (dividend) is calculated on the nominal value.

The principal calculations in stock transactions are :—

1. To find the **buying price** of a certain quantity of a specified stock at a given price.

$$\text{Buying Price} = \frac{\text{Stock} \times (\text{Price} + \text{Brokerage})}{100}$$

Ex. Find the buying price of £650 stock at $80\frac{3}{4}$.

$$\frac{650 \times (80\frac{3}{4} + \frac{1}{8})}{100} = \frac{650 \times 80\frac{7}{8}}{100} = \frac{650 \times 80{\cdot}875}{100}$$

$$= £525{\cdot}6875 = \textbf{£525, 13s. 9d. Ans.}$$

To find **the selling price** instead of the buying price, **deduct the $\frac{1}{8}$ in the numerator,** instead of adding it as above.

2. To find what **quantity of stock** at the quoted price can be **bought** for a certain sum of money.

$$\text{Stock} = \frac{\text{Money} \times 100}{\text{Price} + \text{Brokerage}}$$

Ex. How much stock at $91\frac{7}{8}$ may be bought for £828 ?

$$\text{Stock} = \frac{828 \times 100}{91\frac{7}{8} + \frac{1}{8}} = \frac{828 \times 100}{92} = \textbf{£900 Ans.}$$

To find what quantity of stock must be **sold** to **realise** the given sum, **deduct the $\frac{1}{8}$ in the denominator,** instead of adding it as above.

3. To find the **rate per cent.** per annum arising from investing money in a certain stock at specified price.

$$\text{Rate } \% = \frac{\text{Rate (name of stock)} \times 100}{\text{Price} + \text{Brokerage}}$$

Ex. If I purchase 3 per cent. stock at $95\frac{7}{8}$, what rate per cent. do I get on my money?

$$\text{Rate } \% = \frac{3 \times 100}{95\frac{7}{8} + \frac{1}{8}} = \frac{3 \times 100}{96} = 3\frac{1}{8} \text{ Ans.}$$

4. To find **at what price** a given stock at given rate must be **bought** to give a specified percentage of interest on the money invested.

$$\text{Price (without brokerage)} = \frac{\text{Rate (name of stock)} \times 100}{\text{Rate per cent. required}}$$

Ex. At what price must the 3 per cents. be bought to give a return of 5 per cent. on the investment?

$$\text{Price} = \frac{3 \times 100}{5} = £60$$

Ans. £60 + $\frac{1}{8}$ = **£60, 2s. 6d.**

5. To find the **income** obtained by investing a stated sum in a stock of given price and rate.

$$\text{Income} = \frac{\text{Sum} \times \text{Rate (name of stock)}}{\text{Price} + \text{Brokerage}}$$

Ex. What income will a man derive from investing £899 in the 4 per cents. at $115\frac{7}{8}$?

$$\text{Income} = \frac{899 \times 4}{115\frac{7}{8} + \frac{1}{8}} = \frac{899 \times 4}{116} = £31$$

6. To find **how much money** must be invested in a stock of given price and rate to produce a given income.

$$\text{Money} = \frac{\text{Income} \times (\text{Price} + \text{Brokerage})}{\text{Rate (name of stock)}}$$

Ex. How much money must a man invest in the 3 per cents. at $98\frac{1}{2}$ to secure an annual income of £900?

$$\text{Sum of money required} = \frac{900 \times (98\frac{1}{2} + \frac{1}{8})}{3}$$

$$= \frac{900 \times 98\frac{5}{8}}{3} = £29587, 10\text{s. Ans.}$$

All other problems occurring in stock calculations are compounded of one or other of these, and solutions should be found by methods indicated above.

EXERCISE 86.

(*Brokerage only when mentioned.*)

1. Find the income from £56525 stock in the 3 per cents.?
2. Find the annual income for £10871, 10s. stock in the 3 per cents.
3. Find the value of £800 stock at $95\frac{1}{8}$.
4. Find the value of £450 stock at 88.
5. Find the buying price of £375 stock in the 3 per cents. at $70\frac{3}{8}$, allowing brokerage at $\frac{1}{8}$ per cent.
6. What was paid for £650 stock in the $3\frac{1}{2}$ per cents. at $91\frac{1}{4}$, allowing brokerage at $\frac{1}{8}$ per cent.?
7. Find the selling price of £330 stock in the 3 per cents. at $87\frac{1}{2}$, paying brokerage at $\frac{1}{8}$ per cent.
8. How much was obtained for £570 stock in the $3\frac{1}{2}$ per cents. at $94\frac{5}{8}$, allowing brokerage at $\frac{1}{8}$ per cent.?
9. Find the quantity of stock at $81\frac{7}{8}$ worth £655.
10. Find the quantity of stock at $83\frac{3}{4}$ worth £502, 10s.
11. How much stock at $93\frac{3}{8}$ may be bought for £750, allowing brokerage at $\frac{1}{8}$ per cent.?
12. How much stock at $81\frac{1}{4}$ may be bought for £434, allowing brokerage at $\frac{1}{8}$ per cent.?
13. Find the quantity of stock at $96\frac{1}{8}$ which will realise £576, allowing brokerage at $\frac{1}{8}$ per cent.
14. Find the quantity of stock at $92\frac{1}{2}$ which will realise £739, allowing brokerage at $\frac{1}{8}$ per cent.
15. Find the rate of interest obtained when the $3\frac{1}{2}$ per cents. are at $95\frac{1}{2}$.
16. What rate of interest is obtained when the $3\frac{3}{4}$ per cents. are at $97\frac{1}{2}$?
17. How do the 3 per cents. stand when they yield 4 per cent.?
18. What is the price of the $3\frac{1}{4}$ per cents. when they yield $3\frac{1}{2}$ per cent.?
19. What income is derived from a capital of £611, 5s. invested in the $3\frac{1}{2}$ per cents. at $81\frac{1}{2}$?
20. Find the income derived from £308 invested in the $3\frac{1}{4}$ per cents. at 82.
21. What sum must be invested in the $3\frac{3}{4}$ per cents. at $84\frac{1}{4}$ to produce an income of £50?
22. How much must be invested in the $3\frac{1}{2}$ per cents. at $92\frac{1}{2}$ to produce an income of £504?
23. A legacy of £2000, reduced by a duty of 3 per cent., has been invested in the $3\frac{1}{4}$ per cents. at $97\frac{3}{8}$. Find the amount of the annually derived income.
24. Bought £300 stock at $90\frac{1}{2}$, and sold it at $95\frac{1}{4}$; what was gained, allowing $\frac{1}{8}$ per cent. for brokerage on both buying and selling price?

25. When the 3 per cents. are at 89, at what rate must the $3\frac{1}{2}$ per cents. stand to produce the same rate of interest?

26. Find the difference in the rate of interest between the 3 per cents. at 90 and the $3\frac{1}{2}$ per cents. at 98.

27. A person buys £800 stock at 91, and sells out at $93\frac{1}{8}$. What does he gain, allowing $\frac{1}{8}$ per cent. for brokerage on the buying and selling price?

28. Invested £1380 in stock at $91\frac{7}{8}$, and sold out at $90\frac{1}{8}$. How much was lost, reckoning the usual brokerage on the buying and the selling price?

29. How much India 5 per cents. at $101\frac{1}{4}$ may be purchased for £1593?

30. What should be paid for £3420 bank stock (10 per cent.) at 243?

31. How much stock at $152\frac{1}{2}$ can be purchased for £1822, 7s. 6d.?

32. What is the value of £1260 stock at $65\frac{3}{8}$?

33. What annual income is derived from investing £1058, 18s. 4d. in 10 per cent. bank stock at $242\frac{3}{8}$, including brokerage $\frac{1}{8}$ per cent.?

34. When the 3 per cents. are at $88\frac{3}{8}$, 10 per cent. bank stock at 244, and 5 per cents. at $104\frac{1}{4}$, which is the best investment?

35. What income will a person derive annually by investing £926, 2s. in Great Northern Railway stock ($8\frac{3}{4}$ per cent.) at £135 per cent.?

36. What should be paid for £476, 7s. 6d. South-Eastern and Chatham Railway stock at $92\frac{1}{2}$, including brokerage $\frac{1}{8}$ per cent.?

37. At what price should money be invested in India 5 per cents. to yield 4 per cent.?

38. What is the price of 10 per cent. bank stock, when £6396, 19s. 6d. invested in it produces £263, 5s. yearly?

39. Invested £2297, 15s. in 3 per cents. at $88\frac{3}{8}$, and sold out at $88\frac{7}{8}$; what was the gain?

40. If £3365, 11s. be invested in the 5 per cents. at $103\frac{3}{4}$, and sold out at $104\frac{1}{4}$; what difference will it make in my income to reinvest the proceeds in 10 per cent. bank stock at $242\frac{7}{8}$ per cent., brokerage $\frac{1}{8}$ per cent.?

41. Purchased an estate, which let for £530, 9s. yearly, at 21 years' purchase; how much 5 per cents. at 103 must be sold out to pay for it?

42. If £2153, 6s. be invested in the 3 per cents. at $88\frac{1}{4}$; at what rate should the stock be sold to gain £9, 3s.?

43. A gentleman has £3753, 6s. 8d. in the 3 per cents. at $88\frac{1}{2}$; how much should he invest in the 5 per cents. at 104 to secure from both investments £274, 12s. yearly, brokerage $\frac{1}{8}$ per cent.?

44. How much bank stock at 244 must be sold out to pay a debt of £500, brokerage $\frac{1}{8}$ per cent.?

45. A person has $\frac{1}{4}$ of his money in the 3 per cents., $\frac{1}{6}$ in the 5 per cents., $\frac{1}{6}$ in 10 per cent. bank stock, and £2300 in stock which yields 8 per cent.; what is his annual income?

46. What rate per cent. is derived by purchasing into the 4 per cents. at $104\frac{5}{8}$, brokerage $\frac{1}{8}$ per cent.?

47. Invested £3040, 4s. in $4\frac{1}{2}$ per cents. at 108: at what price should it be sold out, so as to gain £35, 3s. 9d.?

48. If £4134, 7s. 6d. be invested in the 3 per cents. at $87\frac{1}{2}$, and sold out at $88\frac{5}{8}$; what difference will it make in my yearly income to reinvest the proceeds in the 4 per cents. at $102\frac{1}{2}$?

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

In 1790 the National Assembly of France appointed a Commission to devise a more satisfactory system of weights and measures than was then in use, and the outcome was the introduction of the **Metric System.**

The **unit of length (the Metre)** on which the whole system is founded, is the length of a platinum bar, safely kept at Paris, which was originally taken as being the $\frac{1}{10000000}$ part of the distance from the North Pole to the Equator, measured on the French First Meridian. It is a little longer than our standard yard, being equal to 39·37 + inches.

The units of the various quantities (length, area, volume, and weight) are all connected with each other and with the Metre by simple relations.

Thus, the **Are** (standard of *area**) 100 sq. metres

Litre (standard of *volume*) = $\frac{1}{1000}$ cub. metre

Gram (,, *weight*) = $\frac{1}{1000}$ of the weight of 1 litre of water

Latin prefixes are used to denote the *sub-multiples*, and Greek prefixes the *multiples* of these units.

Deci = one-tenth	**Deka** = ten
Centi = one-hundredth	**Hecto** = one hundred
Milli = one-thousandth	**Kilo** = one thousand

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* For land measurements on a small scale, such as fields,

Table of Length.

10 millimetres (mm.)	=	1 centimetre (cm.)
10 centimetres	=	1 decimetre (dm.)
10 decimetres	=	**1 metre** (m.)
10 metres	=	1 dekametre (Dm.)
10 dekametres	=	1 hectometre (Hm.)
10 hectometres	=	1 kilometre (Km.)

It is usual to write, as above, the contracted forms for units and sub-multiples in *small* letters, and for multiples in *capitals*.

Table of Area.

Carrying out the system, the unit of area is *a square, of which the side is one metre.*

100 sq. millimetres	=	1 sq. centimetre
100 sq. centimetres	=	1 sq. decimetre
100 sq. decimetres	=	**1 sq. metre** (1 centiare)
100 sq. metres	=	1 sq. dekametre (1 are)
100 sq. dekametres	=	1 sq. hectometre (1 hectare)
100 sq. hectometres	=	1 sq. kilometre

Measurements of the work of builders and other tradesmen are expressed in **square metres**; of fields and farms in **ares**; and of large areas in **square kilometres.**

An **are** (a.) is the area of a square of 10 metre side
A **hectare** (Ha.) „ „ 100 „
A **centiare** (ca.) „ „ 1 „

Cubic Measure.

1000 cub. millimetres	=	1 cub. centimetre
1000 cub. centimetres	=	1 cub. decimetre
1000 cub. decimetres	=	1 **cub. metre**

The cubic metre is the standard of measurement for excavations, blocks of stone, &c. In measuring wood the cub. metre is called a **stere.**

Table of Capacity.

10 millilitres (ml.)	=	1 centilitre (cl.)
10 centilitres	=	1 decilitre (dl.)
10 decilitres	=	1 **litre** (l.)
10 litres	=	1 dekalitre (Dl.)
10 dekalitres	=	1 hectolitre (Hl.)
10 hectolitres	=	1 kilolitre (Kl.)

Table of Weight.

The unit of weight is the **gram**, which is the weight of a cubic centimetre of distilled water at its greatest density and weighed in a vacuum, but the **standard weight** is the **kilogram of platinum** kept in safe custody in Paris.

10 milligrams (mg.)	=	1 centigram (cg.)
10 centigrams	=	1 decigram (dg.)
10 decigrams	=	1 **gram** (g.)
10 grams	=	1 dekagram (Dg.)
10 dekagrams	=	1 hectogram (Hg.)
10 hectograms	=	1 kilogram (Kg.)

The kilogram (called shortly **kilo**) is the ordinary commercial weight corresponding to the English pound (lb.).

For heavy weights there are used the

quintal (Q.) = 100 kilograms

and **tonne** (*tonneau de mer*) (T.) = 1000 kilograms

The following parallel comparisons of the principal of these quantities with their English equivalents may be found useful.

1 centimetre = ·3937 inch	1 inch = 2·540 cm.
1 kilometre = ·6214 miles	1 mile = 1·609 km.

1 sq. cm.	=	·1550 sq. in.	1 sq. in. = 6·451 sq. cm.
1 sq. m. / 1 centiare	=	10·764 sq. ft. / 1·196 sq. yd.	1 sq. yd. = ·8361 sq. m.
1 hectare	=	2·471 acres	1 acre = ·4047 Ha.

1 cub. cm. = ·061 cub. in.	1 cub. in. = 16·386 cub. cm.
1 cub. metre = 1·308 cub. yd.	1 cub. yd. = ·7645 cub. m.

1 litre = ·2201 gallon 1 gall. = 4·546 litres
1 litre = 1·76 pints (rather more than $1\frac{3}{4}$ pints)
1 pint = ·568 litre (,, ,, $\frac{1}{2}$ litre)

1 gram = 15·432349 grains 1 grain = ·06479 g.
1 Kg. = 2·2046 lb. 1 lb. av. = ·45359 Kg.
1 tonne = ·9842 ton 1 ton = 1·016 tonnes

Reduction in the metric system becomes exceedingly simple, as it requires only multiplication and division by powers of 10, which is done (see decimals) by merely moving the point to right or to left.

Ex. *Reduce* 7 *Hm.* 3 *m.* 8 *dm.* 5 *cm. to metres, to dekametres, to millimetres, and to kilometres.*

7 Hm. 3 m. 8 dm. 5 cm. = 703·85 m. { Since 1 Hm. = 100 m. / 1 dm. = ·1 m. / and 1 cm. = ·01 m.

= 70·385 Dm. Since 1 m. = ·1 Dm.

= 703850 mm. Since 1 m. = 1000 mm.

= ·70385 Km. Since 1 m. = ·001 Km.

In **Addition, Subtraction,** &c., it is most convenient to express all the quantities in one denomination before working.

Ex. 1. *Express as hectolitres and add:* 5 *Hl.* 4 *Dl.* 7 *l.* 8 *cl.*; 215 *l.*; 5427 *dl.*; 27·32 *Dl.*; ·29 *l.*; 18946 *cl.*; 18 *l.* 9 *dl.* 4 *cl.*

5 Hl. 4 Dl. 7 l. 8 cl.	=	5·4708 Hl.
215 l.	=	2·15 ,,
5427 dl.	=	5·427 ,,
27·32 Dl.	=	2·732 ,,
·29 l.	=	·0029 ,,
18946 cl.	=	1·8946 ,,
18 l. 9 dl. 4 cl.	=	·1894 ,,
		17·8667 Hl.

The addition of the above quantities might have been equally well effected by reducing them all to any other denomination, say, litres.

Ex. 2. *A litre of mercury weighs* 13 *Kg.* 598 *g. Find the weight of* 369 *centilitres.*

369 cl. = 3·69 l.

∴ Weight of 369 cl. mercury = 13·598 × 3·69 kilograms.
= 50·17662 kilograms.

In connection with business transactions, it is often necessary to change English weights and measures into the metric equivalents, and *vice versâ*.

The degree of accuracy attained depends upon the *number of correct places* in the equivalent used as multiplier or divisor, and in any case can only be approximate.

Ex. Express 8 tons 8 cwt. 2 qr. 2 lb. in kilograms.

8 tons 8 cwt. 2 qr. 2 lb. = 18874 lb.
1 Kg. = 2·2046 lb.
8561·190 Kg. **Ans.**

22′0′4′6)188740000·
123720
134900
26240
4194
1989
5 &c.

EXERCISE 87.

Express :—

1. 9 m. 2 dm. in m.
2. 7 m. 2 dm. 4 cm. in m.
3. 4 m. 7 cm. in m.
4. 2 m. 17 cm. in m.
5. 9 m. 3 dm. 4 cm. 5 mm. in m.
6. 9 m. 34 cm. 5 mm. in m.
7. 9 m. 3 dm. 45 mm. in m.
8. 9 m. 345 mm. in m.
9. 13 m. 24 mm. in m.
10. 9 m. 7 mm. in m.
11. 5 Km. 216 m. in Km.
12. 8 Km. 19 m. in Km.
13. 23 Km. 7 Hm. in Km.
14. 346732 mm. in Km.
15. 3007 mm. in Km.
16. 298 Dm. in Km.
17. 9 Km. 3 m. 27 mm. in Km.
18. 29 Km. 23 m. 8 mm. in Km.
19. 3 Km. 7 m. 92 mm. in m.
20. 17 m. 9 mm. in cm.
21. 8 dm. 3 cm. in Dm.
22. 3 a. 2 ca. in a.
23. 17 a. 9 ca. in Ha.
24. 24673 ca. in Ha.
25. 198 ds. in s.
26. 3 s. 4 ds. in Ds.
27. 3 l. 4 dl. 5 ml. in l.
28. 17 l. 6 cl. in Hl.
29. 21637 ml. in l.
30. 2467 l. in Kl.
31. 4 Kg. 3 Hg. 9 g. in Kg.
32. 6 Kg. 43 cg. in Kg.
33. 24983 mg. in Kg.
34. 36728 g. in T.
35. 916 Kg. in Q.
36. 3 Kg. 7 mg. in T.
37. 48 sq. m. 73 sq. dm. in sq. Hm.
38. 17 sq. dm. 4 sq. mm. in sq. m.
39. 52984 c.dm. in c.m.
40. 14 c.m. 85 c.dm. 6 c.cm. in c.m.

EXERCISE 88.

1. 8·473 m. + 5 dm. + 273 mm. + ·045 Km.
2. 3·473 m. + 15 dm. + 16 cm. + 83 mm. + ·045 m.
3. 12 dm. + 7·367 Km. + 637 m. + 4 Dm. + 13 Hm.
4. 3 Ha. + 4·67 Ha. + 5 a. + 9 ca. + 207 ca.
5. 483 Hl. + 19·47 Hl. + 346 l. + 19 Dl. + 5 Hl. + 3 Dl. 8 l.
6. 8·498 Kg. + ·09 Kg. + 347 g. + 15 Hg. + 2·7 g.
7. 1·6 T. + 29 Q. + 27·9 Hg. + 21 Kg.
8. 23·4 sq. m. + 538 sq. cm. + 99·8 sq. dm. + ·086 sq. Dm.
9. 25·03 c. dm. + 431829 c.mm. + 8564·3 c.cm. + ·84 c.m.
10. 6 a. 4 ca. − ·7 a.
11. 41 s. 7 ds. − 3·29 Ds.
12. 243 Kg. 4 dg. − 17 Kg. 793 g.
13. 1·7 T. − 346 Kg. 576 mg.
14. 18 Km. 6 m. − 243 m. 691 mm.
15. ·3598 sq. Km. − 273·4 sq. Dm.
16. 4·0287 c.m. − 59847 c.cm.
17. Four rods whose lengths are 7 m. 500 mm.; 3 m. 75 cm.; 4 m. 8 dm.; and 8 m. 6 cm. are laid end to end; how far do they reach?
18. A boy is 1·61 m. in height, and his sister is 193 mm. less. Express her height in mm.
19. On the same railway there are four stations between which the consecutive distances are as follows: 7 Km. 249 m.; 3 Km. 2 Hm.; and 5·007 Km. Find the distance between the first and fourth stations.
20. A goldsmith sold jewels of the following weights respectively: 27 g. 9 mg.; 3 Dg. 7 dg.; 7 g. 4 cg.; and 19 g. 3 dg. 4 cg. 7 mg. Find the total weight.
21. Out of a farm of 340 Ha. 7 a. the area of 119 Ha. 29 a. 3 ca. is laid out in pasture. How much is not?
22. Of two casks, one holds 353 l. 5 dl., and the other 17 l. 93 cl. less. How much does the second hold?
23. Find the difference between 249·58 c.cm. of water and 157·3 cl. of water.
24. An ounce of standard gold (British coinage gold) is worth £3·89375, while an ounce of pure gold is worth 4£ 2fl. 48 mils (100 mils = 1fl.) nearly. What is the difference?

EXERCISE 89.

1. 7 Km. 24 m. × 73.
2. 3 Ha. 4 a. 27 ca. × 279.
3. 4 Kg. 29·007 g. × ·906.
4. 8 l. 9 cl. × 98·7.
5. 19 sq. m. 571 sq. cm. × ·1013.
6. 57 c.cm. 14 c.mm. × 500.
7. 9 Km. 29 m. 25 mm. ÷ 75.
8. 436 Kg. 545 g. ÷ 450.
9. 773 Hl. 5 Dl. 9 l. 5 cl. ÷ 4·5.
10. 132 Ha. 7 a. 5 ca. ÷ 1500.
11. 257 sq. Km. 840 sq. m. ÷ ·018.
12. 15 c.m. 14 c.dm. 917 c.cm. ÷ 4.
13. If a miner earns 5 florins 25 mils in a day, find the amount of his earnings in a month of 31 days having 5 Sundays, supposing that he works every lawful day.
14. A litre of mercury weighs 13 Kg. 596 g. Find the weight of 3 l. 69 cl.
15. 13 Ha. 7 a. 4 ca. of land may be rented for 326·76 florins. Find the rent of 1 Ha.
16. 16 l. 94 cl. of olive oil weigh 15 Kg. 5 Hg. Find the weight of 1 l.
17. A litre of air weighs 1 g. 293·2 mg. Find the weight of 9 l. 72 cl.
18. Find the rent of 87 Ha. 19 a. 75 ca. of land at 312 florins 7 cents (10 cents = 1fl.) per Ha.
19. Find the price of 243 Kg. 500 g. of sugar at 17 mils per Kg.
20. Find the price of 29 l. 9 cl. of wine at 1·07 florins per litre.
21. Divide a length of 25 Km. 20 m. into 30 equal parts.
22. How often may a measure of 3·43 l. be filled out of a cask containing 133 l. 77 cl.?
23. A gentleman divided 10£ 9 florins 4 cents 4 mils (10 mils = 1 cent) among a number of paupers, giving each 4 florins 5 cents 6 mils. How many paupers were there?
24. Into how many lots of 3 a. 75 ca. may 8 Ha. 40 a. be divided?

EXERCISE 90.

1. Express the distance from Berlin to Paris, which is 1308 Km., in chains.
2. The distance from Aberdeen to Glasgow is 152·75 miles. What is that in Km.?
3. The summit of Vesuvius is 1198 m. above the level of the sea. Express this height in feet.
4. The Olympic Stadium was 606·75 feet. Express this in m.
5. 5,130,740 old French toises = 1,000 myriametres, and
   1 myriametre = 10,000 metres.
   Find the number of feet in a toise.
6. A falling body travels 16·1 ft. in the first second of its fall. Give this distance in cm.

7. The weight of a litre of normal hydrogen is ·0896 gram. Find in grains the weight of a litre of normal oxygen, which is 16 times as heavy.

8. The weight of a litre of normal air is 1·2932 grams. Express this as the decimal of 1 lb.

9. Assuming the velocity of the molecules in normal air to be 485 m. per sec., find what their velocity is in miles per hour.

10. The normal pressure of the air is 14·7 lb. per sq. in. Express this in Kg. per sq. cm.

11. What is £1 per acre in florins per Ha.?

12. When wheat is at 30s. per qr., how many cents is it per litre?

DECIMAL COINAGE.

In a **Decimal System of Coinage each separate money denomination is a power of ten of the next lower denomination**; or in other words the various money units are related in the same way as the units of any of the tables of the metric system. The advantages of the metric system all hold good in a decimal system of coinage.

(1) Reduction from one denomination to another becomes merely a question of moving the point.

(2) Were such a system in use, decimals would be understood by everybody.

(3) All calculations would then be made by the *simple* rules in decimals, and would be carried through by methods of approximation.

(4) The custom of ignoring fractions of any quantity being dealt with would no longer be necessary, any degree of correctness wanted being easily obtainable by decimals.

(5) The labour of calculations and the time spent would be greatly reduced.

Almost all countries with the exception of Britain and some of her possessions have now adopted such a system of coinage.

The Coinage of France.

10 centimes = 1 decime
10 decimes = 1 franc

In France the standard coin or **unit** is the **franc**, a coin of 5 grams weight composed of 90 °/₀ of pure silver and 10 °/₀

of copper. Accounts are kept in francs and centimes; and, as 100 centimes = 1 franc, the moving of the point two places to right or to left reduces francs to centimes, or centimes to francs respectively.

Thus, 2485 centimes = 24·85 francs.

In actual dealing, say in speaking of the price of anything, **24·85 francs** would be **read 24 francs 85 centimes.**

The Coinage of America.

10 mills = 1 cent (ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

A 25-cent piece is usually spoken of as a "Quarter."

The unit is the **dollar**, and accounts are kept in dollars and cents. The practice in writing and reading dollars and cents is as in France with francs and centimes.

Thus, 317·75$ would be read 317 dollars 75 cents.

PROPOSED BRITISH DECIMAL COINAGE.

The adoption of the metric system of weights and measures and of a decimal system of coinage has repeatedly been urged on the Government since the first suggestion in 1838. The only result so far is that the use of the metric system is *legalised*, though neither recommended nor enforced.

The coinage has not gone beyond the "proposed" stage.

The proposed coinage system which receives most support and which would least interfere with present trade arrangements, is:—

10 mils (m.) = 1 cent (c.)
10 cents = 1 florin (fl.)
10 florins = 1 pound or sovereign (£)

The Sovereign and the Florin would remain as at present; the cent, being = 2·4 of our present pence, would take the place of our Threepenny piece; while the Mil would be ·24 pence, *i.e.*, practically our present Farthing. As additional coins, we should keep the Half-Sovereign, the Shilling (5 cents or 50 mils), the equivalent of the Sixpenny piece (2½ cents or 25 mils), while the 4-Mil and the 2-Mil would be substituted for the Penny and Halfpenny.

How very much simpler money calculations would be with such a system is seen from the following example.

Ex. Add :—

17£	4fl.	9 cents	2 mils
5£	8fl.	7 cents	5 mils
3£	6fl.	8 cents	3 mils
12£	9fl.	8 mils	

The working would be

£17·492
5·875
3·683
12·908
£39·958

and the sum would be 39£ 9fl. 5 cents 8 mils.

The **advantages** of such a system over our present one are granted by most people; but *objections* are also raised to its adoption.

(1) That much unsettling in certain lines of business would arise through the cent, 4-mil, and 2-mil not being of *quite* the same value as the 3-penny, penny, and halfpenny.

(2) That the unfamiliar names and appearance of the coins would lead to confusion in the case of those long accustomed to the old.

(3) That unless a decimal system of weights and measures were adopted at the same time, the calculation of the prices of some of our weights and measures would be troublesome; as, in the decimal system, division by 12 (articles in a dozen), by 16 (ounces in a pound), and by 8 (stones in a cwt.) cannot be done without remainder.

EXERCISE 91.

Practical Exercises on the Proposed Coinage.

1. A gentleman's annual income is £525, 8fl. 7c. 5m.; his expenses amount to £375, 9fl. 8c. 6m.; how much does he save in the year?
2. What should be paid for 37 lb. of sugar at 3c. 7m. per lb.?
3. If 67 yd. of cloth cost £82, 7c. 5m.; what is the price per yd.?
4. The yearly rent of a house is £92, 3fl.; how much is it weekly?

5. What should be paid for 57 qr. of wheat, at £2, 3fl. 7c. 5m. per qr.?

6. Divide £505, 7c. 5m. among 89 men.

7. What is the weight of a piece of silver, which cost £34, 3fl. 7c. 5m., at 2fl. 7c. 5m. per oz.?

8. If 17 cwt. of sugar cost £36, 1fl. 2c. 5m.; what should 183 cwt. of the same cost?

9. How many yards of cloth, at 2fl. 2c. 5m. per yd., should be given for 243 yd. at 1fl. 8c. 7½m. per yd.?

10. Find the value of 1 cwt. of sugar, when 5 cwt. 18 lb. cost £13, 5m.

11. A bankrupt's effects amount to £5756, and he compounds with his creditors for 6fl. 2c. 5m. per £; what is the amount of his debts?

12. If 45 men earn £125, 1fl. 5c. 6m. in 10 days, how many men will earn £187, 7fl. 3c. 4m. in 15 days?

13. In what time will the interest of £525, 3fl. 7c. 5m. at 5 °/₀ per annum pay a debt of £87, 5fl. 6c. 2½m.?

14. What part of £23, 6fl. 5c. 2m. is £13, 7fl. 9c. 7m.?

15. Reduce £7, 6fl. 1c. 6m. to the decimal of £14, 8fl. 7c. 5m.

16. Find, by Practice, the value of 12 cwt. 3 qr. 14 lb. of tea at £19, 6fl. per cwt.

17. What is the interest on £574, 6fl. for 4½ years at 3½ per cent. per annum?

18. Required the amount of £5760, 8fl. for 2 years 4 months, at 2¼ per cent.

19. Find the interest on £1533 for 240 days at 3¼ per cent.

20. What is the amount of £520, 1fl. 2c. 5m. for 2 years and 120 days at 4 per cent.?

21. What sum will amount to £188, 2c. 5m. in 2 years 3 months at 4 per cent.?

22. What discount should be allowed for present payment of £763, 3fl. 4c. 4m., due six months hence, at 2½ per cent.?

23. Find the present value of £187, 7fl. 7c. 5m., due 219 days hence, at 2½ per cent.

24. If £775, 8fl. 7c. 5m. amounts to £1024, 1fl. 5c. 5m. in 8 years; what rate per cent. is charged for the loan?

25. In what time will the interest of £2053, 1fl. 2c. 5m. at 3½ per cent. pay a debt of £100, 8fl.?

26. What must be paid for insuring £7050, at 1fl. 7c. 5m. per cent. ?

27. How much should be paid for £948, 5fl. 1c. 2m. 3 per cent. stock, at 92½, brokerage 1fl. 2c. 5m. per cent. ?

28. How much must be invested in the 3 per cents. at 92¼ to produce an annual income of £245, 2fl. 5c. 6m. ?

29. A person has £5000 in the 3 per cents. ; how much must he invest in the 3½ per cents. at 93¾ to have an income from both investments of £399, 9fl. per annum ?

30. A grocer mixes 12 lb. of tea at 2fl. 2c. 5m. with 15 at 2 fl., 20 at 1fl. 7c. 5m., and 24 at 1fl. 5c. ; how should the mixture be sold per lb. to clear 8fl. 3c. 2m. ?

31. A bankrupt owes to A, £420, 6fl. 2c. 5m. ; to B, £342, 3fl. 7c. 5m. ; to C, £497, 3fl. 2c. 5m. ; to D, £508, 6fl. 7c. 5m. ; and to E, £542, 1fl. 2c. 5m. ; his effects amount to £1774, 9fl. 4c. 4m. ; how much will each receive ?

32. Bought 3 casks of sugar, each 3 cwt. 2 qr. 14 lb., at 2c. 5m. per lb. ; what should the whole be sold for to gain 6fl. per cwt. ?

33. Bought goods for £613, 2fl. 5c. 6m. ; how should they be sold to gain 7½ per cent. ?

34. A person's income is £525 ; how much has he left after paying a tax of 7c. 5m. per £ upon it ?

35. Find the difference between the simple and compound interest of £450, 6fl. 7c. 5m. for 4 years at 4 per cent.

36. What is the present value of £756, 8fl. 5m., due 3 years hence, at 3 per cent. per annum compound interest ?

37. How many pounds, florins, cents, and mils, and the same number of each, are contained in £809, 9fl. 1c. 9m. ?

38. The sum of the incomes of 2 persons is £1005, 4fl. 8c. 9m., and the difference is £45, 7fl. 5c. 9m. ; what is the income of each ?

39. By selling coffee at £9, 3c. per cwt., 7½ per cent. is gained ; how much is gained per cent. by selling it at £9, 1fl. 5c. 6m. per cwt. ?

40. Lost 8 per cent. by selling an article for £16, 6fl. 7c. 5m., and recovered the loss by selling another for £37, 7fl. ; what was the gain per cent. on the second article ?

EXCHANGE.

Gold Standard and Silver Standard. With regard to their coinage, countries are said to have either a *gold*

standard or a *silver* standard; which means that the standard or **unit coin** is valued as **equal** either (1) to **so much pure gold,** or (2) to the **market value of a certain weight of pure silver.**

The principal money systems are shown in the following tables:—

GOLD STANDARD COUNTRIES.

Name of Country.	Standard Coin.	Grains of Pure Gold.	Exchange Value.	Sub-divisions.
Britain	Sovereign	113·0016	£1	20 shillings
United States	Dollar	23·2199	49·29d.	100 cents
The Latin Union	Franc	4·4803	9·51d.	100 centimes
France, Italy, Switzerland, Belgium, and Greece	In Italy, the unit is called *Lira*;	...	..	100 centesimi
	in Greece, *Drachma*	...	..	100 lepta
Germany	Mark	5·5313	11·745d.	100 pfennige
Austria	Florin	9·4099	19·985d.	100 kreutzers
Holland	Florin	9·3459	19·84d.	100 cents
Norway, Sweden, Denmark	Krone	6·2227	13·2d.	100 öre
Turkey	Pound	102·0804	18s. 0·8d.	100 piastres
Spain	Peseta*	4·4803	9·51d.	100 centimos
Portugal	Milreis	25·0885	53·278d.	1000 reis

* This coin is of equal value with the franc, for Spain and also most of the Balkan States have adopted the standard of the Latin Union.

SILVER STANDARD COUNTRIES.

Name.	Standard Coin.	Grains of Pure Silver.	Exchange Value.	Sub-divisions.
India	Rupee	165	Varies, for the Exchange	16 annas
Russia	Rouble	277·722	Value in gold depends upon	100 copeks
Mexico	Dollar	377·168	the market price of silver.	100 cents
China	Shanghai Tael (a weight)	516·405	The value of the rupee also depends on	1000 cash
Japan	Yen	374·4	other considerations.	100 sen

MONEY OF VARIOUS COUNTRIES.

In practically all the **British possessions** with the exception of Canada, **English money** is the currency. In **Canada,** including Newfoundland, the American coinage is copied, the currency being in **dollars** and **cents.** Most of the **South American States** have a similar system with the **Peso** as the unit sub-divided into 100 cents. In **Servia, Roumania,** and **Bulgaria** the coinage is that of the Latin Union, francs and centimes, but called in these various States, *dinars* and *paras*, *leïs* and *banis*, and *leva* and *stotinkis.*

Just as Stocks and Shares were found to have a **Nominal Value** (which represented the amount of capital originally invested, and was *fixed*), and also to have a **Market Value** (which varied as the demand for a certain stock was great or little); so in converting the money of one country into that of another, two exchange values have to be considered.

The "Exchange Value" of any coin, as given in the foregoing tables, is calculated on the quantity and the purity of the gold it contains, and is spoken of as the **Par of Exchange.** That is the *nominal* value, and is fixed; but if it were necessary to change money—say an Englishman travelling in France wished to exchange his sovereigns for francs—the amount

received would not, in most cases, be quite equal to this nominal value. The market price—called the Course of Exchange, or **Rate of Exchange**—rises or falls a little, just as the price of any kind of goods does.

The same terms are used as in stocks to indicate how the market price stands in relation to the nominal price, and so the Rate of Exchange is also spoken of as being "at par," "at a premium," or "at a discount."

In the case where a merchant transacts business with another in a foreign country, payment may be made by the buyer forwarding the value of the goods in one of three ways :—

(1) In Coin of his own country (**Specie**)
(2) In uncoined or Bar gold (**Bullion**)
or (3) By means of a **Bill of Exchange**

This last is by far the most usual way, being both safer and, as a rule, less expensive.

Another **Bill of Exchange** has been spoken of at an earlier stage in this volume; this is merely **a written order authorising the person to whom it is addressed** (an agent of the sender) **to pay a certain sum of money to a certain named person at a stated time.**

The subjoined shows the form of a Foreign Bill of Exchange, *three* copies of which are made and forwarded separately in case the first should be lost in transmission. The only difference is in the words First, Second, and Third, which, of course, change according as the copy is the first, second, or third.

No. 900. £150, 10s. 0d. Due.....

COPENHAGEN, *May* 1, 1903.

Thirty days after sight of this our First of Exchange (Second and Third of the same tenor and date being unpaid) pay to the order of Messrs Beveridge & Co. the sum of one hundred and fifty pounds ten shillings sterling as advised.

To the Bank of Scotland,
Edinburgh.

C. BERTEL.

Bills of Exchange are arranged for through the agency of Bill-Brokers who charge a commission of from $\frac{1}{16}$ to $\frac{1}{8}$ per cent. for either buying bills from or selling them to merchants who have such business to negotiate. The Rate of Exchange obtainable at any place is intimated by telegraph to other centres and published in the newspapers, so that those interested may know the state of the money market.

The *Rate of Exchange*, or as it is sometimes called, the "Course of Exchange," *can never, however, rise much above the par of Exchange;* for in such a case those who would ordinarily make use of Bills would find it less costly to transmit specie than to buy bills at the high rate.

THE EXCHANGES.

London, *Wednesday Evening.*

The following are the rates of exchange on London cabled from the chief commercial centres.

	To-day.	Previous.
Paris, cheques	25.12½	25.12
Brussels, cheques	25.16½—17	25.16
Geneva, sight	25.15	25.15½
Berlin, cheques	20.37½—38	20.37½—38
Vienna, sight	23.93	23.92½—93
Copenhagen, sight	18.14	18.14
Stockholm, sight	18.15	18.15
Amsterdam, sight	12.06½	12.06¾
Genoa, sight	25.11	25.11
Athens, sight	39.45	39.70
Madrid, sight	34.15	34.15
Lisbon, sight	43d.	43d.
St Petersburg, 3 months	93.70	93.75

The above is a specimen of the form of money market or exchange quotations.

The expressions "sight," "3 months," &c., in such quotations, indicate the "currency" of the bills of exchange.

The figures in the extract above represent the number of the coinage unit of the country given for £1; except in the case of Vienna, where 23.93 means 23.93 florins per £2; of Lisbon, where 43d. means 43 pence English is the exchange value of 1 milreis; and of St Petersburg, where 93.70 means 93.70 roubles per £10.

In some instances a second price is given (Berlin 20.37½—38) which means that sellers of bills get 20.37½ marks per £1 but buyers pay 20.38 marks per £1.

Questions in exchange are solved by ratio.

Ex. 1. How many dollars are equivalent to £850, 16s. 8d., the dollar being equal to 4s. 2d. ?

$$\frac{£850\frac{5}{6}}{50\text{d.}} = \frac{£\frac{5105}{6}}{£\frac{50}{240}} = \frac{5105}{6} \times \frac{240}{50}$$

$$= \mathbf{4084\ dollars.}$$

Ex. 2. A merchant remits 2152 marks 20 pfennige from Berlin. The rate being 20mk. 40pf. per £1, to how much sterling is this equivalent ?

$$\frac{2152{\cdot}20}{20{\cdot}40} = \frac{215220}{2040} = \frac{211}{2} = \mathbf{£105,\ 10s.}$$

Sometimes a merchant finds it costs less to remit money not directly but through another country to the foreign creditor. This involves the use of a compound ratio, or the calculation is made by what is called "Chain Rule."

Ex. 3. How many francs will be cleared off an account in Paris by transmitting £100 *via* Amsterdam, the rates of exchange between London and Amsterdam being £1 = 12fl. 15c. ; and between Amsterdam and Paris, 9fl. 72c. = 20fr.

? francs = £100
£1 = 12·15fl.
9·72fl. = 20fr.

The given equivalents are set down as shown, care being taken that the same denomination does not appear twice in the same column.

$$\frac{100 \times 12{\cdot}15 \times 20}{1 \times 9{\cdot}72}$$

The complete column becomes the numerator, the column containing the ? the denominator.

$$= \frac{100 \times 1215 \times 20}{1 \times 972}$$

Numerator and denominator both multiplied by 100.

$$= \mathbf{2500\ francs}$$

EXERCISE 92.

1. How many $ will an emigrant to Canada receive for £135, 8s. 4d. sterling at 50d. per $1 ?
2. An emigrant on arriving at Toronto changes 6 crowns, 7 half-crowns, 37 shillings, and 6 sixpences sterling to $, at the rate of 4s. 2d. per $1. How many dollars does he receive ?
3. A Calcutta merchant makes a payment of a lac or 100,000 rupees. Find the amount in sterling at 1s. 10¼d. per rupee.
4. Sent to Bombay goods worth £299, 16s. 3d. To how many rupees is this equivalent at 1s. 10½d. each ?
5. How much sterling is = 5400 taels, paid at Shanghai, at the rate of 6s. 6d. each ?

6. A merchant sends goods to Hong Kong to the amount of £846, 13s. 4d. How many dollars does he receive at 4s. 1½d. each?

7. How much sterling is in $3390, remitted from New York, at $4·80 per £1?

8. How much sterling is in $994·25, remitted from Philadelphia, at $4·85 per £1?

9. How many $ are received from Boston for £738, at $4·87 per £1?

10. How many $ are in £7659, received at New Orleans, at a premium of 10 per cent. or $110 for £22½?

11. How many francs are = £525, 10s. 6d., remitted to Marseilles, at 25fr. 22c. per £1?

12. How many francs must be remitted from Brussels to pay a bill of £987, 14s. 6d., at 25fr. 10c. per £1?

13. How much sterling must be remitted to Paris to settle an account of 9900 francs, at 24fr. 75c. per £1?

14. How much sterling must be sent to Antwerp to be equivalent to 25663fr. 75c. at 24fr. 50c. per £1?

15. A London jeweller remits £701, 12s. 6d. to a watchmaker in Geneva at 25fr. 30c. per £1. How many francs does the latter receive?

16. A merchant of Zurich changed 5666fr. 55c. to sterling at the rate of 25fr. 52½c. per £1. How much sterling did he receive?

17. Sent £240 to Basle. How many francs were received at 25fr. 57½c. per £1?

18. Remitted £375, 10s. to Milan. To what amount of lire is this equal at 27l. 40c. per £1?

19. An Englishman, on visiting Leghorn, expends 682½ lire. Express this in sterling at 27l. 30c. per £1.

20. If 34,000 lire are spent at Venice on illuminations, state the equivalent in sterling at 27l. 20c. per £1.

21. A contribution of £1330 is sent to Naples. Give its equivalent in lire at 27l. 9c. per £1.

22. If the course of exchange at Palermo is 27l. 40c., and at Messina is 27l. 45c., on how much sterling will the difference of exchange amount to 10 lire?

23. If when the exchange is at 26l. 80c. a sum of 6700 lire is required in Genoa to discharge a debt in London, what would have been required when the exchange is 40c. greater per £1?

24. A merchant in Liverpool begins to negotiate with an agent in Florence about a debt of 7335l. 96c., but before concluding arrangements, the exchange, which was at 27l. 12c., becomes 27l. 5c. How much sterling has he gained or lost by the delay?

25. How much sterling is in 4347mk. charged by a bookseller in Leipsic at 20mk. 70pf. per £?

26. How much sterling must be remitted to Berlin to discharge a debt of 6890mk. at 20mk. 67pf. per £1 ?

27. A contribution of £1500 is transmitted to Hamburg. Express this in marks at 20mk. 64pf. per £1.

28. If an account of 4151mk. 25pf. is about to be transmitted to Hamburg, how much sterling will be gained or lost by the exchange falling from 20mk. 50pf. to 20mk. 25pf. per £1 ?

29. A Dantzic trader owes £750 to a firm in Hull. How many marks will he lose by a rise in the exchange from 20mk. 30pf. to 20mk. 70pf. ?

30. How much Scandinavian currency is in £432, remitted to Copenhagen at 18kr. 60ör. per £1 ?

31. A Leith merchant pays an account in Copenhagen of 8041kr. Express this in sterling at 18kr. 10ör. per £1.

32. A sum of 7400kr. is raised in Stockholm. To what does this amount at 18kr. 50ör. per £1 ?

33. How many kroner are in £1050, 10s., remitted to the Bank of Norway at Trondheim at 18kr. 40ör. per £1 ?

34. How much sterling is in 18164kr., remitted through a branch of the Bank of Norway at Bergen, exchange 18kr. 20ör. per £1 ?

35. How many florins must be paid at Amsterdam in order to liquidate a debt of £1500, 8s., exchange 12fl. 6c. per £1 ?

36. A merchant at Gouda consigns cheese to the amount of 8993 florins to an agent in Scotland. How much sterling must the latter remit at 11fl. 50c. per £1 ?

37. A tourist for Rotterdam changes 5 five-pound notes and 5 sovereigns at 12fl. 10c. per £1. How much Netherlands currency does he receive ?

38. How many florins will be received at Vienna for £786, exchange 10fl. 80kr. per £1 ?

39. The sum of 15,470fl. is remitted from Trieste. Find the value in sterling at 11fl. 37½kr. per £1.

40. If the rate of exchange rises from 11fl. 37½kr. to 11fl. 42½kr., on how much sterling will the difference amount to 20fl. ?

41. If 4540 florins are required to clear a debt when the exchange is at 11fl. 35kr., how many florins would have been needed when the exchange is at 11fl. 47½kr. ?

42. A British merchant sends £867, 14s. 6d. to an agent at St Petersburg ; what does the latter receive at 3s. 1½d. per rouble ?

43. How much sterling must be remitted to Riga to discharge a bill of 1200rou. 50k. at 3s. 1½d. ?

44. How much sterling will a Hull merchant require to send to Archangel when the rouble is at 33⅜d. to pay an account of 4120 roubles ?

45. If the rouble rises from $32\frac{9}{16}$d. to $33\frac{3}{8}$d., how much has a Leith merchant lost by delaying to forward payment of an account of 4000 roubles to St Petersburg?

46. If the rouble rises from $32\frac{9}{16}$d. to $33\frac{3}{8}$d., how much has a Riga merchant gained by delaying to send payment to Leith of an account of £1159, 4s. 6d.?

47. How much sterling is in a conto or 1000 milreis remitted from Oporto at 56d. per milreis?

48. How many milreis are = £2270, sent to Lisbon at $56\frac{3}{4}$d. per milreis?

49. Change £840 to milreis at the rate of $4\frac{1}{2}$ milreis per £1.

50. A traveller pays an interpreter at Constantinople the sum of 500 piastres. What is the value in sterling at 120 piastres per £1?

51. Change £125, 10s. to piastres at Alexandria at $97\frac{1}{2}$ piastres per £1.

52. Find the difference in sterling and Greek money between £44, 16s. and 1317dr. 42lp., exch. at 28dr. 15lp. per £1.

53. How many francs = 1620 marks, sent from Berlin through London to Paris, £1 being = 20mk. 25pf. = 25fr. $37\frac{1}{2}$c.?

54. How many $ = 8260 marks sent from Berlin through London to New York, £1 = 20mk. 65pf. = $4·80?

55. How much Scandinavian currency must be sent from Copenhagen through London to Florence, to pay an account of 930 lire, £1 = 18kr. 40ör. = 27l. 60c.?

56. How much sterling is = 1590mk. 75pf., exchange between Berlin and Paris 1fr. 20c. per mk., and between Paris and London 25fr. 20c. per £1.

MENSURATION OF PLANE FIGURES AND RECTANGULAR SOLIDS.

Powers and Roots. A Power has been already defined as the number resulting when a given number is multiplied by itself a certain indicated number of times.

Thus the **fourth power** of 3 (written 3^4) $= 3 \times 3 \times 3 \times 3 = 81$.

Powers are indicated by small numeral characters written to the right of, and above the given number; as 2^5; 13^4; 9^3, &c. This power indicator is called the **Index.**

A Root may be defined as the converse of a power. A root is the number which when multiplied by itself the indicated number of times will produce a given number.

Thus, since the third power or **Cube** of 5 (written 5^3) $= 5 \times 5 \times 5 = 125$; the **Cube Root** of 125 (written $\sqrt[3]{125}$) is 5.

The mark $\surd$ is used to signify root, and a small index figure placed inside the arms thus $\sqrt[5]{}$ indicates which root is wanted. Where no index figure is expressed, the *square* root is understood. Thus, $\surd 16$ = the square root of 16.

It was also pointed out that roots of integers could be found by breaking up the members into prime factors; but before the calculation of areas and quantities is entered upon, methods of finding the roots of any number should be studied.

To find the Square Root. The squares of all numbers up to twenty at least should be learned; and of course this conversely gives a table of roots which will be frequently useful.

Thus, the square root of—

1 is 1	36 is 6	121 is 11	256 is 16	441 is 21
4 is 2	49 is 7	144 is 12	289 is 17	484 is 22
9 is 3	64 is 8	169 is 13	324 is 18	529 is 23
16 is 4	81 is 9	196 is 14	361 is 19	576 is 24
25 is 5	100 is 10	225 is 15	400 is 20	625 is 25

Since $1^2 = 1$; $10^2 = 100$; $100^2 = 10000$; $1000^2 = 1000000$, &c., it is evident that the square root of any number between 1 and 100 consists of **one** digit only; of numbers between 100 and 10000 of **2** digits; of numbers between 10000 and 1000000 of **3** digits, &c.; so that for every **two figures** in the number there is **only one in the root.**

This is made use of in finding the root; the method of doing which will be best seen from an example.

Ex. Find $\surd 26715{\cdot}9025$

From the principle seen above, the number of digits in the root can at once be found. The figures are marked off **in pairs** right and left from the decimal point. If the number of digits in the decimal is odd, a cipher is added; and if the root is not exact at that point, any number of pairs of ciphers may be added and the root found **correct to as many places** as may be desired.

In the example the number of digits in the integer part of the answer will be 3. The nearest square to 2 is $1 = 1^2$. 1 is put in the answer **and to the left.**

Subtracting product 1 and taking down the next pair of figures gives a new dividend 167. Adding last divisor to itself, we get $1 + 1 = 2$; but as when the new figure of the answer is added the 1 will be **10**, this is not really 2 but **20**, that is to be used as a **trial** divisor to find the new figure in the dividend. $167 \div 20 = 6$. The 6 is added to the right of 2 and in the answer. $26 \times 6 = 156$.

Subtracting and taking down next pair gives new dividend 1115. Adding 6 to 26 gives new **trial** divisor 320. $1115 \div 320 = 3$. The 3 is added to the right of 32 and to the answer. $323 \times 3 = 969$.

Subtracting and taking down next pair of digits, the new dividend is 14690. As the digits 90 are to the right of the point the point is inserted in the answer at this stage.

1	2'67'15·90'25'(163·45
1	1
26	167
6	156
323	1115
3	969
3264	14690
4	13056
32685	163425
	163425

Adding last new figure to last divisor the new **trial** divisor becomes 3260—the 0 filling meantime the place of the new figure. $14690 \div 3260 = 4$. The 4 is put in the answer and to the right of 326. $3264 \times 4 = 13056$.

Subtracting and taking down next pair gives new dividend 163425; adding last figure 4 to divisor gives **trial** divisor 32680. $163425 \div 32680 = 5$. The 5 is put in the answer and to the right of 3268, making the new divisor 32685, which multiplied by 5 gives 163425. The square root of 26715·9025 is therefore 163·45.

The square root of a vulgar fraction can sometimes be found by inspection.

Ex. 1. $\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$

Ex. 2. $\sqrt{27\frac{9}{16}} = \sqrt{\frac{441}{16}} = \frac{21}{4} = 5\frac{1}{4}$

The roots of both numerator and denominator are found, and these form numerator and denominator of the answer.

In all other cases it is best to reduce the vulgar fraction to a decimal and proceed by the method given above for finding the square root.

EXERCISE 93.

Find the square roots of the following, carrying the decimal to 6 places where the root cannot be expressed exactly :—

(1) 15·7609
(2) ·180625
(3) 2889·0625
(4) ·001296
(5) 152·399025
(6) ·00494209
(7) 7
(8) 2200
(9) ·025
(10) ·0729
(11) 11
(12) 45
(13) 16·675
(14) 28·75
(15) 43·384675
(16) 3·16227766
(17) 7·0030025
(18) ·0000016
(19) ·00784
(20) ·000784
(21) ·042849
(22) ·081
(23) $\frac{64}{169}$
(24) $\frac{225}{361}$
(25) $\frac{7}{12}$
(26) $\frac{3}{4}$
(27) $\frac{1}{2}$
(28) $\frac{1}{256}$ of $114\frac{1}{9}$
(29) $\frac{1}{135}$ of $48\frac{8}{9}$
(30) $2\frac{37}{40}$ of $5\frac{1}{6}$

To Find the Cube Root. Just as with squares so with cubes, a table should be made and committed to memory of the cubes of numbers up to 20. Here is the Converse Table. The cube root of—

1 is 1	512 is 8	3375 is 15
8 ,, 2	729 ,, 9	4096 ,, 16
27 ,, 3	1000 ,, 10	4913 ,, 17
64 ,, 4	1331 ,, 11	5832 ,, 18
125 ,, 5	1728 ,, 12	6859 ,, 19
216 ,, 6	2197 ,, 13	8000 ,, 20
343 ,, 7	2744 ,, 14	9261 ,, 21

As $1^3 = 1$; $10^3 = 1000$; $100^3 = 1000000$; $1000^3 = 1000000000$, &c., it follows that the cube root of numbers between 1 and 1000 consists of **one** digit; of numbers between 1000 and 1000000 of **2 digits,** &c.; so that for **three figures** in the number there is only **one** in the root. In square root the figures were marked off *in pairs*, in cube root they are marked off in *groups of 3 digits* right and left from the decimal point.

Ex. Find $\sqrt[3]{361705{\cdot}078125}$

			Ans. 361′705·078′215′ (71·25
			343
$7 \times 3 = 21$			
211	$7^2 \times 3 =$	14700	18705
	$211 \times 1 =$	211	
		14911	14911
		1	3794078
$71 \times 3 = 213$			
2132	$71^2 \times 3 =$	1512300	
	$2132 \times 2 =$	4264	
		1516564	3033128
		4	760950125
$712 \times 3 = 2136$			
21365	$712^2 \times 3 =$	152083200	
	$21365 \times 5 =$	106825	
		152190025	760950125

Two digits will be in the integer part of the answer. The nearest cube to 361 is 343, the cube of 7. The 7 is put in the answer and 343 under 361. Subtracting and taking down next group gives next dividend, 18705. It is easiest now to work with two columns in the divisor. In the first is put the figures of the answer so far found multiplied by 3. This is **tens**, as was the case in square root, and a 0 may be considered at the right hand till the new figure of the answer is got. The **trial** divisor is got in the second column by multiplying the square of the figures of the answer so far found by 3. This is **hundreds**, and **two ciphers** should be added to the right.

$7 \times 3 - 21$; $7^2 \times 3 = 14700$ will go in the columns here, therefore.

$18705 \div 14700 = 1$. The 1 goes in the answer and also to the right of 21 in the first column.

To the second column add for complete divisor the product of first column figures now completed, and the new figure of answer.

$211 \times 1 = 211$; $14700 + 211 = 14911$, the complete divisor, which multiply by the 1.

Each succeeding step is the same: subtracting and taking down next group gives new dividend, 3794078. For new divisor, in first column 71×3 (**tens**); in second column $71^2 \times 3$ (hundreds). In this and succeeding steps the squaring need not be done, as by adding the square of the last figure got in the answer to the two lines of figures above in the second column the result of $71^2 \times 3$ is got.

1512300 is the **trial** divisor.

$3794078 \div 1512300 = 2$. Add 2 to the answer figures and to the right of 213.

(2132×2) is added to second column.

Complete divisor, 1516564.

This is multiplied by 4, subtraction done, next group taken down, and the process carried through as before.

EXERCISE 94.

Where the exact cube root cannot be found, it should be given correct to five places of decimals. Find the cube root of—

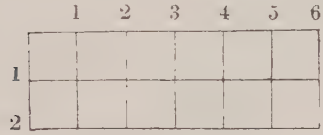
(1) 250·047.
(2) 175·616.
(3) 87528·384.
(4) ·000068921.
(5) ·000405224.
(6) ·000970299.
(7) 2126·781656.
(8) 24212·815957.
(9) ·00027.
(10) ·00008.
(11) $\frac{64}{729}$.
(12) $\frac{125}{343}$.
(13) $\frac{16}{27}$.
(14) $\frac{8}{7}$ of $\frac{3}{8}$ of $1\frac{5}{16}$.
(15) $\frac{5}{21}$ of $\frac{7}{24}$ of $8\frac{1}{3}$.
(16) A cube contains 5832 cub. in. Find the length of its side.

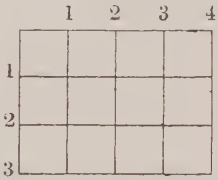
(17) The imperial gallon contains 277·2738 cub. in ; find the side of a cube containing a gallon.

(18) The litre contains 61·027 cub. in. ; find the side of a measure which is an exact cube and contains 1 litre.

Rectangular Areas. In taking the dimensions of the floor area of a room, the length and breadth measurements are taken as so many **feet** and inches, but when the quantity of linoleum, say, to cover it is spoken of, it is as so many **square feet**, &c. The first measurements are of **length**, for which the unit is the **yard**: while the second measurement is the **area**, of which the unit is the **square yard**.

When any surface, *e.g.*, a table top, is spoken of as being so many square feet in area, what is meant is that it might be divided into that number of **squares**, each side of which would in every case measure 1 foot; and this no matter what the shape may be.





In the above figures **the shape** is not the same in both cases, but **the area** is. If measurements are made from the top left hand corner in each case, the one will be found to measure in length 6 of a given unit, whereas the length of the other is only 4 of the same unit. The breadth of the first is 2 units, and of the second 3 units. If division into units of all of the sides is carried out, and the points of division joined, each figure is found to be made up of 12 squares, each of which measures 1 unit along each of its sides. If the unit is called a foot, then the first figure would be said to be **6 feet long** by **2 feet broad**, and to have an **area of 12** ($=6\times2$) **square feet**. The second would be **4 feet long** by **3 feet broad**, and its **area** would also be **12** ($=4\times3$) **square feet**.

To find the area of a rectangular figure, the rule is then to **multiply the length in any denomination by the breadth in the same denomination.**

Ex. A room is 14 ft. 6 in. long and 12 ft. broad. What is the area of the floor?

$$\text{Area} = 14\tfrac{1}{2}\ \textbf{ft.} \times 12\ \textbf{ft.} = 174\ \textbf{square feet}$$

From the above rule expressed as a formula, other rules can be arrived at.

$$\text{Area} = \text{Length} \times \text{Breadth}$$

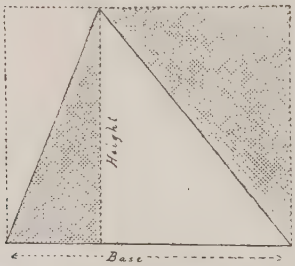
therefore $\quad \text{Length} = \text{Area} \div \text{Breadth}$

and $\quad \text{Breadth} = \text{Area} \div \text{Length}$

Ex. The surface of a sheet of paper is 40 sq. in., and its length is 8 in.; find its width.

$$\text{Width} = \frac{40 \text{ sq. in.}}{8 \text{ in.}} = 5 \text{ in.}$$

Triangular Areas. How to find the area of a triangle will be readily seen from the figure. Using any one of its sides as base, and raising perpendicular lines from its ends, of such an equal height that the line joining their tops will pass through the angular point opposite to the base, a rectangular area is constructed. The relation between triangle and rectangle will be still more easily seen if the angular point spoken of is joined by a perpendicular to the base.



The shaded and the unshaded parts of the triangle are obviously equal in size and shape to the unshaded and the shaded parts respectively of the parts of the rectangle outside the triangle.

In other words, the area of the *triangle* is **half the area** of the *rectangle*. The area of the rectangle is the product of its length and breadth, so the area of the triangle must be half the product of length and breadth (called **base** and **height** in triangular areas).

$$\text{Area of Triangle} = \frac{\text{Base} \times \text{Height}}{2}$$

Ex. Find the area of a triangle whose base is 8 ft. and height 7 ft. 9 in.

$$\text{Area} = \frac{8 \times 7\frac{3}{4}}{2} \text{ sq. ft.} = \mathbf{31 \text{ sq. ft.}}$$

Circular Areas. The measurement *round* a circular figure is called the **Circumference**; the measurement *through the centre* from side to side is called the **Diameter**; and the half of this, that is the measurement *from the centre to the circumference*, is called the **Radius.** No quite exact relation can be found between circumference and diameter, but the **circumference** is 3·1416 +, or nearly **$3\frac{1}{7}$ times, the diameter.**

The **area** stands in a somewhat similar relation to the radius, being **$3\frac{1}{7}$ times the square of the radius.** This measurement given as nearly $3\frac{1}{7}$ is usually expressed by the Greek letter π (pi).

Circumference $= \pi \times d$ (diameter) or $2\pi \times r$ (radius)

Area $= \pi \times r^2$

therefore also Radius = circum. $\div 2\pi$

" $= \sqrt{\frac{\text{area}}{\pi}}$

Ex. 1. Find the circumference of a circle whose diameter is 5 ft.

Circum. $= \pi \times d = 3\frac{1}{7} \times 5$ ft. $= \underline{15\frac{5}{7}}$ ft.

Ex. 2. The area of a circular enclosure being 1 acre, what is the length of a line from the middle to the outer fence?

$$\text{Radius} = \sqrt{\frac{\text{area}}{\pi}}$$

$$= \sqrt{\frac{1 \text{ ac.}}{3\frac{1}{7}}}$$

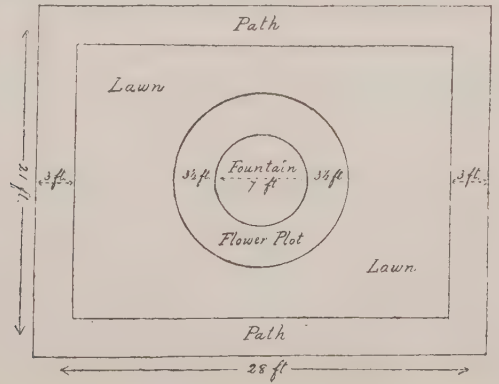
$$= \sqrt{\frac{4840 \text{ sq. yd.}}{\frac{22}{7}}}$$

$$= \sqrt{\frac{4840}{1} \times \frac{7}{22}}$$

$$= \sqrt{1540} = \mathbf{39{\cdot}2 \text{ yd.}}$$

Ex. 3. In the centre of a garden plot measuring 28 ft. long and 21 ft. broad is a fountain with a basin of 7 ft. diameter surrounded by a flower-bed $3\frac{1}{2}$ ft. across. Round the boundary is a walk of 3 ft. width; the rest is lawn. Find the several areas of fountain, flower-bed, lawn, and path.

In such problems as above a plan should be drawn and measurements marked on it. The dimensions of the various parts are then seen at a glance.



Area of fountain basin

$$= (\tfrac{7}{2})^2 \times 3\tfrac{1}{7} = \tfrac{49}{4} \times \tfrac{22}{7} = \mathbf{38\tfrac{1}{2}\ sq.\ ft.}$$

Area of flower-bed + fountain

$$= (3\tfrac{1}{2} + 3\tfrac{1}{2})^2 \times 3\tfrac{1}{7} = 7^2 \times \tfrac{22}{7} = 154 \text{ sq. ft.}$$

Area of flower-bed alone

$$= 154 - 38\tfrac{1}{2} = \mathbf{115\tfrac{1}{2}\ sq.\ ft.}$$

Area of lawn + flower-bed and fountain

$$= (28 - 6\,*) \times (21 - 6\,*)$$
$$= 22 \times 15 = 330 \text{ sq. ft.}$$

Area of lawn alone

$$= 330 - 154 = \mathbf{176\ sq.\ ft.}$$

Area of garden

$$= 28 \times 21 = 588 \text{ sq. ft.}$$

Area of path = Area of garden − Area of lawn, &c.

$$= 588 - 330 = \mathbf{258\ sq.\ ft.}$$

In measuring small dimensions a **foot-rule** is used, and measurements are taken in feet and inches; for larger measurements a **tape** of 50 or 60 feet is made use of. For large areas like fields surveyors use Gunter's Chain of 100 links. The chain is 22 yards long, and measurements made in chains allow of the area (sq. chains) being readily reduced to acres in which such areas are usually expressed, for 1 ac. = 4840 sq. yd. and 1 sq. chain = (22 × 22) = 484 sq. yd.; or 1 acre = 10 sq. chains.

Moving the decimal point one place to the left therefore reduces sq. chains to acres.

EXERCISE 95.

1. The side of a square table-top measures 3 ft. 2 in.; how many square inches are in its surface?

2. The side of a square garden-plot is 36 ft.; how many square yards in its surface?

* Path at both sides, therefore twice the width deducted.

3. Find the area of a square floor whose side is $19\frac{3}{4}$ ft.

4. How many flagstones 14 in. square will be required to floor a kitchen 21 ft. square?

5. How many square yards are there in a rectangular court measuring 326 ft. by 153 ft.?

6. How many acres, &c., are there in a square field whose side measures 3525 links?

7. How many stones of rectangular form each 3 ft. by $2\frac{1}{2}$ ft. will pave a road 40 yd. long and 6 yd. broad?

8. How many panes of glass each 18 in. by 14 in. will be required for 22 windows each 5 ft. by 3 ft. 6 in.?

9. What length must be cut from a board 26 in. broad to contain $1\frac{1}{2}$ sq. yd.?

10. Along one side of a court 47 ft. 9 in. square there is a footpath 4 ft. broad. What will be the expense of laying the rest of the court with stones at 6d. per sq. yd.?

11. A rectangular space 68 ft. 3 in. by 56 ft. 8 in. is to be paved with stones each 2 ft. 3 in. by 10 in. How many stones will it take and what will be the expense at 2s. 3d. per sq. yd.?

12. What is the length of the side of a square field of which the area is 40 acres?

13. What will it cost to fence a square paddock of 4 ac. 26 po. $17\frac{1}{2}$ yd. at 2s. 6d. a yard?

14. How many acres are there in a rectangular field whose length is 6 ch. 25 lk. and breadth 5 ch. 40 lk.?

15. The area of a rectangular field is 36 ac. $3\frac{1}{10}$ ro. Find the width if the length is 228 yd.

16. How many tiles each 6 in. by $4\frac{1}{2}$ in. will pave a corridor 125 yd. long and 4 yd. wide?

17. Find the cost of turfing a lawn 120 ft. long by 100 ft. broad, if sods measuring 3 ft. by 1 ft. cost 6s. 9d. per hundred, laid.

18. What is the area of a triangular garden one of whose sides is 296 yd., and the perpendicular distance from the opposite corner to that side, 176 yd.?

19. What is the length of an oblong twice as long as it is broad whose area is 5 ac.?

20. The area of Great Britain and Ireland is 122091 square miles. Find the side of an equally large square tract of land.

21. How many yards are there in the side of a square equal in area to an oblong 972 yd. long and 1296 ft. broad?

22. A rectangle is 240 yd. long and 450 ft. broad. Find the side of a square 10 times as large.

23. Find the side of a square of equal extent to 3 fields respectively 15 ac. 3 ro. 17 po. ; 11 ac. 3 ro. 36 po. ; and 5 ac. 1 ro. 36 po. in area.

24. How many times is a circle 27 ft. in diameter as large as another 15 in. in diameter?

25. The paving of a circular floor 25·6 ft. in diameter cost £9, 13s. 4d. ; what will be the cost of paving a similar one 38·4 ft. in diameter?

26. Find the diameter of a circle twice as large as another whose diameter is 120 ft.

27. Find the diameter of a circular pond 4 times as large as another whose diameter is 60 yd.

28. A room 27 ft. long requires 52 sq. yd. of carpet to cover the floor ; find the breadth of the room.

29. A courtyard 48 ft. long and 36 ft. 3 in. broad is to be paved at 3s. 6d. per sq. yd. ; what will it cost?

30. A path $4\frac{1}{2}$ ft. wide is made round a lawn 12 yd. long and 5 yd. wide ; how many square yards are there in the surface covered by the path?

31. A square field contains 59 ac. 2 ro. 12 po. 26 sq. yd. ; find the length of one side in yards and reduce the result to furlongs, poles, &c.

32. Turf is taken from a field 2 fur. 36 po. 3 yd. 2 ft. long and 2 fur. 14 po. 2 yd. 2 ft. 3 in. broad, and is found to be exactly sufficient to cover a square garden ; find the length of one side of the garden.

33. A room is 17 ft. long and 13 ft. 6 in. broad ; find how many square yards of matting will cover the floor.

34. If a piece of silk cost £15, 15s. $2\frac{1}{4}$d., and the number of yards in it is the same as the number of pence in the cost of 1 yd., find the number of yards and the price per yard.

35. The ceiling of a room 28 ft. broad contains 114 sq. yd. 6 sq. ft. ; what is the length of the room?

36. A roof is to be covered with lead at 8 lb. to the square foot. If the roof measures 24 ft. 8 in. by 14 ft. 6 in. and lead costs 2 guineas a cwt., find the expense.

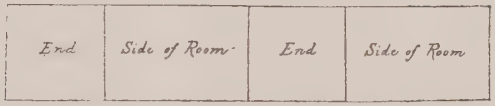
37. A rectangular garden 60 ft. long by 50 ft. wide has a path 3 ft. wide all round it. The rest is turfed with the exception of 3 circular flower beds whose diameters are 8 ft., 16 ft., and 24 ft. respectively. Find the number of square yards of turf in the garden.

38. A square courtyard 32 yd. long is surrounded on the outside by a gravel path $4\frac{1}{2}$ ft. wide. Find (1) the area of the path ; (2) the cost of gravelling it at $10\frac{1}{2}$d. per sq. yd.

39. Find the cost of staining a width of 2 ft. all round the floor of a room 21 ft. long by 16 ft. wide at 6d. per sq. yd.

40. A square pond of $32\frac{2}{3}$ yd. side is set in the middle of a square lawn of $83\frac{1}{3}$ yd. side; find the area of turf round the pond.

Mensuration of the Wall Surface,—Papering of Rooms, &c. The walls of a room of the usual shape form a series of 4 oblongs of the same width or height, the length of the first and third being the same, and the length of the second and fourth also alike.



They might easily be conceived set all in a continuous stretch as shown, and then the calculation of the area is seen to be merely the mensuration of an oblong whose length is the sum of the lengths of the four walls, and whose breadth is the height of the walls.

The measurement round a room, or the length and breadth each taken twice, is called the **perimeter.**

To find the **area of the wall surface** of a room **multiply the perimeter**, which is double the sum of length and breadth of room, by **the height**.

Ex. Find wall area of a room 16 ft. long, 14 ft. broad, and 12 ft. high.

$$\text{Area of wall surface} = 2(16 + 14) \times 12$$
$$= 60 \times 12 = 720 \text{ sq. ft.}$$

To find the wall area for purposes of plastering and papering, the surface area of doors, windows, fireplaces, &c., has to be deducted from the total area found as in the above example.

Wall paper is sold by the "*piece*," which is usually 12 yards long and 21 inches broad, though "pieces" of French paper are of narrower make, and only about 9 yards long. For waste in matching the pattern, one more piece in 10 is usually allowed in actual work.

In making a carpet to fit a room, allowance has to be made both for matching the pattern and also for the width of the room not being an exact multiple of the carpet width, in which case a whole width must be provided, though only a narrow strip may be needed. In the following exercises, however, make these allowances only where expressly stated.

Ex. How many yards of carpet $\frac{3}{4}$ yd. wide will be required to cover the floor of a room 18 ft. long and 15 ft. wide, allowing 4 yd. for matching?

If the carpet is laid lengthwise, then the required number of pieces, each 18 ft. long, will be found by dividing the width of the room by the width of the carpet—

$$15 \text{ ft.} \div \tfrac{3}{4} \text{ yd.} = 5 \times \tfrac{4}{3} = 6\tfrac{2}{3}$$

The number of pieces required is therefore 7; and as each is 6 yd. (= 18 ft.) long, the number of yards required is $7 \times 6 = 42$ yd.

42 yd. + 4 yd. (allowance for matching) = **46 yd.** (**Ans.**)

EXERCISE 96.

1. Find the wall area of a room whose length is 24 ft. 9 in.; breadth 17 ft. 3 in.; and height 11 ft. 6 in.
2. Find the cost of painting the walls of the above room at 5s. per sq. ft.
3. Find the wall area of a room $12\frac{1}{2}$ ft. long, $10\frac{1}{2}$ ft. wide, and 10 ft. high.
4. How many square yards are in the walls of a room 18 ft. 3 in. high, with a perimeter of 96 ft. 8 in.?
5. Find the surface area of the walls and ceiling of a room 15 ft. 6 in. long, 12 ft. 4 in. broad, and 10 ft. 7 in. high.
6. How many square feet are in the walls of a room 15 ft. 6 in. long, 13 ft. 4 in. broad, and 11 ft. 2 in. high?
7. How many square feet of lead will be required to line a cistern, without lid, 4 ft. 6 in. long, 3 ft. 8 in. broad, and 4 ft. 5 in. deep?
8. Find the cost of painting the walls of a room 13 ft. 6 in. long, 12 ft. broad, and 9 ft. high at 1s. 6d. per sq. yd.
9. Find the cost of painting the outside of a box, except the bottom, length and breadth each 3 ft. 4 in., and depth 2 ft. 8 in., at 1s. 3d. per sq. yd.
10. How many square yards of plastering are in the walls and ceiling of a room in the form of a cube 12 ft. each way, deducting for window 6 ft. 3 in. by 3 ft. 2 in., door 7 ft. 6 in. by 3 ft. 6 in., and fireplace 4 ft. 3 in. by 3 ft. 4 in.?
11. How many copies of a pictorial newspaper of 4 pages, each 28 in. by 20 in., will be required to cover the walls of a country barber's shop, 18 ft. 8 in. long, 14 ft. 4 in. broad, and 8 ft. 10 in. high, allowing for 2 windows each 5 ft. 6 in. by 3 ft. 2 in.; 2 doors each 7 ft. by $3\frac{1}{2}$ ft.; and fireplace 3 ft. by $2\frac{1}{2}$ ft.?
12. Find the cost of carpeting a room $10\frac{1}{2}$ ft. by $15\frac{1}{2}$ ft. with patternless carpet 27 in. wide at 4s. 6d. a yd.
13. Find the total area of the walls, ceiling, and floor of a room 12 ft. 6 in. by 11 ft. 6 in., and with walls 9 ft. 3 in. high.

14. How many square feet of deal will be required to make a rectangular box with top, $3\frac{1}{2}$ ft. long, 2 ft. broad, and 20 in. deep?

15. A room is 60 ft. in perimeter and 12 ft. high. How much paper 2 ft. wide will be required for the walls deducting a door 8 ft. by 4 ft., 3 windows each 5 ft. by $3\frac{1}{2}$ ft., and a fireplace 4 ft. square?

16. How much painting on a wall 14 ft. by $9\frac{1}{2}$ ft. deducting the fireplace 4 ft. 6 in. by 3 ft. 10 in.; find also the cost at 10d. per sq. yd.

17. A room is 20 ft. long, 14 ft. 6 in. broad, and 10 ft. 4 in. high. How much painting is in it deducting a fireplace 4 ft. 4 in. by 4 ft., and 2 windows each 6 ft. by 3 ft. 2 in.?

18. How many yards of plain carpet 27 in. wide will cover a room 19 ft. long and 14 ft. 3 in. wide. What will the carpet cost at 3s. $4\frac{1}{2}$d. a yd.?

19. Deducting $\frac{1}{10}$ of wall area for doors, &c., find the cost of size colouring the ceiling and walls of a room 17 ft. square and 11 ft. 6 in. high at $2\frac{1}{2}$d. per sq. yd.

20. Find the cost of the paper required for a room 20 ft. 8 in. long, $18\frac{1}{2}$ ft. broad, and 12 ft. high at 2s. 9d. per piece (12 yd. by 21 in.), allowing 2 extra pieces for waste.

21. Find the price of a carpet for a room 19 ft. 6 in. long and 16 ft. 4 in. broad at 5s. 3d. per sq. yd.

22. What will it cost to paper a room 24 ft. long, $16\frac{1}{2}$ ft. broad, and $13\frac{1}{2}$ ft. high, with paper $\frac{1}{2}$ yd. wide, $7\frac{1}{2}$ yd. in a piece at 5s. a piece? A skirting board 9 in. in height runs along all the walls, and 11 sq. yd are occupied by doors and windows.

23. How many complete pieces of paper $\frac{5}{8}$ yd. wide will cover the walls of a room 16 ft. 6 in. long, 12 ft. 3 in. broad, and 9 ft. 4 in. high; and what will the paper cost at 1s. 6d. per piece of 12 yd.?

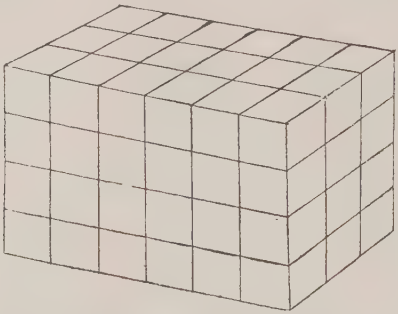
24. Find the cost of painting a room 24 ft. 3 in. long, 17 ft. 9 in. broad, and 16 ft. 4 in. high at 1s. 6d. a square yard.

Rectangular Solids. One rectangular solid with which every one is familiar is the ordinary building brick, and **all Rectangular solids**, like the brick, **have six sides or "faces" each** of which is **a rectangle.**

The length, the breadth, and the thickness of the brick are all different, but some rectangular solids have all three measurements alike, and these solids are called **Cubes.**

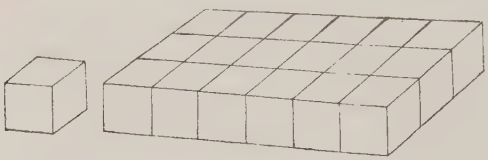
A cube is a solid each of whose six faces is a square.

In the block shown, which is 6 in. long, 3 in. broad, and 4 in. deep, if the top surface be first considered, its area = 3 in. × 6 in. = **18 sq. in.**, which may be counted on the figure.



The block, however, might be cut into slabs of 1 in. thickness, and since the depth is 4 in., there would be four slabs each of which has a top surface area of 18 sq. in.

If one of these slabs is now cut crossways and again lengthways along the lines shown in the figure, each slab would be cut into 18 pieces like the detached one, which measures 1 in. in every direction. It is a cube each face of which is 1 sq. in. and is said to measure **1 cub. in.**



This term *cubic* has no reference usually to shape, but merely indicates into how many cubes of one unit side of the given denomination the figure might be cut.

The total slab represented then consists of 18 of these cubes, or its volume is 18 cub. in., and, as there are 4 such slabs in the block, the total volume or solid content of the block will be 18 × 4 = 72 cub. in.

To find the volume or solid content of a rectangular solid, the **length, breadth, and depth in one denomination must be multiplied together**:—6 in. × 3 in. × 4 in. = **72 cub. in.**

Therefore, Volume = Length × Breadth × Depth

$$\text{Length} = \frac{\text{Volume}}{\text{Breadth} \times \text{Depth}}$$

$$\text{Breadth} = \frac{\text{Volume}}{\text{Length} \times \text{Depth}}$$

$$\text{Depth} = \frac{\text{Volume}}{\text{Length} \times \text{Breadth}}$$

If the solid is a cube all its measurements are alike, and the length of one edge is "*cubed*" to find the volume. So conversely when the volume of a cube is given the length of side is found by *extracting the Cube Root.*

EXERCISE 97.

1. Find the cubic content of a die whose side is $\frac{11}{19}$ in.
2. How many 3 in. cubes can be cut out of a 12 in. cube?
3. If the length, breadth, and thickness of a brick are 9, $4\frac{1}{2}$, and 3 inches respectively; find its surface and volume.
4. How many bricks each 9 in. long, $4\frac{1}{2}$ in. broad, and 3 in. thick must be taken to build a wall 100 ft. long, 20 ft. high, and 1 ft. thick?
5. What is the weight of a cube of cast iron whose side measures $4\frac{1}{2}$ ft., one cubic foot weighing 7248 oz.?
6. A cubic foot of basalt weighs about 2860 oz.; what is the weight of a cubic mass of the same whose side is 10 ft. 3 in.?
7. A wall is covered with plaster $1\frac{1}{4}$ in. thick; how many square yards will be covered by a cubic yard of plaster?
8. A cistern is $13\frac{1}{2}$ ft. long, 11 ft. 3 in. wide, and 4 ft. 2 in. deep. How many cubic feet of water will it contain?
9. How many dice $\frac{1}{3}$ in. in the side can be cut from a cubical piece of ivory $3\frac{1}{3}$ in. in the side?
10. How many gallons each $277\frac{1}{4}$ cub. in. does a cubical cistern of 5 ft. 6 in. side contain?
11. A cubic foot of water weighs 1000 oz.; what weight of water does a cubical cistern contain which measures 7 ft. every way?
12. A cubic foot of chalk weighs 2784 oz.; find the weight of a cubical block of 3 ft. side.
13. Find the side of a cubical vessel that will contain 100 imperial gallons (1 gal. = $277\frac{1}{4}$ cub. in.).
14. A mound of earth is 660 ft. long, 120 ft. broad, and 208 ft. deep. Find the side of a cubic one equal to it.

15. The length of a rectangular cistern is 4 ft., breadth 3 ft. 4 in., and depth 30 in. ; find the edge measurement of a cubical one three times as large.

16. The length of a block of stone is 8 ft., its breadth 6 ft., and its thickness 4 ft. Find the side of a cube of 11 times the solid content of the stone.

17. The edge of a cubical block of stone measures 5 ft. ; how many cubic feet does it contain ?

18. How many $\frac{3}{8}$ in. cubes can be got from a 9 in. cube ?

19. A cubic foot of water weighs 1000 oz. ; what weight of water is there in a cubical cistern of 6 ft. side ?

20. How many gallons each $277\frac{1}{4}$ cub. in. does a cubical cistern of 6 ft. 6 in. side contain ?

21. A gallon of water weighs 10 lb. avoirdupois, and a cubic foot of water 1000 oz. ; find how many gallons are contained in a cistern 12 ft. long, 10 ft. broad, and 6 ft. deep.

22. A gallon of water weighs 10 lb., and a cubic foot of water weighs 1000 oz. If a rectangular cistern be 6 ft. long and 4 ft. broad, and contain 200 gal., what will be the depth of the water in the cistern ?

23. How many cubic feet of air will there be in a room whose dimensions are 12 ft. 6 in. by 15 ft. by 16 ft. 3 in. ?

24. How many small boxes measuring 10 in. by $3\frac{3}{4}$ in. by $3\frac{1}{2}$ in. can be packed into a crate whose measurements inside are 2 ft. 6 in. by 3 ft. 6 in. by 3 ft. 4 in. ?

25. Find the solidity of a block of granite 8 ft. 4 in. long, 6 ft. 6 in. broad, 5 ft. 7 in. thick.

26. Find the cubic content of a slab of marble 5 ft. 6 in. long, 4 ft. 3 in. broad, 1 ft. 10 in. thick.

27. How many cubic feet of air are in a room 35 ft. 6 in. long, 20 ft. 8 in. broad, and 12 ft. 4 in. high ?

28. Find the weight of sea-water in a cistern 11 ft. 3 in. long, 6 ft. 7 in. broad, and 5 ft. 6 in. deep, the weight of a cubic foot of sea-water being 1025 oz.

29. Find the weight of a log of oak 10 ft. 5 in. long and 2 ft. 3 in. square throughout, the weight of a cubic foot of oak being 925 oz.

30. Find the cost of a block of lead 1 ft. 3 in. long, 9 in. broad, $8\frac{1}{2}$ in. thick, taking the weight of a cubic foot of lead at 709 lb., and the price at £23, 10s. per ton.

DUODECIMALS.

In taking measurements of length, all tradesmen, such as joiners, bricklayers, painters, &c., divide the inch into the parts $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$; but in making calculations of the surface area or solid content of their work they do not use these fractions, but proceed by what is called **Cross Multiplication**, or **Duodecimals**.

The name duodecimals (L. *duodecim* = 12) explains the system, which is one of proceeding by denominations which are to one another in the relation of powers or submultiples of 12.

The foot is taken as the standard and is subdivided into 12 primes, a prime into 12 seconds, a second into 12 thirds, &c.; primes, seconds, thirds, &c., being marked ′, ″, ‴, &c.

As 12 lineal inches equal 1 foot; 144 (= 12 × 12) square inches equal 1 square foot; and 1728 (= 12 × 12 × 12) cubic inches equal 1 cubic foot; lineal **inches** are lineal **primes**, **square inches** are **square seconds**, and **cubic inches** are **cubic thirds**.

When the number of feet is greater than 12, it is sometimes for convenience of multiplication expressed by a higher denomination in the duodecimal scale—75 ft. being set down as 6. 3 ft.

To change his lineal measurements in fractions of an inch to the duodecimal scale gives the tradesman no trouble: it is a question of expressing one fraction in the form of another whose denominator is 12, or a power of 12.

$$\text{Thus, } \tfrac{3}{4} \text{ in.} = \frac{3 \times 3}{3 \times 4} = \frac{9}{12} = 9''$$

$$\tfrac{1}{8} \text{ in.} = \frac{1 \times 18}{8 \times 18} = \frac{18}{12 \times 12} = 18''' \text{ or } 1'' \, 6'''$$
(144)

$$\tfrac{1}{16} \text{ in.} = \frac{1 \times 9}{16 \times 9} = \frac{9}{12 \times 12} = 9'''$$
(144)

DUODECIMAL TABLES.

Lineal Measure.

1 foot = 12 inches or primes (′)
1 prime = 12 seconds (″)

Square Measure.

1 square foot = 12 primes (′)
1 prime = 12 square inches or seconds (″)
1 second = 12 thirds (‴)
1 third = 12 fourths (⁗)

Cubic Measure.

1 cubic foot = 12 primes (′)
1 prime = 12 seconds (″)
1 second = 12 cubic inches or thirds (‴)
1 third = 12 fourths (⁗)
1 fourth = 12 fifths (⁗′)
1 fifth = 12 sixths (⁗″)

In multiplying duodecimal quantities together the **product** of any two denominations is **of the denomination indicated by the sum of the denomination marks of multiplier and multiplicand.**

Thus, 2 primes × 3 seconds = 6 thirds.

or 2′ × 3″ = 6‴ (square measure, now).

Again, 8 ft. × 6 in.

8 ft. × 6 primes = 48 primes (in square measure) or 4 sq. ft.

This is correct, as can be seen by proceeding in the usual fractional way—

8 ft. × 6 in. = 8 ft. × $\frac{1}{2}$ ft. = 4 sq. ft.

Ex. 1. Find the area of a surface 3 ft. 4 in. by 2 ft. 3 in.

Ft.	′	″
3	4	.
2	3	.
6	8	.
.	10	0
7	6	0

= 7 sq. ft. 72 sq. in.

3 ft. 4 in. = 40 in.
2 ft. 3 in. = 27 in.

12) 1080 sq. in.
12) 90 + 0 — 72
7 + 6

= 7 sq. ft. 72 sq. in.

Multiplying by 2 ft.: 2 ft. × 4′ = 8′ set down.

2 ft. × 3 ft. = 6 sq. ft. set down.

Multiplying next by 3 primes: 3′ × 4′ = 12″, which divided by 12 gives 1′ 0″; set down 0″, carry 1′.

3′ × 3 ft. = 9′ + 1′ carried = 10′.

Adding 10′ + 8′ = 18′ ÷ 12 gives 1 sq. ft. 6′; put down 6′, carry 1 sq. ft.

1 sq. ft. + 6 sq. ft. = 7.

Ans. 7 sq. ft. 6′ = 7 sq. ft. 72 sq. in.

Ex. 2. Find the area of a surface 75 ft. 9′ 9″ by 16 ft. 4′ 7″.

Break up 75 and 16, dividing by 12:

$75 \div 12 = 6+3$; $16 \div 12 = 1+4$.

		Ft.	′	″	‴	⁗
.	6	3	9	9	.	.
.	1	4	4	7	.	.
6	3	9	9	.	.	.
2	1	3	3	0	.	.
.	2	1	3	3	0	.
.	.	3	8	2	8	3
8	7	5	11	5	8	3
12			12			
103						
12						
1241 sq. ft. 137 sq. in. 8‴ 3⁗						

As the 6 and 1 are of a power higher than feet, the product of these figures with any other gives a denomination *one* higher than the denomination of that other.

The result of multiplying by 4 ft. and 4′ respectively gives same *figures*, but one place right and left respectively.

EXERCISE 98.

Find the area of surfaces of the following dimensions:—

	Ft.	′	Ft.	′		Ft.	′	″	Ft.	′	″		Ft.	′	″	Ft.	′	″
1.	3	2	× 2	3	7.	7	1	6	× 2	4	3	13.	28	9	11	× 11	11	11
2.	5	3	× 6	7	8.	4	4	6	× 5	6	7	14.	34	5	6	× 15	4	7
3.	7	10	× 8	11	9.	8	9	7	× 9	6	5	15.	43	9	10	× 28	11	11
4.	13	6	× 9	8	10.	19	3	6	× 7	4	9	16.	73	6	11	× 18	3	6
5.	18	7	× 7	8	11.	19	8	6	× 11	10	9	17.	64	5	10	× 16	9	9
6.	18	9	× 12	10	12.	32	3	7	× 9	11	9	18.	76	9	5	× 21	11	3

19. Find the content of a board 6 ft. 3 in. long and 4 ft. 7 in. broad.

20. Find the area of a floor 16 ft. 4 in. long and 14 ft. 8 in. broad.

21. Find the area of a square court whose side is 17 ft. 11 in.

22. What is the content of the ceiling of a square room whose walls are 12 ft. 5½ in. broad?

23. How much sheet-iron will be required to line the lower half of 12 window shutters, each 8 ft. 2 in. high and 1 ft. 4 in. broad?

24. How much veneering will be required to cover the surface of 6 counters, of which 2 are each 12 ft. 3 in. by 3 ft. 4 in.; 3 each, 10 ft. 6 in. by 3 ft. 4 in.; and the other 6 ft. 8 in. by 2 ft. 10 in.?

25. Find the price of 12 panes of glass, each 1 ft. 5 in. by 11 in., at 2s. 3d. per sq. ft.

26. How much must be paid for lining the bottom of a reservoir 32 ft. 3 in. long and 14 ft. 8 in. wide with asphalt at 2s. 3d. per sq. yd.?

27. Find the expense of whitewashing the ceiling of a square room, the breadth of the wall being 10 ft. 6 in., at 3d. per sq. yd.

28. What should be paid for causewaying a street 62 yards long and 12 ft. 6 in. broad at 1s. 6d. per sq. yd.?

29. Find the cost of paving a court 58 ft. 9 in. long and $21\frac{3}{4}$ ft. broad at 2s. 3d. per sq. yd.

30. What must be paid for painting a stair of 13 steps, each 2 ft. 7 in. broad and 10 in. wide, at 1s. 6d. per sq. yd.?

Ex. 3. Find the solidity or cubical content of a stone step which measures 4 ft. 5 in. by 2 ft. 9 in. by 1 ft. 8 in.

	Ft.	′	″	‴	
	4	5	.	.	
	2	9	.	.	
	8	10	.	.	The process is exactly the same as in finding the surface area.
	3	3	9	.	
12 sq. ft. 21 sq. in. = (sq. in. are *seconds*)	12	1	9	.	
	1	8	.	.	The denomination of a product is still indicated by the sum of the denomination marks of multiplier and multiplicand.
	12	1	9	.	
	8	1	2	0	
	20	2	11	0	Cubic in. are thirds.

12
35
12

Ans. 20 cub. ft. 420 cub in.

Find the cubic content of solids of the following dimensions:—

	Ft.	′	Ft.	′	Ft.	′
31.	8	11	× 7	8	× 6	7
32.	9	6	× 6	6	× 4	3
33.	9	7	× 6	8	× 5	4

	Ft.	′	″	Ft.	′	″	Ft.	′	″
34.	11	3	4 × 6	9	10 × 5		4	6	
35.	12	4	6 × 8	6	8 × 4		6	6	
36.	9	6	7 × 3	4	5 × 5		4	3	

EXERCISE 99.

General Exercises in Duodecimals.

1. How many square feet in a floor 46 ft. 8 in. long and 25 ft. 4 in. broad?
2. The floor of a room is 74 ft. 10 in. by 53 ft. 3 in.; find its surface area.
3. A window measures 7 ft. 8 in. 6 sec. by 4 ft. $9\frac{2}{3}$ in.; how many square feet does it contain?
4. How many gallons (1 gal. = 277·274 cub. in.) will a cistern contain whose inside dimensions are 38 ft. 6 in., 23 ft. 4 in., and 11 ft. 9 in.?
5. How many gallons will a cistern contain whose dimensions are double those of the one mentioned in previous example?
6. How many cubic feet in a plank of wood 16 ft. 11 in. long, 3 ft. 9 in. broad, and 2 ft. $10\frac{1}{3}$ in. thick?
7. A log of wood, 14 ft. 10 in. long, was sawn into 7 deals, each 2 ft. 11 in. broad; how many square feet did they contain?
8. How many square feet are there in 5 deals, 12 ft. 8 in. long, and their breadths 2 ft. 2 in., 2 ft. 8 in., 3 ft. 5 in., 3 ft. 9 in., and 3 ft. 11 in. respectively?
9. What is the difference between the areas of the floors of two rooms, the one 36 ft. 7 in. by 22 ft. 10 in., and the other 27 ft. 11 in. by 19 ft. 7 in.?
10. What is the cubic content of a box, 7 ft. 9 in. 3 sec. long, 2 ft. 3 in. 6 sec. broad, and 1 ft. 11 in. 11 sec. thick?
11. How many yards of painting are in a room 45 ft. 6 in. long, 24 ft. 10 in. broad, and 13 ft. 4 in. high; and how much does it cost, reckoning the whitewashing of the ceiling at $1\frac{1}{2}$d. per sq. yd., and the rest at $10\frac{1}{2}$d. per sq. yd.?
12. A house has 3 tiers of windows, 4 in each tier; the height of the under tier is 7 ft. 8 in., of the second 6 ft. 10 in., of the third 5 ft. 3 in., and their common breadth is 3 ft. 10 in.; find the expense of glazing them at 1s. 4d. per sq. ft.
13. What is the expense of lining a water-cistern 2 ft. 10 in. long, 2 ft. 6 in. deep, and 2 ft. broad, with sheet lead, 10 lb. to the sq. ft., at £1, 18s. 9d. per cwt.?
14. A court-yard is 50 ft. long and 40 ft. 6 in. broad; what will the paving of it cost, at 3s. $7\frac{1}{2}$d. per sq. yd.?
15. How much plastering on a partition 7 ft. 8 in. long, and 10 ft. 3 in. high, deducting a door 6 ft. 3 in. by 2 ft. 10 in.; and what will it cost at 5d. per sq. yd.?
16. How many square yards of standard brick-work are in a wall 75 ft. long, 15 ft. 9 in. high, and three bricks thick?

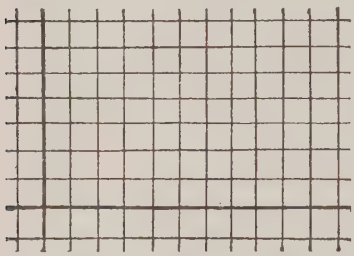
NOTE.—The standard for brick-walls is three half-bricks thick, and walls of any other thickness are reduced to standard by multiplying by the number of half-bricks in the thickness, and dividing by 3.

GRAPHS.

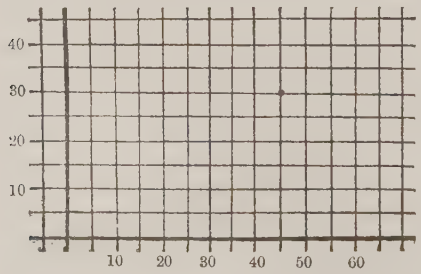
It has already been urged (p. 185) that a sketch is often helpful to the proper understanding of a problem; and the method of illustrating the relation to one another of a series of varying quantities by means of a line, straight or curved, drawn through a succession of fixed points is often found more telling than the mere enumeration of figures.

These line illustrations are called **Graphs**, and are usually, for convenience, drawn on "squared" paper.

Just as the position of any place on the earth's surface is determined by its latitude and longitude, so *the position of any point in a plane is fixed by two measurements.* Longitude is measured in degrees along the line of the Equator, and latitude along the first meridian; and, in like manner, in graphic illustration the measurements are made along two lines (called the **Axes**) at right angles to each other. It is usual to represent the axes by drawing heavier lines along convenient lines of the squared paper, as in the diagram.



Distance measured along the *horizontal* axis is called the **abscissa** (plural, abscissæ), and along the *vertical* axis the



Point whose abscissa is 45, and ordinate 30.

ordinate, of the point to be fixed. The two measurements are called the *co-ordinates* of the point.

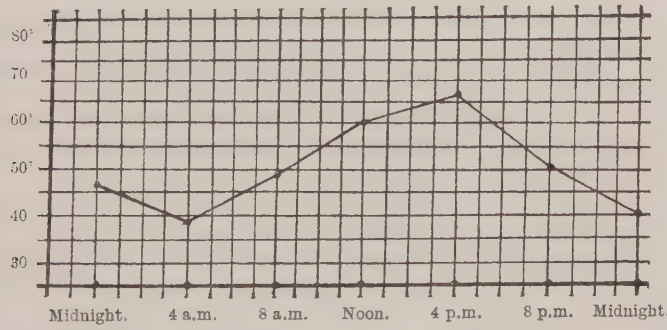
Suppose the abscissa of the point required is 45 and its ordinate 30, then the point is at the intersection of the vertical line through 45 on the horizontal axis, and the horizontal line through 30 marked on the vertical axis.

When the co-ordinates of several points fixed by varying measurements or quantities are given, each point is fixed in turn, as in the example above, and the line joining these various points is called the **graph** of that of which the measurements were given.

Fixing the various points and joining them in this way is called **plotting the graph**.

Ex. To plot the graph of the temperature for a day as shown by the thermometer readings taken : midnight, 46° ; 4 a.m., 38° ; 8 a.m., 48° ; noon, 60° ; 4 p.m., 66° ; 8 p.m., 50° ; midnight, 40°.

Here the abscissæ will be the 24 hours ; and the ordinates,



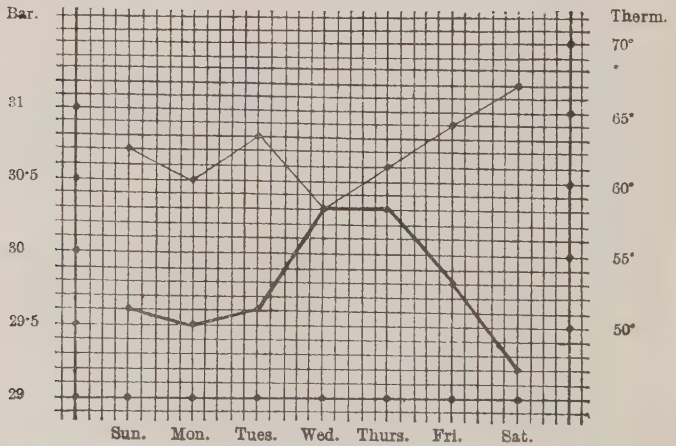
degrees of temperature showing a range a little greater than that indicated by the lowest (40°) and highest (66°) readings.

The intersections of the lines through midnight and 46°, 4 a.m. and 38°, 8 a.m. and 48°, etc., are in turn marked by a dot, and the graph required is drawn by joining all these points.

The advantages of thus graphically showing the variation in temperature are obvious, for, not only can the whole range of temperature be seen at a glance, but also when the highest or lowest or any given temperature within the range was recorded. On the assumption that the rise and fall are gradual the approximate reading can be arrived at for any time within the period, and not only for the hours when actual records were made.

Thus from the diagram we may say the temperature at 9 a.m. was 52° and at 6 p.m. 58°.

This is now the usual method of recording the readings of both thermometer and barometer; but if these are recorded together, a second vertical axis has to be used, as in the illustration below. The height of the mercury in the barometer is marked on the left-hand axis, and the thermometer degrees on the right-hand one. It is also usual to mark the graph of the barometrical readings by a heavier and that of the thermometer by a lighter line.



EXERCISE 100.

Plot graphs to represent the following :—

1. The income-tax during the years 1883 to 1903, both inclusive, was at the following rates :—5d., 6d., 8d., 8d., 7d., 6d., 6d., 6d., 6d., 6d., 7d., 8d., 8d., 8d., 8d., 8d., 1s., 1s. 2d., 1s. 3d., 11d.
2. The changes in the height of the barometer during a week were :—

Mon.	-	29·9	Fri.	-	30
Tues.	-	29·8	Sat.	-	30·1
Wed.	-	29·8	Sun.	-	30·2
Thurs.	-	29·9			

3. Of her manufactured imports, France took from Great Britain :—

	Per cent.		Per cent.
In 1891 -	47·1	In 1897 -	48·0
,, 1892 -	49·4	,, 1898 -	44·9
,, 1893 -	49·2	,, 1899 -	45·2
,, 1894 -	46·6	,, 1900 -	40·3
,, 1895 -	46·5	,, 1901 -	34·4
,, 1896 -	48·9		

4. The readings recorded by the thermometer on three successive days were :—

	9 a.m.	Noon.	3 p.m.
Saturday - -	36°	50°	45°
Sunday - -	29°	35°	33°
Monday - -	30°	33°	32°

From the graph answer the following :—

(*a*) When did the greatest fall of temperature take place?

(*b*) At how many of the readings was the mercury below freezing-point?

(*c*) What was the reading on Saturday at 11 o'clock?

5. Of her manufactured goods, Germany exported to Britain :—

	Per cent.		Per cent.
In 1891 -	11·7	In 1897 -	13·0
,, 1892 -	11·3	,, 1898	12·8
,, 1893 -	11·3	,, 1899 -	12·2
,, 1894 -	12·3	,, 1900 -	11·8
,, 1895 -	11·2	,, 1901 -	13·4
,, 1896 -	12·2		

6. The average price of wheat in the United Kingdom was :—

	s. d.		s. d.
In 1891 -	37 0	In 1897 -	30 2
,, 1892 -	30 3	,, 1898	34 0
,, 1893 -	26 4	,, 1899 -	25 8
,, 1894 -	22 10	,, 1900 -	26 11
,, 1895 -	23 1	,, 1901 -	26 9
,, 1896 -	26 2	,, 1902 -	28 1

(Reduce prices to *pence* for ordinates.)

7. The price of coal per ton has varied thus :—

	s.	d.		s.	d.
In 1893 -	22	10	In 1898 -	19	1
,, 1894 -	19	7	,, 1899 -	21	4
,, 1895 -	18	7	,, 1900 -	27	9
,, 1896 -	18	6	,, 1901 -	24	5
,, 1897 -	18	4	,, 1902 -	23	6

(Reduce prices to *pence* for ordinates.)

8. In the first innings of the fourth test match, the runs made by the English cricketers were :—

Warner - -	0	Hirst - - -	25
Hayward - -	18	Bosanquet - -	12
Tyldesley - -	16	Arnold - -	0
Foster - -	19	Lilley - - -	24
Knight - -	70	Rhodes - -	10
Braund - -	39		

(Show by a graph their varying success.)

9. The church-offerings during the month of August were :—

	£	s.	d.		£	s.	d.
Aug. 2 -	1	19	3	Aug. 23 -	1	16	3
,, 9 -	1	19	4	,, 30 -	1	11	7
,, 16 -	1	17	0				

(Draw a graph representing this variation. Find also the average offering.)

Among other varying quantities which might be graphically represented by pupils are :—

(*a*) The temperature of class-room at time of opening and closing each day.

(*b*) The barometrical readings taken in the same way.

(*c*) The class attendance morning and afternoon.

(*d*) The number of runs made by each member of the cricket eleven in any match.

(*e*) The number of points scored by and against the football team in their season's matches.

(*f*) The variation in the price of consols (or other stock), as shown in Exchange quotations.

(*g*) The variation in the price of any of the commodities quoted in the daily market prices.

EXERCISE 101.

The following questions are selected from recent Civil Service and other Examination Papers :—

Boy Clerks, May 1908.

1. Newspaper wrappers are sold at the rate of 5s. 8½d. for 120. How many at this rate can be bought for 3s. ?

2. A glass tube of uniform bore, 23·8 cm. long, holds 8·7 grams of mercury. 1 c.cm. of mercury weighs 13·6 grams. Find the cross-sectional area of the tube.

3. A cyclist has to go 330 yards through a wood, in which he can only push his cycle at the rate of 3 miles an hour. He can, however, take another route 470 yards long, pushing at 3 miles an hour for 150 yards and cycling the remainder at 9 miles an hour. Which will be the quicker, and by how much, to the nearest minute?

4. The following table gives the amounts expended on the Navy in the years 1896-1906 :—

	£		£
1896 . .	22,272,000	1902 . .	31,004,000
1897 . .	21,170,000	1903 . .	35,709,000
1898 . .	23,881,000	1904 . .	36,860,000
1899 . .	25,731,000	1905 . .	33,152,000
1900 . .	29,999,000	1906 . .	31,870,000
1901 . .	30,981,000		

Find the average amount for the 11 years, and, to the nearest integer, by how much per cent. of the average value the highest amount exceeds the average.

Draw a line near the bottom of a page of your book and draw uprights from it, 1 cm. apart, representing (on the scale of one small division of your book to £1,000,000) the amount expended in each of the above years.

5. (Omitted.)

Boy Clerks, September 1908.

1. A register of 200 pages, which was begun on 18th of November 1900, contained space for two weeks' entries on a page. When was a new register required?

2. Two teas cost a grocer 1s. 7d. and 1s. 1d. a lb. He usually makes 3d. profit out of each 1s. he receives for his wares. If he mixes 9 lb. of the cheaper with 5 lb. of the dearer, at what price per lb., to the nearest penny, must he sell to make his usual profit?

3. Three trains leave Plymouth for London at intervals of 10 minutes and with speeds of 48, 42, and 30 miles an hour respectively. When the first two are 12 miles apart, how far will the third be behind the second?

The question may be answered graphically if you like.

4. In the week ending August the 1st the following passengers arrived in Jersey :—

From Southampton	1375
From St Malo (France)	369
From Cateret (France)	312
From Weymouth	1833
From Granville (France) . . .	534
From other ports	304

Find, to the nearest integer, the percentage that came—

(A) From the two English ports together;
(B) From the three French ports together;
(C) From other ports.

5. (Omitted.)

Boy Clerks, January 1909.

1. A draper during a sale sold 367 yards of cloth at 1s. 6½d. a yard, and 156 blankets at 4s. 11d. each. If he bought these goods for £60, what profit did he make?

2. In 1906 there were 529,995 acres of land under tea in British India, and 461,260 acres in Ceylon. In the same year the production of tea in British India amounted to 240,849,894 lb., and in Ceylon to 170,527,126 lb. Find how much per cent. of the tea produced in India and Ceylon together was produced in Ceylon; find also the yield in pounds per acre in Ceylon. Give each result to the nearest whole number.

3. A man wishes to catch a tram at a point which is half a mile from his house, and a mile and a quarter from the tram's starting-place. If the tram leaves its starting-place at 9.30 and goes at a speed of between 8 and 12 miles an hour, and the man walks at 3½ miles an hour, find the latest time at which he can leave his house (*a*) to be certain of catching the tram, (*b*) to have a chance of catching it. Give your answers to the nearest minute.

4. A charge of shot weighed 25 grams. If the shot are 3 millimetres in diameter, and made of lead, of which 1 c.cm. weighs 11·4 grams, find the number of shot in the charge. A shot of d millimetres diameter contains $0{\cdot}5236 \times d \times d \times d$ cubic millimetres.

5. (Omitted.)

Girl and Lady Clerks, March 1907.

1. Find the amount of the following bill; a fraction of a farthing in any item is to be counted as a whole farthing :—

	s.	d.	
7 yards of cashmere at	2	6½	per yard.
8¼ yards of lining at	0	8¾	per yard.
1⅝ yards of velvet at	3	11	per yard.
½ dozen buttons at	0	2¾	each.

2. Milk is sold at 4 pence per quart. Express this price in centimes per litre. 100 centimes = 1 franc; 25·2 fr. = £1.

3. (Omitted.)

4. In a class there are 4 boys, aged 13 yrs. 5 mo.; 3 boys, 13 yrs. 7 mo.; 3 boys, 13 yrs. 2 mo.; 2 boys, 13 yrs. 6 mo.; 2 boys, 13 yrs. 1 mo.; and five others whose ages are 15 yrs. 4 mo.; 15 yrs., 14 yrs. 11 mo., 12 yrs. 10 mo., and 12 yrs. 4 mo. Find the average age of the class; how much would this average be changed if the three eldest boys were not included?

5. A clerk earns £65 a year and receives £15 a year from other sources. Her necessary expenses at home are £18 a year for clothes and 15 shillings a week for board and lodging (which she does not pay during her fortnight's holiday); her necessary office expenses are 4d. a day for travelling and 9d. a day for food, except on Saturdays, when her food only costs 3d. The days when she does not go to the office are Sundays, a fortnight's holiday, and six other days in the year (none of which are Saturdays). Calculate her office expenses for the year and find what percentage they are of her total necessary expenses. For simplicity you may suppose there are exactly 52 weeks in a year. If she allows £5 for her holiday, how much has she left for each week of the year, to use for pocket money, &c.?

6. The population dwelling within 35 miles of the centre of Manchester is about 8 millions. Assuming the average family to consist of 5 persons, find how many acres (to the nearest tenth) each family occupies.

A circle of radius r miles has an area of $3{\cdot}1416 \times r \times r$ square miles.

Girl and Lady Clerks, September 1908.

1. A dressmaker buys the following materials for a dress:—10½ yards of cloth at 2s. 8d. a yard; 6¾ yards of lace at 4s. 6d. a yard; 5½ yards of silk at 2s. 3d. a yard. Linings and sundries cost her 8s., and she estimates that the labour will cost her 15s. If she charges 6 guineas for the dress, what profit does she make?

2. At the end of April 1907 there were employed in the Transvaal mines 91,824 natives and 53,588 Chinese, at the end of April 1908 the figures were 130,991 natives and 24,059 Chinese. Find at each date, to the nearest whole number, how many Chinese there were to every 100 natives, and the increase or decrease per cent. in the numbers of (*a*) natives, and (*b*) Chinese employed at the end of April 1908 as compared with the numbers for April 1907.

3. The area of the County of London is 75,000 acres. How many square inches does this cover on a map drawn to a scale of an inch to 2 miles?

4. A cloth manufacturer prices some cloth at *either* 4 francs a metre *or* 2s. 11d. a yard. If he intends these prices to be the same, find how many francs, to the nearest hundredth, he has taken as equal to £1.

5. A manufacturer sells jam in 1-pound and 3-pound pots. The inside of the 1-pound pot measures 2·9 inches across and 3·6 inches in depth, the corresponding measurements for the 3-pound pot are 3·9 inches and 5·8 inches. A pot measuring inside a inches across and b inches deep has a capacity of $0{\cdot}7854 \times a \times a \times b$ cubic inches. Find, to the nearest cubic inch, the capacities of the two jars, and express the less as a decimal fraction of the other to two decimal places.

6. (Omitted.)

Abstractors, January 1908.

1. Wax candles 0·9 inch in diameter and 6·5 inches long are sold at $\frac{3}{4}$d. each. Find to the nearest penny the cost at this rate of a cubic foot of wax. The volume of a candle d inches in diameter and l inches long is $\frac{1}{4} \times d \times d \times \pi \times l$ cubic inches.

2. A bucket full of water weighs 30 pounds; when half the water has been poured off the weight is $16\frac{1}{4}$ pounds. Find the weight of the bucket when empty and the number of gallons it will hold.

3. (Omitted.)

4. The following table gives the population and birth-rate per 1000 for several towns. Find the birth-rate per 1000 for the three towns taken together:—

	Population.	Birth-rate.
London	4,684,794	27·1
Glasgow	809,986	30·0
Liverpool	730,143	33·3

5. Nitrogen is 14 times and oxygen 16 times as heavy as hydrogen. Assuming that $\frac{1}{5}$ of air is oxygen and $\frac{4}{5}$ nitrogen, compare the weight of air with that of hydrogen.

Check your result by making use of the following data:—

(a) 1 litre of air weighs 1·2932 grams.
(b) 1 litre of hydrogen weighs 0·0896 grams.

6. (Omitted.)

Abstractors, January 1909.

1. One man's expenses from 1st March to 25th July, both inclusive, were £57, 5s. 2d.; another's were £82, 13s. 5d. from 18th May to 5th December, both inclusive; find which of these is the higher rate of expenditure. If you prefer it, answer the question graphically.

2. A manufacturer makes cloth at a cost of 2s. $10\frac{1}{2}$d. a yard and sells it in England at 3s. 6d. a yard, and in France at 5 francs a metre. If he incurs an additional expense of $1\frac{1}{2}$d. a yard by selling in France, which of these prices gives him the higher rate of profit per cent. on his expenditure? State the percentage profit on sales in England and in France. Take 25 francs = £1.

3. Below are given the area and the population in 1901 of the most densely populated county and of the largest county in England:—

County.	Area in Acres.	Population.
London . . .	74,839	4,536,541
Yorkshire . . .	3,721,094	3,596,325

Estimate mentally, and quite roughly, the number of persons to the square mile in each of these counties. Write down your results, and explain as briefly as possible how you arrive at your figures.

Now calculate the numbers correct to the nearest integer.

4. A bucket D inches across at the top, d inches across at the bottom, and h inches high, has a volume of

$$0{\cdot}2618 \times h \times (D \times D + D \times d + d \times d) \text{ cubic inches.}$$

Find to the nearest tenth of a gallon how much water a bucket 1 foot across at the top, 8 inches across at the bottom, and 15 inches high can hold.

5. If 5 kilograms of a liquid which weighs 1·13 times as much as water are mixed with 2 kilograms of another liquid which weighs 0·87 times as much as water, find how many times as heavy as water the mixture will be.

6. (Omitted.)

Royal Society of Arts Examination—Grade I.—1906.

Arithmetic.

1. Find the H.C.F. of 10395, 5115, 8415.

2. Multiply 40·1870378 by 2·149173945 by a *contracted* method, giving the answer true to the seventh decimal place.

3. A grocer having 1161 cwt. of sugar buys 567 cwt. at £1, 19s. 10½d. per cwt. to mix with it. He sells the whole at 5d. per lb. and gains 12 per cent. on the cost. What was the cost of the first quantity of sugar?

4. A decimetre is equal to 3·937 in., and a cubic inch of water weighs 252·45 grains; express a kilog. in lbs. avoir. correct to three places of decimals.

5. The area of a field is 30479·805 square ft. Its length is twice its breadth; find its length.

6. If 24 horses are fed for 17 days at the cost of £22, 2s., how many days can 8 horses be fed for £23, 8s., the price of food and the rate of consumption being the same in both cases?

7. What decimal multiplied by 115 will give the sum of $\frac{5}{7}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375 and 2·46?

8. Two fields measuring 4 ac. 2 ro. 17 po. and 3 ac. 1 ro. 5 po. together cost £299, 14s. 6d. What is the value of the smaller field?

9. At what rate per cent. per annum did a banker discount a bill for £3730 due 6 months hence, if he paid £3627, 6s. 8d. as its present worth?

10. Make the following six additions, the first four horizontally, the last two vertically:—

(1) 7284, 13650, 7219, 38467, 851302.

(2) 17184, 6545, 2138, 10409, 27351.

(3) 7357, 8492, 17655, 4213, 102013.

(4) 2·15, ·0052, 45·2164, 3·1285, 25·1341.

(5)

£	s.	d.
5,632	4	2
4,187	16	$6\frac{1}{2}$
437	15	$7\frac{1}{2}$
596	13	$1\frac{3}{4}$
5,910	7	8
4,728	6	11
59	12	$10\frac{3}{4}$
19,403	13	8
27,391	4	$6\frac{1}{2}$
40,572	16	4

(6)

£	s.	d.
492	4	11
4,016	3	9
51,200	1	3
6,271	18	3
9,416	9	8
1,067	13	8
25,643	9	9
27,768	12	4
5,712	14	4
975	17	7

Royal Society of Arts—Grade I.—1907.

Arithmetic.

1. The average of 7 results is 16, the average of the first two is 18, and of the last two is 12. What is the average of the rest?

2. Express £15, 11s. $7\frac{3}{4}$d. in pounds, and 5 tons 16 cwt. 3 qr. 12 lb. 4 oz. in tons, each true to the fourth place of decimals.
Employ these figures to find the value of 5 tons 16 cwt. 3 qr. 12 lb. 4 oz. at £15, 11s. $7\frac{3}{4}$d. per ton.

3. Express the weight of a cubic metre of water in decimals of a ton, having given that a cubic foot of water weighs 62·4 lb., and the circumference of the earth is 40,000,000 metres or 25,000 miles. (Answer to four places of decimals only.)

4. Tin weighs 3·2 lb. per cubic inch, and copper 3·8 lb. per cubic inch. What will be the weight of a cubic foot of the mixture when there are 6·5 parts of copper to 4·5 parts of tin?

5. A man having three sons left £19,312 to be divided among them in proportion to their ages at the time of his death. When he died their ages were 21, 22, and 25 respectively. What was the share of each (approximately)? What difference would it have made if the father had lived five years longer?

6. A field was sold for £561, and a loss sustained thereby of $6\frac{1}{2}$ per cent. on the cost. Find what would have been the gain per cent. if it had been sold for £612.

7. A bicycle going at 5 miles an hour passed a milestone, and 14 minutes later a motor bicycle going in the same direction at a rate of 12 miles an hour passed the same milestone. Find when and where the second will overtake the first.

8. A banker's assets are £1,226, 5s. 4d. and his debts are £7,357, 12s. How much can he pay in the £?

9. Find the sum which will amount to £185, 12s. in 4 years, simple interest, being reckoned at 4 per cent. per annum.

10. Make the following six additions: the numbers in the first four are arranged in horizontal lines, and those in the last two in vertical columns:—

(1) 6347, 21586, 13954, 2713, 179426.

(2) 83674, 217586, 3842, 98751, 63718.

(3) £25, 16s. 8d., £187, 14s. 5d., £19, 15s. 9d., £84, 8s. 5d., £19, 12s. 11d.

(4) 2·1384, 65·6, 720·7, 6·435, 79·125.

(5)

£	s.	d.
13,874	15	10
649	7	8
2,735	5	0
10,867	11	11
2,935	19	5
4,672	4	6
19,863	6	4
274	16	9
8,553	7	8
21,758	3	5

(6)

£	s.	d.
28,374	14	10
12,681	7	7½
13,955	9	6
267	6	4½
385	13	9
462	18	3
8,793	7	4½
1,956	19	7
728	6	5½
375	5	8

Royal Society of Arts—Grade I.—1908.

Commercial Arithmetic.

(*Three hours allowed.*)

1. In a book of 604 pages there are 33 lines on each page, and the number of letters is 697416. If the average length of a word is 5 letters, find the average number of words in a line.

2. Find, by practice or otherwise, the value of £517, 13s. 5d. in dollars and cents, the rate of exchange being 4 dollars 86 cents to the £. (Answer to the nearest cent.)

3. A gallon is 0·1606 cub. ft. and a yard is 0·9144 metres. Express a litre as the decimal of a pint correct to two places.

4. Find, to the nearest penny, the simple interest on £84, 10s. for 314 days at 5½ per cent.

5. Find, to the nearest penny, the cost of a carpet 16 ft. 8½ in. long and 13 ft. 7 in. wide at 17s. 3d. per square yard.

6. The rateable value of property in a borough is £317,592, and it is necessary to raise a sum of not less than £20,000. How many pence in the £ must the rate be?

7. If a bookseller sells a book at the published price he makes 40 per cent. profit on what it cost him. What percentage of profit will be made if he takes 3d. in the shilling off the published price?

8. A tonne of 1000 kilograms is about 36 lbs. less than an English ton. Find, to two places of decimals, the number of grains in a gram.

9. A, cycling at 10 miles an hour, starts from London at 9 a.m. to go to Hastings, 64 miles. B starts at 11.30 a.m. from Hastings and, cycling at a uniform speed, passes A at 1.50 p.m. When will B reach London?

10. Make the following six additions: the numbers in the first four are arranged in horizontal lines, and those in the last two in vertical columns:—

(1) 217, 1508, 35, 407, 2094, 77, 132, 908.

(2) 215417, 8417296, 1874382, 707405.

(3) £13, 14s. 6d., £98, 12s. 7d., £213, 4s. 11d., £77, 17s. 7d., £8, 15s. 9d.

(4) 12·407, 8·1275, 6·098, 135·46, 84·076.

	£	s.	d.
(5)	21,772	14	4
	8,259	6	11
	545	12	1
	1,728	0	5
	15,419	8	7
	3,742	11	6
	85	5	2
	137	15	8
	1,027	13	9
	416	8	4

	£	s.	d.
(6)	13,245	13	$4\frac{1}{2}$
	8,027	8	10
	592	11	$3\frac{3}{4}$
	1,728	0	$8\frac{1}{2}$
	24,317	13	7
	4,056	18	$4\frac{3}{4}$
	809	9	9
	2,056	13	5
	92	19	$7\frac{1}{2}$
	186	4	4

EXERCISE 102.

Miscellaneous Exercises.

1. Bought 55 yd. of broadcloth for £64, 17s. 8d.; at what rate must it be sold per yard to gain 25 per cent. upon the whole?

2. A garrison of 1560 men being besieged, has provisions for 9 months; after 7 months they receive notice that they cannot be relieved till the end of 12 months; how many men must be dismissed to enable the remainder to subsist at the same rate on the remaining provisions to the end of that time?

3. Divide 1120 acres of land among A, B, and C, giving B 84 more than A, and C 116 less than B.

4. A gentleman on Christmas day gave £1, 17s. 6d. among his labourers: to every boy he gave 8d., to every woman 1s., and to every man 1s. 6d.; now there were 3 boys for 2 women, and 4 women for 3 men; how many were there of each?

5. A garden containing exactly an acre of land is 88 yd. in length: what is its breadth?

6. A and B enter into a partnership, and the business is to be conducted by A, whose services are to be considered as so much stock; A contributed £250, and is to have $\frac{3}{5}$ of the gain, and B contributed £800; what are A's services valued at, and how much of the gain is awarded for them?

7. The cost of building and fitting up a church for 1800 persons was £1500; $\frac{1}{6}$ of the seats are let at 10s. 6d. each, $\frac{1}{3}$ at 7s. 6d., $\frac{1}{8}$ at 5s., and the remainder are set aside for the poor; now if 5 per cent. is paid for the interest of the outlay, and £133, 6s. 8d. for repairs and other annual expenses, and the remainder given for a stipend to the minister; how much does he receive?

8. A garden wall was measured by a chain, and was found to measure 3276 links; but upon examination the chain was found to have been stretched $1\frac{1}{4}$ ft.; what was the length of the wall in feet?

9. A gentleman planted 156 trees in the breadth of a plantation, 4 ft. 2 in. 3 sec. distant from each other; what was the breadth of the plantation, allowing the same distance between the trees and the fence on each side?

10. Arrange 1875 men so that the number in front may be 3 times as many as in depth.

11. A farmer mixes wheat at 38s. 6d., 40s. 6d., 42s. 9d., 45s., and 47s. together, so as to form 78 qr. of a mixture worth 44s. per qr.; how much must he take of each?

12. The cost of covering a floor 24 ft. long with linoleum at 4s. 3d. a sq. yd. was £7, 13s.; what was the width of the room?

13. If 60 yd. of carpet $\frac{3}{4}$ yd. wide cover a floor 15 ft. broad, what is its length?

14. Divide £10,000 among A, B, C, and D, giving A £300 more than B, B £250 more than C, and C £350 more than D.

15. Three horses are worth £284; the second and third are worth £196, and the second is worth £18 less than the first; what are their values?

16. A room 25 ft. 10 in. long, 20 ft. 6 in. broad, and 13 ft. 3 in. high is to be papered; how many pieces of paper, each 9 ft. 10 in. long and 2 ft. 4 in. broad, will it require?

17. What is the price of 615 doz. bottles of brandy, each containing $2\frac{3}{11}$ pt., at 26s. 8½d. per gallon?

18. A, B, and C enter into partnership; B puts in 30 casks of whisky, and C £900; they gain £640, of which A's share is found to be £250, and B's £156; what is C's share of the gain, A's share of the joint stock, and the value of a cask of whisky?

19. Divide £8492 among A, B, C, and D, giving A 6 as often as B 5, B 2 as often as C 3, and C 4 as often as D 3.

20. A room 27 ft. 6 in. long, 20 ft. 9 in. broad, and 13 ft. 6 in. high is to be covered with paper $\frac{2}{3}$ yd. wide; how many yards will be sufficient?

21. What principal will clear £43, 12s. in $1\frac{3}{8}$ years at $3\frac{1}{2}$ per cent. per annum?

22. A merchant bought 10 casks of wine, each containing 130 gallons, which, having received damage, he sells for £1, 3s. 4d. per gallon, and thereby loses at the rate of 25 per cent.; whereas his gain should have been 20 per cent.; how much under their first value were they sold, and what was the prime cost per gallon?

23. Bought 3255 lb. of dried fish, and am allowed 5 lb. free on every 100 lb.; how much do I pay at $2\frac{1}{4}$d. per lb.?

24. If I spend on an average £5, 18s. $8\frac{1}{2}$d. a week, and save annually £216; what is my yearly income?

25. Multiply £17, 16s. 9d. by 74·75, add the product to £745·8475, and divide the sum by £12, 6s. 6d. × 3·1415 – £18·3714.

26. Bought a quantity of sugar for £375, 10s., and 4 months after sold it for £400; what did I gain per cent. per annum by this transaction?

27. An agent receives for selling goods £45, 12s. 6d. at the rate of $2\frac{1}{2}$ per cent.; what was the amount of the sale?

28. A merchant sells to another 25 cwt. 2 qr. 24 lb. of wool, but allows him 3 per cent. discount; how much does he receive at £1, 3s. 4d. per cwt.?

29. If £100 is due in 53 days, £180 in 95 days, £210 in 10 days, £240 in 156 days, and £219 in 354 days; what is the equated time for paying the whole at once?

30. A mixture of 49 gallons of wine and brandy contains only 1 gallon of brandy; how much wine must be added to it that it may contain in each 49 gallons $\frac{1}{5}$ gallon of brandy?

31. Two parties make a joint stock of £840, of which A contributed £650, and B the rest, and he is to have $\frac{2}{7}$ of the gain in consequence of spending time in the management of the business; at how much stock are his services valued?

32. If a person saves $3\frac{1}{4}$d. each day, in what time will he save £52?

33. If 30 gallons of whisky, worth 8s. 6d. a gallon, be mixed with 5 gallons of water, what will be the price of a pint of the mixture?

34. If I gain 25 per cent. by selling sugar at 56s. per cwt., what do I gain or lose per cent. by selling the same at 7d. per lb., and 6 months' credit, interest at 4 per cent.?

35. A person who had $\frac{1}{3}$ of a coal mine, sold $\frac{3}{5}$ of $\frac{3}{5}$ of his share for £5000; what was the value of the whole, and of his remaining share at that rate?

36. A merchant began business 20 years ago with a capital of £1600, and he is now worth £10,764; at what rate per cent. per annum has he increased his capital?

37. A person bought 240 apples at 2 a penny, and the same number at 3 a penny; if he sold the whole at 5 for 2d., how much did he gain or lose?

38. What sum will amount to £1920 in 5 years at 4 per cent. per annum simple interest?

39. If by selling cloth at 8s. 6d. per yd. I gain 5 per cent., what do I gain or lose per cent. by selling it at 7s. 1d. per yd.?

40. A and B gain by trade £120, of which A's share is £50, and it is known that B advanced £50 more than A; required the stock of each party.

41. Insured £15,000 on a ship at $2\frac{1}{2}$ guineas per cent., and commission $\frac{3}{4}$ per cent.; she received damage to the extent of £4000; what is the net recovery, allowing $2\frac{1}{4}$ per cent. discount on the damage?

42. In what time will the simple interest of £525 at 4 per cent. per annum pay a debt of £75?

43. What is the annual income derived from investing £4155 in the $3\frac{1}{4}$ per cents. at $103\frac{3}{4}$, brokerage $\frac{1}{8}$ per cent.?

44. Bought 12 cwt. 3 qr. 16 lb. of tea at £25, 13s. 4d. per cwt.; what do I gain or lose upon the whole by selling it at 5s. 3d. per lb.?

45. A merchant mixes together 75 lb. sugar at 4d. per lb., $87\frac{1}{2}$ lb. at 5d., 95 lb. at $5\frac{1}{2}$d., and 100 lb. at 6d. per lb.; what should the selling price of a lb. of the mixture be?

46. Find the amount of £500 in 7 years at $3\frac{1}{2}$ per cent. per annum compound interest.

47. In what time will any sum of money double itself at 4 per cent. simple interest?

48. In what time will the interest of £750, 12s. 6d. at 4 per cent. pay a debt of £135, 2s. 3d.?

49. Bought 72 stones of butter at 23s. per stone of 16 lb. (1 lb. = 24 oz. avoir.), and sold it at 18d. per lb. avoir.; what was gained?

50. A person after paying 25 per cent. of his income has remaining £480, 12s. 6d.; what was his original income?

51. If the freight of a ship of 170 tons for 3 months is £102, what should be the freight of a ship of 118 tons for 5 months?

52. How many gallons of water must be run off from a cistern which is 8 ft. 6 in. long, 5 ft. 4 in. broad, and 3 ft. 4 in. deep, to make the surface sink 1 ft. 4 in.?

53. Suppose bell-metal to be composed of 3 parts of copper and 1 of tin; how much of each is there in a bell of 150 lb. weight?

54. How much water must be added to a cask containing 84 gal. of spirits at 13s. 6d. to reduce the value to 11s. $4\frac{1}{2}$d.?

55. A rectangular tank is 18 ft. long, 14 ft. broad, and 10 ft. deep; find the cost of painting the sides and bottom of the inside at 1s. $1\frac{1}{2}$d. per sq. yd.

56. How much water must be added to a cask containing 248 gal. of spirits at 18s. to reduce the price to 15s. 6d.?

57. Paid £975, 16s. 3d. for £750, 12s. 6d. borrowed 7½ years since; what rate per cent. per annum was paid for it?

58. What is the present value of £494, 13s. 9¾d., due 15 months hence, at 2 per cent. per annum?

59. Reduce £2, 14s. 6d. to the decimal of £7, 5s. 4d.

60. Find the discount on £630, 15s., due 5 years hence, at 2½ per cent.

61. If 25 yards of cloth cost £31, 5s., how should it be sold per yard to gain 12½ per cent.?

62. How many sovereigns, half-sovereigns, crowns, half-crowns, and florins, and an equal number of each, are contained in £248, 17s.?

63. How many yards of plastering are in a wall 24 ft. 6 in. broad and 10 ft. 8 in. high?

64. Bought goods for £400, 8s. 3d., and 8 months' credit; what discount should be allowed for present payment, interest at 4½ per cent.?

65. What length of a board that is 9 in. broad will contain 12 sq. ft.?

66. Insured £7560 on a ship at 7½ guineas per cent.; £12,500 on the cargo at 5 guineas per cent.; and £750 on the net freight at 75s. per cent.; the commission is ¼ per cent.; what is the whole expense of the insurance?

67. A man's income is £250 per annum; what has he remaining after paying income tax at the rate of 9d. per £1?

68. A bill of £430, 14s., dated 15th May, at 3 months, was discounted 1st July at 4 per cent.; find the net proceeds.

69. Bought goods for £45, and sold ⅓ of them at a profit of 20 per cent.; what should the remainder be sold for to gain 17½ per cent. on the whole?

70. How many dice of ½ in. side can be cut from a cube of ivory of 4 in. side?

71. I invested £3042, 6s. 6d. in the 3 per cents. at 92½, and sold out at 93; what difference will it make in my income if I reinvest the proceeds in the 4 per cents. at 97½?

72. I invested £1749, 14s. 2d. in the 4 per cents. at 98, but had to sell out when the stock had fallen ½ per cent.; what was lost?

73. Bought goods for £500, and sold them for £512, 10s. payable 6 months hence; what was gained or lost, interest at 5 per cent.?

74. A cubic foot of water weighs 1000 oz.; what weight of water is in a cistern 12 ft. 6 in. long, 7 ft. 8 in. wide, and 1 ft. deep?

75. A piece of ground 55 yd. broad contains exactly 2½ acres; what is its length?

76. A legacy of £575 to come into possession in 5 years is improved at 2½ per cent. per annum compound interest; how much will then be received?

77. Find the difference between the simple and compound interest of £250 for 4 years at 4 per cent.

78. In what proportion should whisky at 15s. and 21s. per gallon be mixed so as to sell it at 17s. 6d. per gallon?

79. Find the side of a square field equal in area to a rectangular one, which is 600 yd. broad and 2400 yd. long.

80. What is the difference between the simple and compound interest of £575, 17s. 6d. for 5 years at 5 per cent.?

81. How many bricks 8 in. long, $3\frac{1}{2}$ in. broad, and 3 in. thick will be required for a wall which is 98 yd. long, 6 ft. high, and 1 ft. 6 in. broad?

82. How many yards of carpet 2 ft. 6 in. wide will cover a square floor 18 ft. 6 in. in the side?

83. A bankrupt can pay 16s. 8d. per £1, but the expense attending his failure is £313, 1s. $0\frac{1}{2}$d., and then he can only pay 15s. per £1; what is the amount of his debts?

84. Find by practice the value of 73 cwt. 3 qr. $24\frac{1}{2}$ lb. at £5, 16s. 8d. per cwt.

85. Find by decimals the value of 1450 ac. 2 ro. 25 po. at £43, 13s. 6d. per acre.

86. A merchant bought 40 yd. of cloth for £28, 2s. 6d.; he gave 12s. 6d. a yard for part of it, and 15s. a yard for the rest; how many yards of each did he buy?

87. Bought a quantity of cloth for £412, 10s.; 85 yd. getting damaged, were sold at 15s., whereby I lost £6, 7s. 6d., but sold the remainder so as to gain £17, 16s. 8d. upon the whole; find the quantity bought, and at what rate the undamaged part was sold.

88. Bought goods to the value of £360, and sold them at £400, payable in 6 months; if the rate of interest was 5 per cent., what was the gain or loss?

89. What is that gentleman's yearly income who lets his estate of 24642 ac. at 11s. 8d. an acre?

90. A bankrupt's effects amount to £1850; divide it among his six creditors, whose claims are to each other as the numbers, 3, 5, 7, 9, 11, 13.

91. How much timber is there in a log 3 ft. 8 in. by 2 ft. 11 in., and $37\frac{1}{4}$ ft. long?

92. How many roods (rood = 36 sq. yd.) of mason work in a wall 36 ft. long and 15 ft. high?

93. A piece of ground 88 yd. long has an area of $\frac{3}{4}$ of an acre; what is its breadth?

94. How many square yards of brickwork are in a wall 56 ft. long, 24 ft. high, and $2\frac{1}{2}$ bricks thick?

95. In what proportions should tea at 3s. 4d. be mixed with tea at 4s. a lb. so as to sell at 3s. 6d. a lb.?

96. After paying a tax of 7d. in the £1, and other taxes of 1s. 3d. per £1 on the remainder, £2184, 7s. 6d. was left; what was the original income?

97. What is the difference between the simple and the compound interest on £336, 13s. 4d. for 4 years at 2½ per cent.?

98. What will be the cost of paving a street ¾ mile long and 66½ ft. broad at 6½d. a square yard?

99. What will the glazing of nine windows cost at 9¾d. a foot; three are 6½ ft. high, three 5½ ft. high, and three 4¾ ft. high, and their breadth in every case is 3 ft. 8 in.?

100. How many deals 33 ft. by 8 in. will floor a room 33 ft. by 28 ft.?

101. A wall that is to be built to a height of 21 ft. is raised 6 ft. high by 9 men in 12 days; how many men will be required to finish it in 18 days?

102. If an army of 75,000 men lose 4950 killed and wounded, what percentage of the original army does this loss represent?

103. A debtor owed £5146 and had sufficient assets to pay 9s. 4½d. in the pound, if the cost of liquidation were not deducted, but, after the process of liquidation, a creditor, to whom £800 was owed, received only £338, 6s. 8d.; find the whole cost of the liquidation.

104. Two cyclists, A and B, start from two places 35 miles apart and ride to meet each other. The circumference of A's driving wheel is 88 in., and it makes 150 revolutions per minute; that of B's is 90 in., and makes 145 revolutions per minute. Find where A and B will meet.

105. Rain fell to the depth of 0·8 inch on a field containing 4·48 acres. If the weight of a cubic foot of water be 62½ lb., find how many tons of rain have fallen on the field.

106. Find, by working in decimals, the true value of $20{\cdot}875 \times 12{\cdot}256 - 4{\cdot}125 \times 37{\cdot}8 \div 7{\cdot}875$.

107. A news sheet is 6 ft. by 5 ft. 6 in. How many miles of paper 6 ft. wide would be required for the copies issued in a week, if the daily circulation amounts to 55,000 copies every week day?

108. A cyclist rides up hill at an average rate of 6 miles an hour, on the level at an average rate of 12 miles an hour, and down hill at an average rate of 15 miles an hour. Find the time he takes to ride 50 miles, one-third of which is level and two-fifths up hill.

109. The length and breadth of a board are, to the nearest centimetre, 95 centimetres and 15 centimetres; between what limits do the actual length and breadth lie.

110. A clerk deducted 5 % from a bill instead of 4 %, and received in consequence £3, 2s. too little. What was the amount owing?

111. A bar of iron 10 metres long and one square centimetre in cross-section is drawn into a wire one-tenth of a square millimetre in cross-section; find the length of the wire.

112. The Government Grant paid to a school is at the rate of 22s. per head on the average attendance throughout the year. The children on the register attend, on the average, only 85 per cent. of the times that the school is open, and the consequent

loss of grant through this irregularity of attendance is £41, 5s. per annum. How many children are there on the registers?

113. A surveyor's chain is 22 yards long, and consists of 100 equal links. A rectangular field measures 7 chains 20 links in length, and 6 chains 25 links in breadth; find its area in acres.

114. A motor car on its way to York overtook a cyclist at 20 minutes past 3: the car reached York at 4, and immediately returned, meeting the cyclist at 4.16. Find when the cyclist would reach York, supposing the speeds of car and cyclist uniform throughout.

115. A book contains 650 pages, each 18·5 centimetres long and 12·25 centimetres broad. If 100 pages make a thickness of 0·42 centimetre, find the volume of the book correct to the nearest cubic centimetre.

116. A sells an article to B at a profit of 20 per cent. B sells it to C at a profit of 5 per cent. If C pays 70 shillings, what did it cost A (to the nearest shilling)?

117. A train has increased its speed in the last 10 minutes from 22 feet per second to $27\frac{1}{2}$ feet per second; if this rate of increase is maintained, find the speed of the train 15 minutes from now.

118. (1) Add $1\frac{1}{6}$, $2\frac{2}{15}$, $3\frac{3}{5}$, and divide their sum by the difference between $4\frac{1}{2}$ and $5\frac{1}{3}$.

(2) Simplify 2·879 of £1 + $9{\cdot}0\dot{6}$ of 1s. + ·24 of 1d.

119. Name one principal coin in use in each of the following countries—Germany, France, India, United States of America; and express £100 as nearly as you can in terms of each of these coins.

120. The average profit per annum made in a business during three years was £5744, but during the first two of these years the average profit was £4843, while during the last two it was £6417; find each separate year's profit.

121. A map is on the scale of 6 in. to a lineal mile; how many acres does a square inch represent?

122. Express in tons and vulgar fraction of a ton the weight of the lead required to cover 294 sq. yd. with lead $\frac{1}{8}$ in. thick, a cubic foot of lead weighing 710 lb.

123. A sum of money lent at $4\frac{1}{2}$ per cent. interest, after deduction of income tax at 1s. 3d. in £1, yields £71, 14s. $4\frac{1}{2}$d. annually; find the sum lent.

124. The traffic receipts of a railway in 1900 were £1,987,000, and its expenses were £1,144,000. In 1901 it was found that the receipts had fallen $2\frac{1}{2}$ per cent., while the expenses had risen $\frac{1}{2}$ per cent.; find to two decimal places by what percentage of itself the profit in 1900 exceeded the profit in 1901.

125. Find the cost price of goods which, sold for £456, yield a profit of 15 per cent.

126. Show how to convert a recurring decimal to a vulgar fraction. Take as an example $0{\cdot}4\dot{1}\dot{7}\dot{2}$.

127. A 4 per cent. loan is issued at 97. If I invest in it £4850, and sell after three years at par, how much per cent. have I made per annum? If I invest the proceeds in a 3 per cent. stock at 88½, what is the difference in the income?

128. A man's net income is £760 a year after paying 1s. in the £ income tax. Suppose his gross income is increased by 20 per cent., and the income tax rises to 1s. 4d. in the £, what is his net income?

129. A pound sterling exchanges for 26·4 Italian *lire*, and for 25·2 French *francs*. Convert 1000 *francs* into *lire*.

130. Find the present value of £5000 due 2½ years hence, reckoning 4 per cent. compound interest, payable half-yearly.

PRINTED BY OLIVER AND BOYD, TWEEDDALE COURT, EDINBURGH

ANSWERS

Commercial Arithmetic

A COMPLETE MANUAL OF

APPLIED ARITHMETIC

FOR SENIOR CLASSES

ANSWERS

OLIVER AND BOYD
EDINBURGH: TWEEDDALE COURT
LONDON: 33 PATERNOSTER ROW, E.C.

ANSWERS.

EXERCISE 1.

(*a*) 4870832 (*b*) 3196987 (*c*) 4025306
(*d*) 3864040 (*e*) 1406063

EXERCISE 2.

(*a*) 6179660 (*b*) 5016167 (*c*) 5061037
(*d*) 6234880 (*e*) 6117780

EXERCISE 3.

(1) 197978 (4) 99837 (7) 3612976
(2) 355481 (5) 153411 (8) 1111095
(3) 697099 (6) 179523 (9) 10991003334
(10) 8336897788

EXERCISE 4.

(*a*) 19933 (*e*) 15948 (1) 29807 (4) 24707
(*b*) 15825 (*f*) 21405 (2) 23127 (5) 31611
(*c*) 16627 (*g*) 23046 (3) 38929 Total 148181
(*d*) 19812 (*h*) 15585

EXERCISE 5.

(*a*) 1862487 (*d*) 2506606 (3) 204672 (6) 996612
(*b*) 713328 (1) 553346 (4) 1697734 (7) 1813053
(*c*) 1785923 (2) 555647 (5) 1047280 Total 6868344

EXERCISE 6.

(*a*) 2850726	(*d*) 2407292	(1) 2039267	(4) 3492712
(*b*) 3387383	(*e*) 3174837	(2) 3815046	(5) 3483802
(*c*) 2405403	(*f*) 3506292	(3) 2271591	(6) 2629515
		Total 17731933	

EXERCISE 7.

(*a*) 558278	(1) 229075	(7) 276312
(*b*) 614844	(2) 275129	(8) 291932
(*c*) 699555	(3) 314241	(9) 280406
(*d*) 711174	(4) 224775	(10) 184758
(*e*) 682986	(5) 298642	(11) 363387
Total 3266837	(6) 260804	(12) 267376

EXERCISE 8.

(*a*) 397557 (*b*) 456837 (*c*) 566751 (*d*) 619061 (*e*) 595868

EXERCISE 9.

(*a*) 424725 (*b*) 553954 (*c*) 619357 (*d*) 592086 (*e*) 572511

EXERCISE 10.

(*a*) 365394	(1) 108641	(11) 110073
(*b*) 218013	(2) 190528	(12) 195437
(*c*) 271702	(3) 193116	(13) 256823
(*d*) 279247	(4) 174444	(14) 204622
(*e*) 439521	(5) 161976	(15) 70823
(*f*) 277615	(6) 224943	(16) 169895
(*g*) 278368	(7) 144014	(17) 307908
(*h*) 253658	(8) 156506	(18) 109630
(*i*) 261081	(9) 121532	Total 3041760
(*j*) 397161	(10) 140849	

EXERCISE 11.

(*a*) 855036	(*i*) 711633	(7) 275536	(15) 438217
(*b*) 670127	(*j*) 891523	(8) 570731	(16) 538383
(*c*) 826104	(1) 265584	(9) 383097	(17) 528883
(*d*) 953176	(2) 501119	(10) 552055	(18) 236224
(*e*) 874876	(3) 433862	(11) 257208	(19) 123258
(*f*) 680204	(4) 304356	(12) 508702	(20) 371668
(*g*) 801266	(5) 311372	(13) 607818	Grand Total 7938635
(*h*) 674690	(6) 226058	(14) 504504	

EXERCISE 12.

(1) 23740 ; 237400 ; 2374000 ; 237400000
(2) 906000 ; 9060 ; 9060000 ; 906000000
(3) 38900 ; 3890000 ; 389000 ; 38900000
(4) 1420840 ; 4262520 ; 35521000 ; 497294000
(5) 10770 ; 2872000 ; 251300000 ; 32310000
(6) 3140000 ; 471000000 ; 7065000 ; 157000

(7) 6915555	(8) 27212724	(9) 822724452
(10) 6573632571	(11) 51289138	(12) 484725094
(13) 3487415420	(14) 4746767440	(15) 617280
(16) 39450600	(17) 172839000	(18) 22530850
(19) 70986250	(20) 77160000	

EXERCISE 13.

(1) 40996494	(6) 109492752	(11) 155308260
(2) 55654140	(7) 97636032	(12) 55555515
(3) 77202073	(8) 124822404	(13) 498469104
(4) 96490058	(9) 169336664	(14) 380486367
(5) 86838210	(10) 181472829	(15) 182962884
	(16) 802098705	

EXERCISE 14.

(1) 315414 ; 2896074 ; 3584250
(2) 789200 ; 3125232 ; 1104880
(3) 1406767 ; 82502747 ; 20687750
(4) 4589375 ; 36898575 ; 477295
(5) 7885592 ; 1895575 ; 7430654
(6) 15543500 ; 1181306 ; 62360522
(7) 794200 ; 508288 ; 3081496
(8) 5921125 ; 710535 ; 4831638
(9) 15992750 ; 1151478 ; 2878695
(10) 4594875 ; 1286565 ; 36685482

EXERCISE 15.

(1) 24127632	(11) 193625747	(21) 57783767936
(2) 17819543	(12) 65720808	(22) 24685316040
(3) 3597705	(13) 331824675	(23) 6070595256
(4) 18866175	(14) 307736000	(24) 22307329341
(5) 15468441	(15) 4806273384	(25) 23985264576
(6) 22648432	(16) 955945452	(26) 12018873792
(7) 21286356	(17) 17394083928	(27) 57218121072
(8) 40155786	(18) 29720058996	(28) 18680854125
(9) 618176268	(19) 12861344175	(29) 448187974590708
(10) 531916875	(20) 38346523875	(30) 44832223896372

EXERCISE 16.

(1) 1491930848
2331141950
1678422204

(2) 438065978
844841529
1501940496

(3) 536135910
1286726184
786332668

(4) 681825774
1428587336
1461055230

(5) 4991390136
3396918287
3812867465

(6) 1123038144
1637763960
4632532344

(7) 1883287476
1150897902
1953038864

(8) 942954496
2475255552
3565546688

(9) 4544477721
4760881422
6924918432

(10) 4757052352
5723328611
8027525844

(11) 2641150092
7640469909
12451136148

(12) 1918240716
4019171024
6576825312

(13) 374277618
(14) 388393856
(15) 488526920
(16) 675826011
(17) 234913524
(18) 731104084
(19) 194539008
(20) 4631512230
(21) 6696908120
(22) 5764827879
(23) 1992639966
(24) 5190103485
(25) 36180721665
(26) 575610088830
(27) 62634005490
(28) 283950000000
(29) 601224600000
(30) 8485840102080
(31) 24504402000
(32) 21553600000
(33) 84283000000
(34) 78763200000
(35) 69565200000
(36) 68730000000
(37) 12795174396
(38) 45839656861
(39) 28777715404
(40) 81942907651
(41) 56197589304
(42) 72294112302
(43) 54012068792
(44) 22397800057
(45) 212869936864
(46) 940056059943
(47) 365486477103
(48) 791031048808
(49) 569247764085
(50) 187882537392
(51) 291242362752
(52) 358391469792

EXERCISE 17.

(1)	441	(9)	41209	(17)	729	(25)	144
(2)	1681	(10)	1004004	(18)	42436	(26)	1596
(3)	1089	(11)	3136	(19)	1521	(27)	2491
(4)	2809	(12)	11025	(20)	1020100	(28)	621
(5)	2209	(13)	5476	(21)	91	(29)	247500
(6)	9801	(14)	7921	(22)	1600	(30)	4875
(7)	998001	(15)	4624	(23)	17800		
(8)	10201	(16)	7225	(24)	755		

EXERCISE 18.

1. (1) (*a*) 2×3 (*h*) $2 \times 3 \times 11$ (*o*) $2^4 \times 3 \times 13$
(*b*) $2^2 \times 3$ (*i*) $2 \times 5 \times 7$ (*p*) $2^2 \times 3^2 \times 11$
(*c*) 3×5 (*j*) 3×5^2 (*q*) 11^3
(*d*) 3×7 (*k*) $2^6 \times 11$ (*r*) $2^2 \times 3 \times 11$
(*e*) $2 \times 3 \times 5$ (*l*) $2^2 \times 3^3 \times 7$ (*s*) $2^4 \times 3^2 \times 11$
(*f*) $2 \times 3 \times 7$ (*m*) $2^3 \times 3^2 \times 11$ (*t*) $2^6 \times 3^3$
(*g*) 5×11 (*n*) $2^5 \times 3 \times 11$

(2) (*a*) $2^2 \times 3 \times 5$ (*f*) $3^2 \times 5 \times 7$ (*l*) $2^4 \times 7 \times 13$
(*b*) $2 \times 3 \times 13$ (*g*) $3 \times 5^2 \times 7$ (*m*) $3 \times 5^3 \times 7^2$
(*c*) $2 \times 3 \times 17$ (*h*) $2^3 \times 3^2 \times 17$ (*n*) $2^2 \times 3 \times 5 \times 7 \times 11$
(*d*) $2 \times 3 \times 5 \times 7$ (*i*) $5 \times 11 \times 13$ (*o*) $2 \times 3 \times 5^3 \times 7$
(*e*) $2^2 \times 3^2 \times 7$ (*j*) $2 \times 7 \times 61$ (*p*) $3^2 \times 5^2 \times 7^2$
(*k*) 11^3

2. 503, 509, &c. - - - - 599 (14 prime numbers)
601, 607 - - - - - - 691 (16 ,, ,,)
701, 709 - - - - - - 797 (14 ,, ,,)
809, 811 - - - - - - 887 (15 ,, ,,)
907, 911 - - - - - - 997 (14 ,, ,,)

4. 1002, 1002, 1004, 1005, 1002, 1001, 1008, 1008, 1010, 1001

EXERCISE 19.

(1) $372649\frac{7}{10}$, $3726\frac{497}{1000}$, &c. &c.

(2) $107367\frac{10}{100}$, $1073\frac{6710}{10000}$, &c.

(3) $1046371\frac{23}{70}$, $244153\frac{33}{300}$, $9155\frac{5993}{8000}$

(4) $38667\frac{305}{400}$, $257785\frac{5}{80}$, $309\frac{17105}{50000}$

(5) $1626343\frac{10}{30}$, $243951\frac{100}{200}$, $4065\frac{10300}{12000}$

(6) $821347\frac{40}{100}$, $7466\frac{8740}{11000}$, $684456\frac{20}{120}$

(7) $5473\frac{3}{5}$, $547\frac{18}{50}$, $54\frac{368}{500}$, $5\frac{2368}{5000}$

(8) $12\frac{11}{25}$, $2\frac{61}{125}$

(9) $28\frac{23}{25}$, $5\frac{98}{125}$, $2\frac{223}{250}$

(10) $325\frac{16}{25}$, $65\frac{16}{125}$, $13\frac{16}{625}$

EXERCISE 20.

(1) $514660495\frac{3}{16}$
$548971194\frac{13}{15}$
$457475995\frac{13}{18}$

(2) $3294264497\frac{17}{24}$
$316249391\frac{18}{25}$
$292823510\frac{23}{27}$

(3) $4454137731\frac{3}{14}$
$194868526\frac{2}{32}$
$173216467\frac{22}{36}$

(4) $2198710382\frac{6}{44}$
$214985015\frac{23}{45}$
$201548452\frac{2}{48}$

(5) $151993226\frac{52}{54}$
$14656489\frac{724}{66}$
$13027990\frac{852}{63}$

(6) $123804030\frac{3}{74}$
$12005239\frac{251}{56}$
$1100480\frac{51}{72}$

(7) $1097934\frac{1}{7}\frac{14}{5}$
$8402557\frac{8}{9}\frac{5}{8}$
$3812271\frac{8}{2}\frac{3}{16}$

(8) $57894274\frac{13}{125}$
$5743479\frac{5}{12}\frac{3}{6}$
$5653737\frac{7}{128}$

(9) $59957316\frac{3}{13}\frac{3}{5}$
$5506284\frac{1}{1}\frac{0}{4}\frac{7}{7}$
$4215748\frac{1}{1}\frac{8}{9}\frac{9}{3}$

(10) $53485662\frac{13}{15}\frac{9}{4}$
$5084439\frac{5}{16}\frac{97}{2}$
$3268568\frac{2}{2}\frac{2}{6}\frac{3}{2}$

(11) $4722771\frac{14}{16}\frac{5}{8}$
$3526336\frac{4}{2}\frac{37}{2}\frac{}{5}$
$2313194\frac{14}{34}\frac{5}{3}$

(12) $2559714\frac{8}{3}\frac{8}{2}\frac{6}{4}$
$2632849\frac{5}{3}\frac{13}{1}\frac{3}{5}$
$1178050\frac{6}{7}\frac{14}{0}\frac{}{4}$

EXERCISE 21.

(1) $2763688\frac{9}{3}\frac{3}{1}\frac{14}{}$
$208961\frac{8}{4}\frac{4}{1}\frac{6}{11}$
$16798\frac{8}{5}\frac{9}{1}\frac{3}{12}$

(2) $1524683\frac{5}{5}\frac{0}{2}\frac{35}{}$
$18438033\frac{1}{4}\frac{2}{8}$
$1495915\frac{8}{6}\frac{9}{3}\frac{8}{}$

(3) $1150459\frac{0}{5}\frac{2}{3}\frac{1}{}$
$17933625\frac{9}{3}\frac{1}{4}$
$1418007\frac{6}{4}\frac{2}{3}\frac{41}{}$

(4) $1277968\frac{8}{6}\frac{0}{2}\frac{13}{}$
$121898\frac{5}{6}\frac{6}{5}\frac{2}{4}\frac{3}{}$
$118259\frac{7}{6}\frac{9}{7}\frac{9}{4}\frac{40}{}$

(5) $8963843\frac{1}{9}\frac{3}{2}\frac{13}{}$
$8867457\frac{9}{9}\frac{2}{3}$
$850178\frac{9}{9}\frac{5}{7}\frac{34}{}$

(6) $9439232\frac{4}{8}\frac{3}{3}\frac{4}{}$
$900524\frac{4}{8}\frac{7}{7}\frac{6}{87}$
$11354439\frac{0}{6}\frac{2}{9}\frac{6}{}$

(7) $9923790\frac{1}{8}\frac{5}{3}\frac{53}{}$
$11937312\frac{8}{6}\frac{4}{9}$
$111307\frac{3}{7}\frac{7}{4}\frac{6}{12}\frac{}{4}$

(8) $1085556\frac{6}{7}\frac{9}{3}\frac{57}{}$
$139027\frac{4}{6}\frac{3}{7}\frac{6}{42}$
$16860774\frac{2}{4}\frac{0}{7}\frac{}{20}$

(9) $1775674\frac{9}{4}\frac{7}{7}\frac{38}{}$
$106995\frac{7}{7}\frac{9}{8}\frac{9}{75}$
$128394\frac{6}{6}\frac{5}{5}\frac{9}{62}$

(10) $1351356\frac{3}{5}\frac{4}{1}\frac{54}{3}$
$113275\frac{4}{6}\frac{6}{1}\frac{1}{2}\frac{21}{1}$

(11) $1949583\frac{8}{4}\frac{7}{2}\frac{72}{3}$
$190896\frac{7}{4}\frac{5}{3}\frac{2}{2}\frac{46}{}$

(12) $1462288\frac{2}{6}\frac{3}{2}\frac{33}{4}$
$120964\frac{0}{6}\frac{9}{3}\frac{4}{1}\frac{90}{}$

(13) $1490629\frac{1}{2}\frac{2}{3}\frac{32}{3}$
$4015184\frac{0}{2}\frac{3}{3}\frac{5}{5}$

(14) $952351\frac{8}{8}\frac{3}{3}\frac{17}{2}$
$139745\frac{4}{5}\frac{2}{6}\frac{7}{7}\frac{8}{}$

(15) $139959\frac{1}{6}\frac{5}{6}\frac{47}{7}$
$1037344\frac{3}{7}\frac{6}{6}\frac{3}{5}\frac{84}{}$

(16) $980042\frac{7}{8}\frac{3}{6}\frac{86}{4}$
$969939\frac{2}{8}\frac{9}{7}\frac{8}{3}$

(17) $858504\frac{1}{9}\frac{9}{9}\frac{4}{3}\frac{0}{}$
$837628\frac{7}{9}\frac{1}{8}\frac{5}{7}\frac{4}{}$

(18) $959809\frac{4}{8}\frac{6}{7}\frac{12}{9}$
$857390\frac{7}{9}\frac{7}{8}\frac{5}{4}\frac{0}{}$

(19) 691431

(20) 510547

(21) 962595

(22) 149857

(23) 858513

(24) 1275353

(25) 574164

(26) 581756

(27) 1174691

(28) 873013

(29) 1756459

(30) 1129802

(31) 1001473

(32) 1020572

(33) 44352

(34) 141884

(35) 102548

(36) 86107

EXERCISE 22.

1. (*a*) £2210, 9s. 5d. (*b*) £2231, 14s. (*c*) £1489, 8s. 5¼d.
(*d*) £2729, 12s. 8d. (*e*) £2516, 7s. 3½d. Total £11177, 11s. 9¾d.

		£	s.	d.
2.	(*a*)	482	10	4¼
	(*b*)	593	18	6½
	(*c*)	804	5	0¾
	(*d*)	2234	9	8
3.	(*a*)	8634	18	0½
	(*b*)	9624	14	11¼
	(*c*)	8020	13	6¼
	(*d*)	7675	4	9½

		£	s.	d.
4.	(*a*)	3734	18	0¼
	(*b*)	4726	15	3¾
	(*c*)	4827	17	8
	(*d*)	6228	6	10
5.	(*a*)	9117	8	4¾
	(*b*)	10218	13	5¾
	(*c*)	8824	18	7
	(*d*)	9909	14	5½

EXERCISE 23.

1.	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(*a*)	38159	19	3	(*b*)	29076	8	7	(*c*)	595	11	0	(*d*)	33022	18	9
(1)	8612	11	10	(2)	8763	10	11	(3)	15656	6	3	(4)	1648	1	6
(5)	6844	9	3	(6)	3718	4	11	(7)	5970	16	3	(8)	6485	2	11
(9)	10000	5	9	(10)	413	9	9	(11)	8267	10	2	(12)	4672	6	9
(13)	9386	6	4	(14)	9383	15	1	(15)	1031	19	11				

Gross Total £100854, 17s. 7d.

2.	£	s.	d.		£	s.	d.		£	s.	d.
(*a*)	20427	13	11	(*b*)	15047	2	11	(*c*)	11716	3	1
(*d*)	1127	12	8	(1)	5013	3	11	(2)	5665	17	7
(3)	4650	11	10	(4)	2149	9	8	(5)	2347	9	8
(6)	3483	15	10	(7)	1970	1	9	(8)	4457	15	6
(9)	3702	15	10	(10)	7312	18	9	(11)	1171	0	6
(12)	778	9	9	(13)	3486	6	10	(14)	1076	17	5
(15)	1051	17	9								

Grand Total £48318, 12s. 7d.

3.	£	s.	d.		£	s.	d.		£	s.	d.
(*a*)	7337	11	1	(*b*)	6086	14	10	(*c*)	6605	1	9
(*d*)	6780	16	5	(1)	2497	11	4	(2)	1704	11	3
(3)	892	0	6	(4)	1300	2	4	(5)	1840	1	4
(6)	488	19	7	(7)	2499	17	1	(8)	1964	6	0
(9)	1009	11	9	(10)	1649	8	1	(11)	1487	15	5
(12)	614	5	8	(13)	2467	11	6	(14)	738	10	4
(15)	1223	6	2	(16)	1942	0	0	(17)	785	12	3
(18)	1704	13	6								

Gross Total, £26810 4s. 1d.

4.

£	s.	d.	£	s.	d.	£	s.	d.
(*b*) 5464	1	11	(*c*) 6111	6	11	(*d*) 6532	16	1
(*e*) 5431	5	2	(4) 2154	8	6	(5) 1530	2	7
(6) 668	5	5	(7) 1801	1	10	(8) 1133	19	5
(9) 1267	7	9	(10) 2079	12	0	(11) 1386	1	3
(12) 774	14	11	(13) 1855	6	8	(14) 489	10	4
(15) 1523	5	1	(16) 1922	14	0	(17) 1292	9	1
(18) 777	3	2	(19) 528	2	9	(20) 1004	3	4
(21) 1351	2	0	Total £23539, 10s. 1d.					

5.

£	s.	d.	£	s.	d.	£	s.	d.
(*a*) 8342	6	3	(*b*) 6703	12	6	(*c*) 7257	6	4
(*d*) 7775	9	4	(*e*) 5670	10	1	(1) 2624	5	11
(2) 1723	4	11	(3) 985	17	2	(4) 2237	3	2
(5) 2157	5	0	(6) 697	9	4	(7) 2585	11	5
(8) 2003	18	11	(9) 1745	4	2	(10) 2222	2	10
(11) 1559	1	11	(12) 786	19	6	(13) 2683	2	2
(14) 772	19	3	(15) 1795	18	11	(16) 1960	13	8
(17) 1611	11	7	(18) 1708	11	5	(19) 573	4	7
(20) 1240	18	1	(21) 2074	0	7	Total £35749, 4s. 6d		

EXERCISE 24.

£	s.	d.	£	s.	d.	£	s.	d.
(*a*) 56319	1	$5\frac{1}{2}$	(2) 25783	3	$8\frac{1}{4}$	(9) 19976	3	$3\frac{1}{4}$
(*b*) 44104	17	$2\frac{1}{2}$	(3) 33289	2	$2\frac{1}{4}$	(10) 11661	11	$10\frac{1}{2}$
(*c*) 68501	6	5	(4) 35599	15	$5\frac{1}{2}$	(11) 25297	4	$6\frac{1}{4}$
(*d*) 79458	4	$4\frac{1}{4}$	(5) 25302	8	$5\frac{1}{2}$	(12) 19204	13	9
(*e*) 55142	5	$6\frac{1}{2}$	(6) 23548	8	$4\frac{1}{4}$	(13) 28233	14	$8\frac{3}{4}$
(*f*) 62528	5	0	(7) 20514	7	$10\frac{1}{4}$	(14) 15180	16	$7\frac{3}{4}$
(1) 40990	7	$3\frac{1}{2}$	(8) 41472	1	$10\frac{3}{4}$			
			Grand Total £366053, 19s. $11\frac{3}{4}$d.					

EXERCISE 25.

£	s.	d.	£	s.	d.	£	s.	d.
(*a*) 43505	6	$1\frac{3}{4}$	(2) 21352	6	$0\frac{3}{4}$	(11) 24270	1	$10\frac{1}{3}$
(*b*) 68190	14	$3\frac{3}{4}$	(3) 52716	7	10	(12) 26511	13	$7\frac{1}{3}$
(*c*) 77123	5	$10\frac{1}{2}$	(4) 25181	9	$10\frac{1}{2}$	(13) 25481	15	$1\frac{1}{2}$
(*d*) 71271	4	$7\frac{3}{4}$	(5) 23148	15	$4\frac{1}{3}$	(14) 33693	2	$1\frac{3}{4}$
(*e*) 61501	10	3	(6) 25573	12	$8\frac{3}{4}$	(15) 18159	8	$2\frac{1}{4}$
(*f*) 65694	9	$9\frac{1}{3}$	(7) 26398	17	$11\frac{1}{4}$	(16) 33187	15	8
(*g*) 72678	19	$0\frac{1}{2}$	(8) 16259	17	11	(17) 22206	10	$1\frac{3}{4}$
(*h*) 66126	17	9	(9) 43210	2	$11\frac{1}{4}$	(18) 35792	15	$7\frac{3}{4}$
(1) 36632	12	$5\frac{1}{2}$	(10) 36115	2	$3\frac{1}{4}$			
			Grand Total £526092, 7s. $9\frac{3}{4}$d.					

EXERCISE 26.

(1) £14, 6s. 9d.
(2) £1, 7s. 6d.
(3) £319, 4s. 4d.
(4) £40, 16s. 1d.
(5) £376, 15s. 7d.
(6) £8604, 13s. 8d.

EXERCISE 27.

	£	s.	d.		£	s.	d.
(1)	75	5	$6\frac{1}{2}$	(2)	140	16	$11\frac{1}{4}$
	112	18	$3\frac{3}{4}$		234	14	$10\frac{3}{4}$
	150	11	1		187	15	11
	188	3	$10\frac{1}{4}$		93	17	$11\frac{1}{2}$
	225	16	$7\frac{1}{2}$		328	12	$10\frac{1}{4}$
	263	9	$4\frac{3}{4}$		375	11	10
	301	2	2		281	13	$10\frac{1}{2}$
	338	14	$11\frac{1}{4}$		469	9	$9\frac{1}{2}$
	376	7	$8\frac{1}{2}$		422	10	$9\frac{3}{4}$
	414	0	$5\frac{3}{4}$		563	7	9
	451	13	3		516	8	$9\frac{1}{2}$
(3)	414	7	$1\frac{1}{2}$	(4)	244	12	3
	452	0	6		380	10	2
	470	17	$2\frac{1}{4}$		434	17	4
	508	10	$6\frac{3}{4}$		475	12	$8\frac{1}{2}$
(5)	1006	3	3	(6)	1572	19	0
	1117	19	2		1605	14	$4\frac{3}{4}$
	1173	17	$1\frac{1}{2}$		1802	6	$9\frac{1}{4}$
	1257	14	$0\frac{3}{4}$		2097	5	4
(7)	2963	0	$8\frac{1}{4}$	(8)	3855	15	0
	3104	2	$7\frac{1}{2}$		4123	10	$2\frac{1}{2}$
	3292	5	$2\frac{1}{2}$		4284	3	4
	3950	14	3		4337	14	$4\frac{1}{2}$
(9)	5784	12	6	(10)	9674	14	$1\frac{1}{2}$
	6507	14	$0\frac{3}{4}$		7675	16	0
	7888	2	6		8635	5	6
	9465	15	0		10554	4	6

	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(11)	174	0	$2\frac{1}{4}$	219	8	$0\frac{3}{4}$	234	10	$8\frac{1}{2}$	279	18	$6\frac{3}{4}$
(12)	413	6	7	445	19	$2\frac{1}{2}$	511	4	$5\frac{1}{2}$	576	9	$8\frac{1}{2}$
(13)	48	5	$9\frac{3}{4}$	53	19	$5\frac{1}{4}$	63	8	$9\frac{3}{4}$	71	0	$3\frac{3}{4}$
(14)	66	19	$9\frac{1}{2}$	78	6	$10\frac{1}{2}$	69	5	$2\frac{1}{2}$	84	0	5
(15)	2495	4	$11\frac{1}{2}$	2621	11	$9\frac{1}{2}$	2684	15	$2\frac{1}{2}$	2811	2	$0\frac{1}{2}$
(16)	4024	0	$5\frac{1}{4}$	4301	10	$9\frac{3}{4}$	4532	16	$1\frac{1}{2}$	4671	11	$3\frac{3}{4}$
(17)	4907	5	$4\frac{3}{4}$	5230	16	$6\frac{1}{4}$	5554	7	$7\frac{3}{4}$	5877	18	$9\frac{1}{4}$
(18)	7128	7	$3\frac{1}{2}$	8028	15	$9\frac{1}{2}$	8779	2	$10\frac{1}{2}$	11030	4	$1\frac{1}{2}$

EXERCISE 28.

	£	s.	d.
(1)	103	11	6
(2)	158	16	10
(3)	2244	4	0
(4)	913	5	$11\frac{1}{2}$
(5)	16	5	0
(6)	118	12	6
(7)	60	19	0
(8)	17	4	$4\frac{1}{2}$
(9)	117	9	$11\frac{1}{2}$
(10)	7	5	$3\frac{3}{4}$
(11)	16938	5	$7\frac{1}{2}$
(12)	32401	14	10
(13)	149	0	$8\frac{1}{4}$
(14)	298627	6	8
(15)	9	3	1
(16)	93	2	$8\frac{1}{2}$
(17)	154	13	9
(18)	24	4	0
(19)	100	0	2
(20)	4	5	$10\frac{3}{4}$

EXERCISE 29.

	£	s.	d.
(1)	159	15	$1\frac{1}{5}$
(2)	21	15	$0\frac{1}{4}$
(3)	6	14	4
(4)	1	8	$5\frac{1}{2}\ \frac{4}{12}$
(5)	0	18	0
(6)	0	18	$11\frac{1}{2}$
(7)	0	18	$11\frac{3}{4}\ \frac{39}{108}$
(8)	4	16	$5\frac{3}{4}\ \frac{70}{132}$
(9)	5	10	$2\frac{1}{4}\ \frac{48}{88}$
(10)	7	12	$5\frac{1}{4}\ \frac{42}{64}$
(11)	8	6	$11\frac{9}{40}\ \frac{12}{40}$
(12)	14	3	$4\frac{1}{2}\ \frac{30}{60}$
(13)	24	17	$10\frac{1}{2}\ \frac{34}{66}$
(14)	92	7	$0\frac{1}{2}\ \frac{41}{81}$
(15)	97	17	$4\frac{1}{4}\ \frac{42}{65}$
(16)	55	13	$10\frac{1}{2}\ \frac{107}{121}$
(17)	27	9	$9\frac{1}{2}\ \frac{46}{144}$
(18)	176	7	$7\frac{1}{2}\ \frac{6}{49}$
(19)	0	6	1
(20)	1	16	1
(21)	0	15	$8\frac{1}{2} + \frac{32}{48}$
(22)	9	1	2
(23)	4	18	$6\frac{1}{2}$
(24)	166	1	$5\frac{3}{4} + \frac{15}{60}$
(25)	8	8	0
(26)	0	15	9
(27)	0	4	$11\frac{1}{2}$
(28)	1	4	$10\frac{1}{4}$
(29)	0	1	$10\frac{1}{2}$
(30)	0	7	$10\frac{1}{2}$
(31)	2	14	0
(32)	1	1	4
(33)	3	2	0
(34)	1	11	6
(35)	1	16	0
(36)	0	18	$11\frac{1}{2}$
(37)	0	18	0
(38)	0	7	6
(39)	0	1	$10\frac{3}{4}$
(40)	0	5	6
(41)	0	4	10
(42)	0	3	$9\frac{1}{2}$
(43)	1	8	5
(44)	1	16	4
(45)	2	3	2
(46)	4	16	1
(47)	2	14	8
(48)	2	18	8

EXERCISE 30.

	£	s.	d.
(1)	1	1	$7\frac{1}{4}\ \frac{11}{17}$
(2)	1	4	0
(3)	0	8	$0\frac{1}{4}$
(4)	0	11	$0\frac{1}{4}$
(5)	0	17	$8\frac{3}{4}$
(6)	4	8	$1\ \frac{6}{38}$
(7)	6	8	$9\frac{3}{4}\ \frac{5}{13}$
(8)	14	9	10
(9)	4	16	$3\frac{1}{2}$
(10)	6	12	$1\frac{1}{4}$
(11)	6	8	$5\frac{1}{4}$
(12)	10	14	3
(13)	5	7	$6\frac{3}{4}$
(14)	1	10	$0\frac{3}{4}\ \frac{11}{17}$
(15)	9	9	$5\frac{1}{2}$
(16)	11	8	$3\frac{1}{2}\ \frac{23}{45}$
(17)	3	4	$0\frac{1}{4}$
(18)	0	18	$9\frac{3}{4}$
(19)	0	17	$11\frac{1}{2}$
(20)	12	18	$11\frac{1}{2}$
(21)	0	13	$7\frac{1}{4}$

EXERCISE 31.

(1) 43
(2) 42
(3) 56
(4) 108
(5) 703
(6) 7081
(7) 103
(8) 4880
(9) 365
(10) 348

EXERCISE 32.

	£	s.	d.
(1)	2	11	6
(2)	0	11	6
(3)	1	7	$5\frac{1}{2}$
(4)	0	3	$11\frac{3}{4}$
(5)	54	3	$8\frac{1}{4} + \frac{4}{17}$
(6)	1118	6	$11\frac{3}{4}$

(7) $121\frac{1}{3}\frac{3}{4}\frac{2}{6}\frac{3}{0}\frac{5}{0}$ tons.
(8) £3, 11s. $9\frac{1}{4}$d.
(9) £1, 15s. $11\frac{1}{2}$d.
(10) £100, 11s. $0\frac{3}{4}$d. + $\frac{1}{3}$ monthly.

	£	s.	d.
(10)	23	4	$1\frac{9}{4} + \frac{3}{5}\frac{4}{3}$ weekly.
(10)	3	6	$1\frac{1}{2} + \frac{3}{5}\frac{4}{3}\frac{1}{8}$ daily.
(11)	18	14	5
(12)	0	3	$9\frac{3}{4}$

(13) 45 yards.
(14) 37 ounces.
(15) 17
(16) 648
(17) In 59 weeks.
(18) 1s. $5\frac{1}{5}\frac{1}{7}$d. per week.

EXERCISE 33.

(1) 83185 dol. 2s. 6d.
(2) 697342 yd. 3 in.
(3) 66880 francs.
(4) 6175 dollars.
(5) 19395 fl. 1s. 8d.
(6) £23989, 10s. 3d. lost.
(7) 83 lb. 1 oz. 2 dwt.
(8) 110 hhds. sugar.
(9) 240 canisters.
(10) 1424087568 pieces.
(11) 1585267200 in.
(12) 431942346 in.
(13) 66 of each.
(14) 232 qr. 2 bu. 2 pk.
(15) 88 days in 1 revolution.
(16) 137280 revolutions.
(17) 6911136 letters.
(18) 65 cwt. 3 qr. 6 lb.
(19) £3, 2s. $10\frac{1}{2}$d. a man's share.
£1, 11s. $5\frac{1}{4}$d. a woman's share.
£0, 10s. $5\frac{3}{4}$d. a child's share.
(20) 75 of each.
(21) 1 cwt. 1 qr. 17 lb.
(22) 16 lb. 1 oz. 12 dwt.
(23) 3 cwt. 1 qr. 26 lb.
(24) 24 cwt. 1 qr. 21 lb.
(25) 172 cwt.
(26) 309 cwt. 17 lb. 13 oz.
(27) 32 cwt. 3 qr. 25 lb. 12 oz.
(28) 14s. $6\frac{1}{2}$d. per yard.
(29) 4s. $4\frac{1}{2}$d. per yard.
(30) £3, 11s. 0d. gain.
(31) £6, 11s. 1d. saves.
(32) £11, 8s. $1\frac{1}{2}$d. saves.
(33) £11, 1s. 0d. saves.
£1, 7s. $4\frac{1}{2}\frac{1}{0}$ weekly exp.
(34) £574, 12s. 6d. gain.
(35) £87, 3s. 9d. loss.
(36) £59635, 6s. 0d. stock.

EXERCISE 34.

2. (1) 4 (2) 1 (3) 17 (4) $3\frac{7}{31}$ (5) $3\frac{16}{113}$
(6) $46\frac{22}{43}$ (7) 139 (8) $91\frac{23}{47}$ (9) $85\frac{18}{35}$ (10) $37\frac{1}{27}$
(11) $30\frac{70}{81}$ (12) $27\frac{29}{73}$ (13) $3\frac{177}{1250}$ (14) $45\frac{15}{133}$ (15) $15\frac{15}{199}$

3. (1) $\frac{31}{4}$ (2) $\frac{59}{5}$ (3) $\frac{97}{7}$ (4) $\frac{94}{11}$ (5) $\frac{108}{13}$
(6) $\frac{271}{17}$ (7) $\frac{4517}{50}$ (8) $\frac{5553}{70}$ (9) $\frac{839}{36}$ (10) $\frac{4948}{15}$
(11) $\frac{16649}{18}$ (12) $\frac{8009}{89}$ (13) $\frac{82604}{89}$ (14) $\frac{355}{113}$ (15) $\frac{7725}{331}$

4. 63 **5.** 101 **6.** 147 **7.** $22\frac{1}{4}$ **8.** $7\frac{5}{16}$ **9.** $365\frac{97}{400}$

10. $\frac{69}{3}$, $\frac{161}{7}$, $\frac{46}{2}$, $\frac{253}{11}$, $\frac{115}{5}$, $\frac{299}{13}$, $\frac{575}{25}$, $\frac{2875}{125}$

EXERCISE 35.

(1) 6 (2) 14 (3) 11 (4) 19 (5) 112 (6) 29 (7) 75
(8) 9 (9) 3 (10) 7 (11) 12 (12) 4 (13) 2 (14) 11
(15) 73 (16) 6

EXERCISE 36.

(1) 60 (2) 360 (3) 720 (4) 432 (5) 60 (6) 120
(7) 420 (8) 14112 (9) 5400 (10) 10080 (11) 5880 (12) 2016
(13) 6048 (14) 7560 (15) 50400 (16) 30240

EXERCISE 37.

(1) 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199
(2) $5^2 \times 7 \times 11 \times 107$ (3) $5 \times 7 \times 11 \times 19$; $5 \times 3^2 \times 11 \times 17$; 55
(4) 17 reckoning the number itself and unity
(5) 840 (6) £5, 5s. (7) 1680 in. (8) 60 gal. (9) 1683
(10) 4749 (11) 119, 239, 359, 479, 599, 719, 839, 959 (12) 18
(13) 1507 (14) 5040 (15) 660660
(17) $2^7 \times 11 \times 13$; $3^2 \times 5^2 \times 7 \times 13$; $3^2 \times 5 \times 7^2 \times 11$; 5148 or $(2^2 \times 3^2 \times 11 \times 13)$
(18) 1 hour (19) 6 minutes (20) £14, 14s.; 2s. 4d. (21) 319, 377
(22) 40 (23) 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72 (24) 3

EXERCISE 38.

(1) $\frac{2}{3}$
(2) $\frac{8}{9}$
(3) $\frac{24}{25}$
(4) $\frac{7}{9}$
(5) $\frac{2}{3}$
(6) $\frac{2}{3}$
(7) $\frac{7}{9}$
(8) $\frac{1}{2}\frac{3}{5}$
(9) $\frac{4}{7}$
(10) $\frac{7}{3}$
(11) $\frac{8}{9}$
(12) $\frac{11}{14}$
(13) $\frac{7}{8}$
(14) $\frac{5}{6}$
(15) $\frac{4}{15}$
(16) $\frac{9}{32}$
(17) $\frac{68}{341}$
(18) $\frac{41}{100}$
(19) $\frac{112}{229}$
(20) $\frac{37}{66}$
(21) $\frac{7}{187}$
(22) $\frac{37}{41}$
(23) $\frac{10}{11}$
(24) $\frac{87}{110}$
(25) $\frac{6}{8}, \frac{15}{20}, \frac{21}{28}, \frac{27}{36}, \frac{9}{12}$
(26) $\frac{8}{12}, \frac{10}{15}, \frac{14}{21}, \frac{22}{33}, \frac{32}{48}$
(27) $\frac{18}{2}, \frac{81}{9}, \frac{99}{11}, \frac{117}{13}, \frac{126}{14}$
(28) $\frac{15}{21}, \frac{25}{35}, \frac{45}{63}$
(29) $\frac{20}{30}, \frac{25}{30}, \frac{18}{30}, \frac{120}{30}$
(30) $\frac{80}{86}, \frac{28}{86}, \frac{27}{86}, \frac{72}{86}$

EXERCISE 39.

A. (1) $\frac{3}{4}, \frac{2}{3}, \frac{1}{2}$
(2) $\frac{2}{13}, \frac{1}{7}, \frac{1}{8}$
(3) $\frac{24}{25}, \frac{5}{6}, \frac{4}{5}$
(4) $\frac{11}{15}, \frac{7}{10}, \frac{3}{5}$
(5) $\frac{1}{7}, \frac{3}{22}, \frac{2}{15}$
(6) $\frac{13}{66}, \frac{2}{11}, \frac{5}{33}$
(7) $\frac{14}{15}, \frac{13}{14}, \frac{11}{12}, \frac{9}{10}$
(8) $\frac{11}{15}, \frac{2}{3}, \frac{13}{21}, \frac{17}{27}, \frac{6}{9}$
(9) $\frac{9}{19}, \frac{7}{13}, \frac{5}{11}, \frac{4}{9}, \frac{3}{7}$
(10) $\frac{17}{20}, \frac{5}{6}, \frac{49}{60}, \frac{4}{5}, \frac{11}{15}$
(11) $\frac{19}{20}, \frac{14}{15}, \frac{15}{19}, \frac{13}{17}$
(12) $\frac{19}{20}, \frac{16}{17}, \frac{15}{16}, \frac{12}{13}, \frac{9}{10}$
(13) $\frac{7}{12}, \frac{4}{7}, \frac{10}{21}, \frac{3}{7}, \frac{5}{12}, \frac{5}{14}$
(14) $\frac{5}{9}, \frac{9}{20}, \frac{4}{9}, \frac{13}{30}, \frac{19}{45}, \frac{2}{5}$
(15) $\frac{79}{152}, \frac{39}{76}, \frac{17}{38}, \frac{42}{95}, \frac{8}{19}, \frac{7}{17}$
(16) $\frac{6}{31}, \frac{7}{38}, \frac{11}{60}, \frac{2}{11}, \frac{9}{50}, \frac{3}{17}$

B. (1) $\frac{6, 8, 9,}{12}$
(2) $\frac{54, 45, 66, 28}{72}$
(3) $\frac{210, 125, 54}{440}$
(4) $\frac{48, 27, 20, 26}{72}$
(5) $\frac{24, 35, 36, 26}{56}$
(6) $\frac{1428, 765, 1547, 1680}{1785}$
(7) $\frac{20, 30, 5, 48}{80}$
(8) $\frac{15, 30, 9, 40}{45}$
(9) $\frac{300, 315, 320, 324}{360}$
(10) $\frac{2288, 7865, 1980, 780}{8580}$
(11) $\frac{264, 270, 272, 280, 279, 276}{288}$
(12) $\frac{1755, 3640, 1638, 1260}{4095}$
(13) $\frac{216, 168, 80, 165, 315, 136}{360}$
(14) $\frac{2618, 2448, 2805, 594, 1680, 21}{3366}$

EXERCISE 40.

(*a*) (1) $1\frac{1}{12}$ (2) $2\frac{19}{72}$ (3) $2\frac{21}{40}$ (4) $2\frac{52}{495}$ (5) $1\frac{117}{221}$
(6) $1\frac{7}{24}$ (7) $2\frac{43}{84}$ (8) $2\frac{23}{60}$ (9) $1\frac{19}{294}$ (10) $1\frac{139}{144}$
(11) $2\frac{1}{144}$ (12) $1\frac{688}{2205}$ (13) $1\frac{811}{972}$ (14) $2\frac{13}{264}$

(*b*) (15) $\frac{17}{56}$ (16) $\frac{46}{255}$ (17) $\frac{4}{45}$ (18) $\frac{9}{304}$ (19) $\frac{16}{117}$
(20) $\frac{16}{221}$ (21) $\frac{1}{42}$ (22) $\frac{5}{42}$ (23) $1\frac{1}{4}$ (24) $2\frac{2}{15}$
(25) $4\frac{1}{2}$ (26) $3\frac{5}{14}$ (27) $8\frac{7}{20}$ (28) $7\frac{1}{12}$ (29) $4\frac{9}{16}$
(30) $2\frac{17}{45}$

(*c*) (31) $\frac{1}{24}$ (32) $\frac{3}{4}$ (33) $\frac{15}{16}$ (34) $1\frac{13}{18}$ (35) $\frac{29}{40}$
(36) $1\frac{17}{24}$ (37) $5\frac{277}{720}$ (38) $1\frac{319}{1260}$ (39) $4\frac{91}{120}$ (40) $1\frac{3}{16}$
(41) $3\frac{97}{360}$ (42) $6\frac{39}{40}$

EXERCISE 41.

(1) $\frac{11}{16}$ (2) $\frac{7}{15}$ (3) $\frac{103}{240}$ (4) $\frac{87}{1300}$ (5) $\frac{211}{320}$
(6) $\frac{19}{20}$ (7) $\frac{2955}{4556}$ (8) $\frac{11}{16}$ (9) $\frac{1}{5}$ (10) $\frac{5}{7}$
(11) $\frac{4}{13}$ (12) $\frac{29}{75}$ (13) $\frac{91}{180}$ (14) $\frac{13}{35}$ (15) $\frac{1}{7}$
(16) $\frac{1}{8}$ (17) $\frac{3}{40}$ (18) $\frac{42506}{64117}$

EXERCISE 42.

(1) $\frac{25}{18}$, $\frac{5}{3}$, 10 (7) $\frac{5}{22}$ (13) $1\frac{47}{77}$ (19) 1
(2) $\frac{11}{25}$, $2\frac{1}{5}$, 11 (8) $\frac{2}{9}$ (14) $4\frac{3}{28}$ (20) $\frac{49}{64}$
(3) $\frac{5}{64}$, $\frac{5}{16}$, $1\frac{1}{4}$ (9) $\frac{2}{7}$ (15) $1\frac{73}{77}$ (21) $\frac{2252}{2332}$
(4) $2\frac{3}{8}$, $2\frac{5}{7}$, $3\frac{1}{6}$ (10) $\frac{3}{16}$ (16) $73\frac{4}{7}$ (22) $3\frac{1}{4}$
(5) $\frac{5}{8}$ (11) $\frac{8}{9}$ (17) $323\frac{1}{5}$ (23) $1\frac{1}{2}$
(6) $\frac{3}{11}$ (12) $\frac{14}{15}$ (18) 76 (24) $12\frac{4}{9}$

EXERCISE 43.

(1) $\frac{4}{27}$ (9) $91\frac{2}{3}$ (17) $\frac{2}{35}$
(2) $\frac{55}{2116}$ (10) $\frac{221}{264}$ (18) $\frac{1}{12}$
(3) $4\frac{23}{24}$ (11) 2 (19) $1\frac{3}{4}$d.
(4) $\frac{77}{108}$ (12) $\frac{1}{248}$ (20) £6008
(5) $4\frac{5}{7}$ (13) $9\frac{1}{2}$ (21) 3220
(6) $\frac{91}{842}$ (14) $5\frac{10}{18}$ (22) $37\frac{1}{60}$ min.
(7) $2\frac{4}{9}$ (15) $\frac{1}{88}$ (23) 1127 yards
(8) $\frac{20}{27}$ (16) $\frac{7}{240}$ (24) $7\frac{1}{4}$ miles

EXERCISE 44.

(1) $\frac{3}{28}$, $\frac{2}{28}$, $\frac{1}{28}$
(2) $\frac{3}{19}$, $\frac{2}{19}$, $\frac{1}{57}$
(3) $\frac{8}{135}$, $\frac{2}{135}$, $\frac{1}{405}$
(4) $\frac{5}{36}$, $\frac{1}{108}$, $\frac{1}{216}$
(5) $1\frac{13}{16}$
(6) $1\frac{1}{36}$
(7) $\frac{22}{23}$
(8) $4\frac{1}{2}$
(9) $1\frac{3}{5}$
(10) $\frac{13}{14}$
(11) $1\frac{7}{13}$
(12) 20
(13) $17\frac{8}{11}$
(14) $\frac{8}{9}$
(15) $\frac{4}{9}$
(16) $\frac{7}{10}$
(17) $\frac{1}{8}$
(18) $\frac{22}{189}$
(19) $\frac{13}{16}$
(20) $\frac{33}{56}$
(21) $\frac{29067}{29088}$
(22) $\frac{16790}{16880}$
(23) $1\frac{1001}{11270}$
(24) $1\frac{29}{24001}$

EXERCISE 45.

(1) $3\frac{11}{15}$
(2) 16
(3) $1190\frac{1}{4}$
(4) 240
(5) 73
(6) $13\frac{7}{11}$
(7) $1\frac{15}{34}$
(8) $1\frac{5}{28}$
(9) 850
(10) $6077\frac{1}{2}$ miles
(11) 13080 men
(12) 400 miles
(13) $\frac{2}{28}$
(14) £21000
(15) $\frac{42}{69}$
(16) $10\frac{5}{7}$ feet

EXERCISE 46.

(1) $1\frac{61}{115}$
(2) ·3
(3) $\frac{1338}{2997}$
(4) 3
(5) $\frac{1}{2}$
(6) $\frac{12}{35}$
(7) $13\frac{1}{14}$
(8) $\frac{116}{125}$

EXERCISE 47.

(1) 8s. 4d.
(2) £2, 5s.
(3) 8s. $3\frac{1}{9}$d.
(4) £3, 3s. $8\frac{3}{4}$d.
(5) $11\frac{1}{4}$d.
(6) 2s. $0\frac{2}{7}\frac{1}{6}$d.
(7) 13 cwt. 3 qr.
(8) 9 yd. $8\frac{1}{9}$ in.
(9) 2 tons 5 cwt. 3 qr. 21 lb.
(10) 16s. $10\frac{3}{5}$d.
(11) £12, 10s. $10\frac{1}{2}$d.
(12) £83, 19s. $2\frac{50}{65}$d.
(13) £5
(14) £3
(15) 6s. $3\frac{1}{4}$d.
(16) 1d.
(17) 1 ton 3 cwt. $0\frac{16}{21}$ qr.
(18) 1 cwt. 0 qr. 3 lb.
(19) $£\frac{23}{40}$
(20) $£\frac{5380}{}$
(21) $\frac{8}{9}$ yd.
(22) $\frac{27}{32}$ acres
(23) $\frac{51}{64}$ miles
(24) $\frac{41}{80}$ lb. Troy
(25) $\frac{13}{41}$
(26) $\frac{41}{71}$
(27) $\frac{14415}{29}$
(28) 3780
(29) $8\frac{8}{38}$
(30) $\frac{1}{4}$
(31) $2\frac{7}{10}$
(32) $\frac{1}{3}$
(33) $\frac{16}{57}$
(34) $1\frac{5}{27}$
(35) $\frac{21}{200}$
(36) $1\frac{7}{8}$

EXERCISE 48.

(1) $\frac{89}{120}$ (2) $\frac{27}{182}$ (3) $\frac{45}{112}$ (4) $1\frac{1297}{1439}$ (5) $\frac{18}{121}$

(6) 209 (7) 1570 (8) 31 (9) $\frac{7}{60}$ of the work

(10) £16632 (11) $\frac{7}{27}$; £1166, 13s. 4d. (12) $94\frac{2}{25}$

(13) $7\frac{1}{2}$ hr. after B starts; 200 miles from the starting-point.

(14) 240 (15) (A), $\frac{1}{16}$; (B), $\frac{1}{48}$; (C), $\frac{1}{24}$

(16) $1\frac{22}{45}$ (17) 5; $\frac{20}{23}$ (18) $\frac{1}{27}$

EXERCISE 49.

(1) ·3 ·01, ·12, 1·2, 1·2, ·246, 109·09, ·0047, 51·2, 4·002

(2) ·5, ·6, ·84, ·75, ·95, ·22, ·015

(3) $\frac{9}{10}$, $\frac{9}{100}$, $\frac{19}{100}$, $1\frac{1}{10}$, $30\frac{3}{1000}$, $10\frac{1}{100}$, $203\frac{7}{10000}$

(4) $\frac{2}{10}+\frac{1}{100}+\frac{3}{10000}$, $\frac{5}{100}+\frac{1}{1000}+\frac{6}{100000}$, $3+\frac{2}{10}+\frac{7}{100}+\frac{1}{1000}+\frac{5}{10000}$

EXERCISE 50.

A. (1) 46; 460; 4600
(2) 41·9; 419; 4190
(3) 10·5; 105; 1050
(4) 6·28; 62·8; 628
(5) 20·06; 200·6; 2006
(6) ·005; ·05; ·5
(7) 370·007; 3700·07; 37000·7
(8) 280·3205; 2803·205; 28032·05
(9) 170·5706; 1705·706; 17057·06
(10) 437·6201; 4376·201; 43762·01
(11) 300·012; 3000·12; 30001·2
(12) 830·0156; 8300·156; 83001·56
(13) 41·9; 419; 4190
(14) 10·5; 105; 1050
(15) 53·45; 534·5; 5345
(16) 20·06; 200·6; 2006
(17) 11·003; 110·03; 1100·3
(18) 53·72; 537·2; 5372
(19) 5; 50; 500
(20) 8·7; 87; 870
(21) 6·28; 62·8; 628
(22) ·4; 4; 40
(23) ·15; 1·5; 15
(24) ·05; ·5; 5

B. (1) ·46; ·046; ·0046
(2) ·419; ·0419; 00419
(3) ·105; ·0105; ·00105
(4) ·0628; ·00628; ·000628
(5) ·2006; ·02006; ·002006
(6) ·00005; 000005; ·0000005
(7) 3·70007; ·370007; ·0370007
(8) 2·803205; ·2803205; ·02803205
(9) 1·705706; ·1705706; ·01705706
(10) 4·376201; ·4376201; ·04376201
(11) 3·00012; ·300012; ·0300012
(12) 8·300156; ·8300156; ·08300156

(13) ·419; ·0419; ·00419
(14) ·105; ·0105; ·00105
(15) ·5345; ·05345; ·005345
(16) ·2006; ·02006; ·002006
(17) ·11003; ·011003; ·0011003
(18) ·5372; ·05372; ·005372
(19) ·05; ·005; ·0005
(20) ·087; ·0087; ·00087
(21) ·0628; ·00628; ·000628
(22) ·004; ·0004; ·00004
(23) ·0015; ·00015; ·000015
(24) ·0005; ·00005; ·000005

EXERCISE 51.

(1) 2·07918 (2) 97·0752 (3) 414·4726 (4) 190·5266 (5) 156·869
(6) 4154·16232 (7) 357·605 (8) ·811103 (9) 716·7948316

EXERCISE 52.

(1) ·1263
(2) ·35
(3) ·077
(4) 1·375
(5) 1·1875
(6) 3·0375
(7) 1·075
(8) ·048
(9) 2·0075
(10) 1·0991
(11) 1·74991
(12) 8·12997

EXERCISE 53.

(1) 25·4541
(2) ·95484
(3) 1·4364
(4) 8·2586
(5) 1·19706
(6) ·152066
(7) ·0254541
(8) ·000095484
(9) ·0014364
(10) ·0082586
(11) ·00000119706
(12) ·00000152066
(13) 2545410
(14) 954840
(15) 1436·4
(16) 825·86
(17) 119·706
(18) 15·2066
(19) 97·1285610258
(20) ·080779853376
(21) ·00441726642
(22) 5835·4321
(23) 59958824·4
(24) ·000000008847

EXERCISE 54.

(1) ·0023
(2) ·00000637
(3) 7·46
(4) ·000123
(5) ·000000725
(6) ·0000229
(7) 3·723
(8) ·007
(9) ·064
(10) 24000
(11) 3·1416
(12) 7·09
(13) ·23
(14) 6·37
(15) 74·6
(16) 12·3
(17) 72·5
(18) ·229
(19) 23000
(20) 63700
(21) 74600
(22) 123
(23) 72500
(24) 22900

EXERCISE 55.

A. (1) 71·443 (2) 554·6773 (3) 3·00795
(4) 9856·356 (5) 345700 (6) 1462·1
(7) 1553200 (8) 18·076 (9) 3·652
(10) ·0022 (11) ·0184 (12) ·0565

B. (1) 2·0769 (2) 2·105 (3) 6·219
(4) 127·892 (5) 86·54 (6) ·00613
(7) ·000224 (8) ·0000005485 (9) 1·563
(10) 2115·1921 (11) ·0228 (12) 129·73216

EXERCISE 56.

A. (1) ·625 (13) $\cdot 00 5\dot{3}$ (25) $\cdot \dot{8}46153\dot{8}$
(2) ·5 (14) $\cdot 6\dot{3}$ (26) $1 \cdot 0\dot{8}$
(3) ·25 (15) $\cdot 7\dot{2}$ (27) $7 \cdot \dot{3}$
(4) ·75 (16) $\cdot \dot{1}$ (28) $\cdot 3\dot{2}\dot{1}$
(5) ·875 (17) $\cdot 8\dot{1}$ (29) $\cdot 1\dot{3}\dot{5}$
(6) $\cdot \dot{3}$ (18) $\cdot 1\dot{4}2857\dot{2}$ (30) $1 \cdot 5\dot{3}7\dot{0}$
(7) $\cdot \dot{1}$ (19) $\cdot 2\dot{7}$ (31) $2 \cdot 17857142\dot{2}$
(8) $\cdot 8\dot{3}$ (20) $\cdot 0769230\dot{8}$ (32) $6 \cdot 0\dot{5}$
(9) $\cdot 1\dot{6}$ (21) $\cdot 0\dot{1}$ (33) $1 \cdot 6\dot{4}28571\dot{1}$
(10) ·5625 (22) $\cdot 20\dot{3}\dot{7}$ (34) $1 \cdot 0\dot{8}\dot{5}$
(11) ·2 (23) $\cdot \dot{6}$ (35) $6 \cdot 5\dot{8}1\dot{0}$
(12) $\cdot 01\dot{3}$ (24) $\cdot \dot{9}41176470588235\dot{2}$ (36) $2 \cdot 10\dot{7}14285\dot{7}$

B. (1) $\frac{1}{2}$ (10) $\frac{5}{32}$ (19) $\frac{5}{64}$ (28) $\frac{7}{12}$; $\frac{13}{44}$; $\frac{1}{6}$
(2) $\frac{1}{4}$ (11) $\frac{1}{32}$ (20) $\frac{249}{40000}$ (29) $\frac{1}{22}$; $\frac{1763}{2700}$; $\frac{1}{150}$
(3) $\frac{3}{4}$ (12) $\frac{3}{160}$ (21) $\frac{7}{9}$; $\frac{1}{3}$; $\frac{5}{9}$ (30) $\frac{1179}{1850}$; $\frac{338}{22725}$
(4) $\frac{1}{8}$ (13) $3\frac{6}{25}$ (22) $\frac{5}{11}$; $\frac{6}{11}$; $\frac{7}{111}$ (31) $\frac{5}{6}$; $\frac{3}{20}$; $\frac{3199}{6650}$
(5) $\frac{5}{8}$ (14) $5\frac{16}{125}$ (23) $\frac{5}{27}$; $\frac{2}{27}$; $\frac{26}{27}$ (32) $\frac{16}{2775}$; $\frac{1198}{126225}$
(6) $\frac{17}{50}$ (15) $7\frac{64}{125}$ (24) $\frac{3}{7}$; $\frac{9}{1111}$ (33) $\frac{7}{27000}$; $\frac{25}{74}$
(7) $\frac{3}{8}$ (16) $6\frac{3}{40}$ (25) $\frac{8}{13}$; $\frac{2}{21}$ (34) $\frac{889}{1100}$; $5\frac{31}{55}$
(8) $\frac{1}{200}$ (17) $\frac{21}{4000}$ (26) $\frac{4}{7}$; $\frac{27}{37}$ (35) $6\frac{493}{2100}$; $\frac{81}{260}$
(9) $\frac{39}{500}$ (18) $\frac{1}{64}$ (27) $\frac{124}{333}$; $\frac{25}{111}$ (36) $27\frac{821}{2100}$; $\frac{857}{1050}$

EXERCISE 57.

A. (1) $92 \cdot 9712\dot{2}46337$ (2) $374 \cdot 2346776549\dot{2}$
(3) $178 \cdot 725932$ (4) $\cdot 01\dot{3}1838409066$
(5) $92 \cdot 02\dot{3}11584$ (6) $30 \cdot 6855239\dot{5}8083195706$

(7) $894{\cdot}70\dot{4}2\dot{4}1787\dot{}$ (8) $860{\cdot}394\dot{1}67\dot{7}\dot{}$
(9) $5{\cdot}04\dot{8}09627$ (10) $46{\cdot}0\dot{3}70\dot{9}$
(11) $109{\cdot}93\dot{6}30175\dot{}$ (12) $83{\cdot}39\dot{0}6979\dot{7}\dot{}$
(13) $355{\cdot}6050\dot{0}8335\dot{5}\dot{}$ (14) $215{\cdot}8409\dot{2}1\dot{6}$
(15) $84{\cdot}82\dot{7}32641\dot{}$ (16) $470{\cdot}\dot{7}95571449036\dot{}$

B. (1) 24·928 (2) ·1091 (3) 3·1690
(4) 2·369 (5) ·449455 (6) 1·6174
(7) 12·605 (8) 8·245 (9) ·0486
(10) ·25655 (11) ·0539 (12) ·021
(13) ·192 (14) ·34242 (15) ·0928
(16) ·00017 (17) ·00998 (18) 1·78846

EXERCISE 58.

A. (1) $1{\cdot}1\dot{6}$
(2) $243{\cdot}74\dot{9}$
(3) $21{\cdot}14\dot{6}$
(4) $243{\cdot}9791\dot{6}$
(5) $\cdot208\dot{3}$
(6) 6·78395+
(7) $\cdot413\dot{1}$
(8) $233{\cdot}877\dot{1}$
(9) ·0415
(10) ·1442
(11) ·3
(12) 49·725
(13) $9{\cdot}2495\dot{5}3\dot{6}$
(14) $\cdot3\dot{9}36117$
(15) $\cdot204\dot{9}\dot{6}\dot{0}$
(16) 6·026008+

B. (1) $1{\cdot}291\dot{6}$
(2) $\cdot48\dot{1}$
(3) $\cdot1134\dot{2}$
(4) $11{\cdot}74\dot{6}$
(5) $10{\cdot}0\dot{4}$
(6) $3386{\cdot}\dot{6}$
(7) $15{\cdot}05\dot{3}$
(8) ·1458
(9) 52·875
(10) $26{\cdot}\dot{8}$
(11) 7·521532567+
(12) 19·8
(13) ·006544828125
(14) $20{\cdot}5\dot{7}142\dot{8}$
(15) $\cdot52\dot{1}1\dot{7}$
(16) ·83818125

EXERCISE 59.

(1) 9d.
(2) 10d., nearly
(3) 17s. 6d.
(4) 12s. 8¼d.
(5) 19s. 11¾d., almost
(6) 14s. 3½d., almost
(7) 3s. 2·4d.
(8) 6s.
(9) 12s.
(10) 11d., almost
(11) 4½d.
(12) ¾d.
(13) £2, 2s. 1½d.
(14) £51, 6s. 4½d.
(15) 1s. 9d.
(16) £33, 10s. 10d.
(17) £670, 1s.
(18) £1250, 0s. 5d.
(19) 11 lb. 3 oz. 3·2 dr.
(20) 5 cwt. 2 qr. 9 lb.
(21) 15 poles
(22) 141 tons 5 cwt.
(23) 1 yd. 2 ft. 11·28 in.
(24) 16 gal. 3 qt.
(25) £35, 16s. 1¾d.+
(26) 2s. 3·1d.
(27) £14
(28) £4, 15s. 4·6d.
(29) 3s. 0·6d.
(30) £11, 17s.
(31) £30, 19s.
(32) £1, 13s. 5¼d.
(33) 1s. 3d.
(34) £16, 2s. 6d
(35) 17 yd. 1 ft. 9 in.
(36) 1 qr. 6 lb. 11 oz.

EXERCISE 60.

(1) £·0375
(2) £·041$\dot{6}$
(3) £·875
(4) £·634375
(5) £·9989583
(6) £·714583
(7) £·1$\dot{6}$
(8) £·3
(9) £·6
(10) £·04583
(11) £·01875
(12) £·003125
(13) ·0625 cwt.
(14) ·0714285 cwt.
(15) ·007589$\frac{2}{7}$ ton
(16) ·905803$\frac{1}{2}$ ton
(17) ·9875 lb. tr.
(18) ·0569$\frac{1}{4}$ lb. tr.
(19) ·275 oz. tr.
(20) ·00439453125 cwt.
(21) ·671875 lb. av.
(22) ·3097$\frac{1}{2}$ lb. tr.
(23) ·75 yd.
(24) ·765625 ml.
(25) ·56875 ac.
(26) ·91$\dot{6}$ sh.
(27) ·8$\frac{1}{3}$ sh.
(28) ·625 sh.
(29) ·5902$\dot{7}$ gu.
(30) ·61328125 qr.
(31) 1·08$\frac{1}{3}$ pl.
(32) ·019863$\frac{1}{7}$ yr.
(33) ·29716$\dot{6}$ day
(34) ·3675 hr.
(35) ·3875 ac.
(36) ·12665289$\frac{3}{121}$ ac.
(37) ·01524334$\frac{274}{1080}$ ro.
(38) ·009589$\frac{3}{7}$ yr.
(39) 4·8$\frac{1}{4}$
(40) ·15

EXERCISE 61.

	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(1)	5	0	(2) 3	18	4½	(3) 0	2	5½	(4) 15	19	9
	2	6	2	10	9	2	6	10¾	159	17	6¼
	15	0	1	1	3¾	0	17	10	1	11	11¾
	15	6	8	17	9	8	3	6½	0	3	2¼

(5) £3·575; £·425; £·125; £·0375; £·05
(6) £1·731; £2·5125; £4·054; £9·489
(7) £·5; £·25; £·125; £·0625; £·03125; £·1; £·05; £·025; £·0125
(8) £5·883; £4·724; £1·476; £1·024
(9) £·672; £·051
(10) £512·477. The error is less than ½d.
(11) £7, 12s. 5½d.
(12) 6s. 8½d.

EXERCISE 62.

	£	s.	d.		£	s.	d.		£	s.	d.
(1) £2972·011	(7)	10172	6	5½	(13)	3481	10	3¾			
(2) £28569·928	(8)	7967	12	2¼	(14)	5791	16	10½			
(3) £775131·321	(9)	5378	19	10½	(15)	4346	11	0¼			
(4) £10605·468	(10)	232572	13	1½	(16)	352668	0	6¾			
(5) £346849·9864083	(11)	129667	19	6¾	(17)	104113	12	$6\frac{1}{2}\frac{3}{8}$			
(6) £1298, 5s. $0\frac{3}{16}$d.	(12)	184663	1	4½	(18)	40762	6	$4\frac{1}{2}\frac{1}{10}$			

£1416, 11s. $11\frac{3}{4}$d. $\frac{5}{8}$d.

EXERCISE 63.

	£	s.	d.
(1)	13	9	2
	9	15	$5\frac{9}{11}$ $\frac{39}{77}$
	8	11	$10\frac{1}{2}$ $\frac{19}{33}$
(2)	5	3	$1\frac{1}{4}$ $\frac{3}{7}\frac{9}{3}$
	5	15	$9\frac{1}{2}$ $\frac{4}{8}\frac{9}{8}$
	6	7	7
(3)	6	11	$3\frac{1}{3}$
	5	8	$6\frac{3}{4}$ $\frac{1}{5}\frac{4}{3}$
	4	4	$3\frac{9}{4}$ $\frac{3}{8}\frac{3}{7}$
(4)	23	17	$4\frac{3}{4}$
	22	6	$7\frac{9}{4}$ $\frac{5}{6}\frac{7}{5}$
	21	17	$2\frac{1}{4}$ $\frac{3}{5}\frac{9}{5}$
(5)	23	17	$4\frac{8}{4}$
	27	19	$0\frac{1}{2}$ $\frac{5}{7}\frac{1}{6}$
	27	4	$8\frac{1}{2}$ $\frac{4}{7}\frac{7}{5}$

	£	s.	d.
(6)	0	19	$0\frac{5}{4}$ $\frac{1}{1}\frac{6}{6}\frac{6}{5}$
	0	14	$9\frac{3}{4}$ $\frac{2}{3}\frac{3}{3}\frac{8}{3}$
(7)	13	9	$0\frac{9}{2}$ $\frac{1}{4}\frac{9}{7}\frac{9}{3}$
	7	8	$11\frac{3}{4}$ $\frac{1}{1}\frac{0}{6}\frac{8}{3}$
(8)	5	0	$2\frac{3}{4}$ $\frac{7}{9}\frac{9}{3}\frac{1}{4}$
	4	14	$10\frac{1}{4}$ $\frac{4}{9}\frac{5}{8}\frac{4}{7}$
(9)	11	6	$4\frac{9}{1}$ $\frac{4}{1}\frac{3}{6}\frac{3}{7}$
	6	8	$10\frac{1}{3}$ $\frac{3}{8}\frac{3}{7}\frac{6}{3}$
(10)	2	10	$8\frac{3}{4}$
	2	1	$2\frac{5}{3}$
(11)	6	2	$2\frac{3}{4}$
	1	16	$11\frac{1}{4}$
(12)	4	2	$11\frac{1}{4}$
	2	7	$5\frac{3}{4}$

(13) 7s. $8\frac{3}{4}$d. (15) $11\frac{3}{4}$d. (17) 10s.; 3s. 4d.
(14) 7s. $8\frac{3}{4}$d. (16) 19s. $4\frac{1}{2}$d. (18) 12s. $9\frac{1}{2}$d.

EXERCISE 64.

	£	s.	d.
(1)	0	5	0
	0	10	3
	0	15	0
	1	12	0
(2)	0	18	0
	0	15	6
	2	17	0
	6	17	6
(3)	0	16	0
	0	15	0
	2	2	0
	4	11	0
(4)	0	9	0
	0	18	0
	4	19	0
	9	0	0

	£	s.	d.
(5)	7	0	0
	5	5	0
	0	12	1
	2	7	6
(6)	2	18	4
	3	17	1
	11	5	0
	99	11	8
(7)	8	0	0
	7	15	0
	23	5	0
	67	5	0
(8)	3	0	0
	12	0	0
	31	0	0
	37	10	0

	£	s.	d.
(9)	3	0	0
	12	0	0
	38	0	0
	53	0	0
(10)	3	3	0
	9	19	6
	15	15	0
	18	7	6
(11)	12500	0	0
	3375	0	0
	4137	10	0
(12)	0	2	$0\frac{3}{4}$
	0	12	$4\frac{1}{2}$
	1	2	$8\frac{1}{4}$

	£	s.	d.
(13)	1	8	0
	5	2	8
	1	3	4
	6	13	0
(14)	0	2	0
	0	5	0
	0	7	4
	0	17	8
(15)	73	0	0
	109	10	0
	136	17	6
(16)	19	11	3
	39	2	6
	46	19	0
	70	8	6

	£	s.	d.
(17)	0	18	0
	0	12	0
	2	6	0
	7	10	0
(18)	0	0	9
	0	1	3
	0	2	$7\frac{1}{2}$
(19)	27	0	0
	36	0	0
	54	0	0
	72	0	0
(20)	13	10	0
	18	0	0
	21	12	0

	£	s.	d.
(21)	0	0	$2\frac{1}{4}$
	0	0	$5\frac{3}{4}$
	0	2	$3\frac{1}{2}$
	0	3	$8\frac{1}{4}$
(22)	0	2	0
	0	2	9
(23)	0	0	5
	0	1	$2\frac{3}{4}$
(24)	0	0	2
	0	0	$1\frac{1}{2}$
	0	0	$1\frac{1}{4}$

EXERCISE 65.

	£	s.	d.
(1)	149	1	3
(2)	278	16	8
(3)	631	6	8
(4)	324	0	0
(5)	220	0	$4\frac{1}{2}$
(6)	380	16	$4\frac{1}{2}$
(7)	1679	17	4
(8)	859	14	8

(9) 1482 oz. 15 dwt.
(10) 2578 oz. 2 dwt. 16 gr.
(11) 149 tons 10 cwt.
(12) 332 tons 8 cwt. 2 qr. 8 lb.
(13) 866 lb. 8 dr.
(14) 8 tons 14 cwt. 2 qr. 3 lb. 13 oz. 4 dr.
(15) 1428 yards
(16) 488 ac. 3 ro. 4 po.

	£	s.	d.
(17)	1821	12	11
(18)	1207	14	2
(19)	324	14	$1\frac{3}{4}$
(20)	379	14	$3\frac{1}{2}$
(21)	90	4	$2\frac{1}{2}$
(22)	794	16	$9\frac{3}{4}$
(23)	183	8	$7\frac{3}{4}$
(24)	2430	14	$3\frac{1}{2}$

EXERCISE 66.

	£	s.	d.
(1)	73	3	$5\frac{1}{2}$
(2)	104	16	4
(3)	25	18	9
(4)	4	5	$9\frac{1}{4}$
(5)	76	14	$0\frac{1}{4}$
(6)	101	6	7
(7)	73	14	$8\frac{1}{2}$
(8)	72	19	$9\frac{3}{4}$

	£	s.	d.
(9)	19	8	10
(10)	6	18	$9\frac{1}{4}$
(11)	38	14	$1\frac{1}{2}$
(12)	32	19	$11\frac{3}{4}$
(13)	19	19	$2\frac{3}{4}$
(14)	7	12	$5\frac{3}{4}$
(15)	76	13	9
(16)	12	3	$6\frac{3}{4}$

	£	s.	d.
(17)	15	4	$5\frac{1}{4}$
(18)	39	13	$6\frac{1}{4}$
(19)	25	8	8
(20)	38	3	$0\frac{1}{4}$
(21)	114	11	3
(22)	18	3	0
(23)	33	16	$5\frac{1}{2}$
(24)	8	18	3

EXERCISE 67.

	£	s.	d.
(1)	1442	14	0
(2)	4	11	$10\frac{1}{2}$
(3)	2721	3	$11\frac{1}{4}$
(4)	99	16	$10\frac{1}{3}$
(5)	304213	8	$10\frac{1}{2}$
(6)	42389	7	4
(7)	2919 qr. 3 bush.		

	£	s.	d.
(8)	352637	13	$3\frac{3}{4}$
(9)	40762	6	$4\frac{1}{2}$
(10)	6	11	1
(11)	3	7	$0\frac{3}{4}$
(12)	22	11	$8\frac{1}{4}$

	£	s.	d.
(13)	28	4	8
(14)	467	1	$6\frac{3}{4}$
(15)	2	1	$1\frac{1}{2}$
(16)	21	17	6
(17)	113	11	9
(18)	1361	17	4
(19)	12847	13	$1\frac{1}{2}$

(20) 53 tons 8 cwt. 3 qr. 6 lb.
(21) £355, 12s. $4\frac{3}{4}$d.
(22) 4 tons 16 cwt. 1 qr. 20 lb.
(23) £2300299, 0s. $1\frac{1}{2}$d.
(24) £157891, 8s. 9d.

EXERCISE 68.

(1) 15 ml.
(2) 320 hhd.
(3) 65 acres
(4) 4 days
(5) 154 men
(6) 12 days $4\frac{3}{5}$ hr.
(7) £19, 5s. 4d.
(8) $58\frac{1}{3}$ min.
(9) 1 hr. 57 min.
(10) £13, 19s. $1\frac{1}{2}$d.
(11) £288
(12) 5d.
(13) 119 ft. $5\frac{1}{3}$ in.
(14) 56 cwt.
(15) 111 cwt.
(16) 6 cwt. 3 qr. 19 lb.
(17) £13, 14s. $3\frac{1}{2}$d. $\frac{1}{100}$
(18) 45 men
(19) 7 hr.
(20) $3\frac{5}{8}$ hr.
(21) 8 cwt. 3 qr. $16\frac{1}{3}$ lb.
(22) 107·2 ft.
(23) $31\frac{3}{7}$ ft.
(24) $46\frac{1}{2}$ min.
(25) 12 days more
(26) 28 tailors more
(27) 66 days after
(28) 101 days
(29) 16 additional men
(30) $6\frac{1}{2}$ days
(31) 70 loaves nearly
(32) 2 pots
(33) $33\frac{1}{11}$ bush.
(34) £409, 10s.
(35) £12, 15s.
(36) 4 days; 135 sheets; 10 compositors
(37) 160 labourers; 6669 yd.; 5 days
(38) $1\frac{1}{13}$ days
(39) £896, 8s.
(40) £8, 17s. $6\frac{1}{2}$d. +
(41) 147 lb.
(42) 270·513 oz. troy
(43) $100\frac{1}{380}$ lb.
(44) 150 men additional
(45) 20 bush. additional
(46) $206\frac{6}{55}$ qr.
(47) 1350 ml.
(48) $43\frac{1}{6}$ days
(49) $26\frac{3}{4}$ ml.
(50) 5s. $11\frac{1}{2}$d.
(51) £15
(52) $21\frac{1}{3}$ days
(53) 100 ft.
(54) £2, 10s. $10\frac{1}{2}$d.
(55) 12 oz. $3\frac{27}{55}$ dr.
(56) 23 ml. 5 fur.
(57) $7\frac{1}{2}$ hr.
(58) 320 yd.
(59) 20 horses
(60) 50 men
(61) 61875 bricks
(62) 14 men
(63) 49 additional men
(64) 10 days
(65) $16\frac{7}{8}$ oz.
(66) $3\frac{1}{5}$ days
(67) 12 men
(68) 160 strokes per min.
(69) $9\frac{7}{20}$ hr.
(70) 120 inches
(71) £103, 4s.
(72) £50, 8s.

EXERCISE 69.

(1) £4; £10; £40
(2) £8; £30; £2, 10s.; £5, 5s.
(3) £3; £9; £19, 10s.; £25
(4) £10; £30; £44; £64
(5) $8\frac{1}{3}$ %; 10 %; $66\frac{2}{3}$ %; 125 %
(6) 2 %; $3\frac{1}{3}$ %; $16\frac{2}{3}$ %; 60 %
(7) 4 %; $7\frac{1}{2}$ %; $3\frac{1}{3}$ %; $1\frac{1}{4}$ %
(8) 5s.; 8s.; £1, 1s. $1\frac{1}{2}$d.; £2, 17s. 6d.
(9) 1s. $1\frac{1}{2}$d.; 2s. $9\frac{3}{4}$d.; 9s. $4\frac{1}{2}$d.; £1, 11s. 6d.
(10) 16s. 6d.; £1, 2s. 11d.; £3, 1s. $10\frac{1}{2}$d.
(11) $8\frac{1}{3}$ %
(12) 76·6066 %
(13) ·8813 %
(14) 5·42091 % increase; 4·67896 % decrease
(15) 104·6406 %
(16) 19·8497 %
(17) £550
(18) $56\frac{75}{176}$ %; $43\frac{453}{1276}$ %
(19) 16s. 8d. (20) 5 % (21) £250
(22) $12\frac{28}{31}$ % (23) 380 (24) $77\frac{1}{2}$ %
(25) $78\frac{1}{3}$ % (26) $95\frac{5}{21}$ % milk; $4\frac{16}{21}$ % water (27) £50; £500
(28) $\frac{1}{8}$ % (29) $13\frac{1}{3}$ %; $46\frac{2}{3}$ %; 40 % (30) $44\frac{4}{9}$ %; $33\frac{1}{3}$ %; $22\frac{2}{9}$ %

EXERCISE 70.

	£	s.	d.		£	s.	d.		£	s.	d.
(1)	20	15	$7\frac{3}{4}\frac{1}{8}$	(11)	0	18	$6\frac{3}{4}$	(21)	175	0	0
(2)	12	0	$1\frac{3}{4}\frac{1}{2}\frac{1}{8}$	(12)	4	2	0	(22)	13	7	4
(3)	13	10	$1\frac{1}{4}\frac{5}{8}$	(13)	26	14	$9\frac{1}{2}\frac{1}{8}$	(23)	23	11	$2\frac{3}{4}$+
(4)	1	11	$0\frac{1}{2}\frac{3}{8}$	(14)	15	10	$10\frac{1}{2}$	(24)	2680	0	0
(5)	1	11	$10\frac{3}{4}\frac{1}{8}$	(15)	86	4	$7\frac{1}{2}$	(25)	0	10	$2\frac{3}{4}$
(6)	3	10	$3\frac{1}{4}\frac{1}{8}$	(16)	33	18	$11\frac{3}{4}\frac{1}{8}$	(26)	1	19	0
(7)	3	4	$2\frac{3}{4}\frac{5}{8}$	(17)	7	16	3	(27)	30000	0	0
(8)	0	16	$10\frac{1}{4}\frac{31}{80}$	(18)	1825	0	0	(28)	5796	14	4
(9)	1	10	$2\frac{1}{4}\frac{7}{128}$	(19)	17	13	2	(29)	1512	8	3
(10)	1	6	$10\frac{3}{4}\frac{17}{128}$	(20)	$4\frac{1}{2}$ %			(30)	3125	10	$11\frac{1}{4}$

EXERCISE 71.

(1)

£	s.	d.
0	4	7
0	0	10
0	2	0
0	1	3
0	0	6
0	9	2

(2)

£	s.	d.
0	8	8
0	1	8
0	2	1
0	2	6
0	2	4
0	5	9
1	3	0

(3)

£	s.	d.
0	0	4
0	0	5
0	0	$4\frac{1}{2}$
0	1	$7\frac{1}{2}$
0	0	$7\frac{1}{2}$
0	0	3
0	3	$7\frac{1}{2}$

(4)

£	s.	d.
0	1	3
0	0	8
0	0	9
0	0	11
0	1	6
0	1	9
0	2	3
0	1	0
0	0	$10\frac{1}{2}$
0	10	$11\frac{1}{2}$

(5)

£	s.	d.
0	0	3
0	0	$7\frac{1}{2}$
0	0	11
0	2	6
0	1	7
0	1	11
0	7	$9\frac{1}{2}$

(6)

£	s.	d.
0	13	10
0	1	$4\frac{1}{2}$
0	2	9
0	1	9
0	1	$6\frac{1}{2}$
0	3	2
1	4	5

(7)

£	s.	d.
1	14	6
1	7	10
0	9	11
0	17	9
1	5	0
5	15	0

(8)

£	s.	d.
0	3	$1\frac{1}{2}$
0	1	$7\frac{1}{2}$
0	1	$11\frac{1}{2}$
0	0	6
0	1	$5\frac{1}{2}$
0	0	6
0	0	$7\frac{1}{2}$
0	9	$9\frac{1}{2}$

(9)

£	s.	d.
0	1	9
0	0	$7\frac{1}{2}$
0	1	3
0	1	$5\frac{1}{2}$
0	4	$4\frac{1}{2}$
0	1	$7\frac{1}{2}$
0	2	4
0	13	5

(10)

£	s.	d.
0	7	11
0	4	$9\frac{1}{2}$
0	6	$10\frac{1}{2}$
0	5	$3\frac{3}{4}$
0	1	6
1	6	$4\frac{3}{4}$

(11)

£	s.	d.
0	5	9
0	3	$9\frac{1}{2}$
0	2	10
0	2	3
0	1	$5\frac{1}{2}$
0	5	3
1	1	4

(12)

£	s.	d.
1	11	6
0	19	0
0	17	6
1	1	0
0	14	0
0	5	6
5	8	6

(13)

£	s.	d.
0	13	6
0	6	$10\frac{1}{2}$
0	2	0
0	0	$4\frac{1}{2}$
0	9	7
1	12	$3\frac{3}{4}$

(14)

£	s.	d.
3	15	1
2	18	0
0	9	9
0	13	6
0	11	10
2	11	1
0	7	8
9	6	11

(15)

£	s.	d.
6	15	2
0	2	4
0	8	8
4	8	10
0	2	2
0	6	10
3	7	2
0	5	2
15	16	4

(16)

£	s.	d.
0	3	$3\frac{3}{4}$
2	0	$0\frac{1}{4}$
3	12	$10\frac{1}{2}$
10	13	$1\frac{1}{2}$
16	9	4

(17)

£	s.	d.
4	3	4
0	8	0
0	4	2
0	18	8
5	14	2

(18)

£	s.	d.
0	2	1
0	15	10
7	16	3
0	4	0
8	18	2

(19)

£	s.	d.
2	17	2
0	0	$1\frac{3}{4}$
0	7	4
0	9	2
3	13	$9\frac{3}{4}$

(20)

£	s.	d.	
48	17	6	
12	0	0	
25	19	9	
86	17	3	
21	14	$3\frac{3}{4}$	Disct.
65	2	$11\frac{3}{4}$	

(21)

£	s.	d.
5	7	$9\frac{3}{4}$
0	17	5
1	6	$9\frac{3}{4}$
0	2	1
7	14	$1\frac{1}{2}$

(22)

£	s.	d.	
3	2	6	
0	10	10	
2	10	0	
6	7	6	
12	10	10	
0	12	$6\frac{1}{2}$	Disct.
11	18	$3\frac{1}{2}$	

(23)

£	s.	d.	
50	17	4	
0	15	10	
3	17	6	
6	4	8	
61	15	4	
7	14	5	Disct.
54	0	11	

(24)

£	s.	d.
7	1	0
1	8	$1\frac{3}{4}$
2	18	7
12	7	6
23	15	$2\frac{3}{4}$

(25)

£	s.	d.
2	9	6
7	2	$4\frac{1}{2}$
0	15	$5\frac{1}{4}$
50	17	4
61	4	$7\frac{3}{4}$

(26)

£	s.	d.
0	13	5
1	0	$6\frac{1}{2}$
2	3	4
1	2	11
5	0	$2\frac{1}{2}$

(27)

£	s.	d.
0	18	8
2	4	8
1	4	9
0	9	3
5	0	0
3	17	0
13	14	4

(28)

£	s.	d.
5	9	4
4	9	10
10	7	$7\frac{1}{2}$
9	11	0
1	18	3
20	13	4
52	9	$4\frac{1}{2}$

(29)

£	s.	d.
2	2	9
1	3	11
5	10	8
3	8	0
2	0	6
2	7	3
16	13	1

(30)

£	s.	d.
61	8	4
79	13	0
8	7	$0\frac{3}{4}$
37	5	$8\frac{1}{2}$
186	14	$1\frac{1}{4}$

(31)

£	s.	d.
8	18	9
3	15	10
2	19	4
1	16	9
1	16	8
19	7	4

(32)

£	s.	d.	
481	19	0	
945	0	0	
266	5	0	
904	1	0	
332	10	0	
820	0	0	
3749	15	0	
1249	18	4	Disct.
2499	16	8	

(33)

£	s.	d.	
54	7	6	
33	6	8	
17	14	2	
11	17	6	
26	5	0	
14	3	4	
25	16	8	
4	11	8	
188	2	6	
15	13	$6\frac{1}{2}$	Disct.
172	8	$11\frac{1}{2}$	

(34)

£	s.	d.	
7	19	6	
231	0	0	
9	15	10	
16	16	7	
8	7	8½	
4	14	9½	
278	14	5	
69	13	7¼	Disct.
209	0	9¾	

(35)

£	s.	d.	
37	0	9	
21	12	3	
37	8	4	
25	18	4	
121	19	8	
15	4	11½	Disct.
106	14	8½	

(36)

£	s.	d.
1026	17	6
463	2	11
630	17	9¾
288	4	8¼
2409	2	11

(37)

£	s.	d.	
273	8	9	
362	18	2	
613	8	11½	
1180	11	1½	
2430	7	0	
607	11	9	Disct.
1822	15	3	

(38)

£	s.	d.
345	13	2¾
200	4	7½
163	14	10½
369	6	1
1078	18	9¾

(39)

£	s.	d.
82	2	6
203	7	11
20	19	4½
306	9	9½

(40)

£	s.	d.
36	11	3
35	15	0
32	15	6
27	6	6¾
107	7	9
141	15	0
200	5	0
78	10	10
660	6	10¾

EXERCISE 72.

	£	s.	d.	
(1)	0	18	0	
(2)	135	0	0	
(3)	1	4	6	
(4)	0	12	0	
(5)	74	2	9	
(6)	31	0	0	nearly
(7)	333	12	2	
(8)	16	0	7$\frac{3}{5}$	
(9)	143	6	6	
(10)	1600	0	0	
(11)	8000	0	0	
(12)	5200	0	0	
(13)	2514	5	8½	$\frac{3}{7}$
(14)	14	10	11	
(15)	2360	10	2	
(16)	5253	6	8	
(17)	5460	0	0	
(18)	0	9	4½	
(19)	97	10	0	
(20)	118	7	6	
(21)	0	8	2½	
(22)	4	6	10½	$\frac{3}{5}$
(23)	452	17	3	
(24)	620	0	0	

EXERCISE 73.

(1) £2322, 15s.
(2) £57699
(3) 1s. 10d.
(4) £226, 18s. 8½d.
(5) £3072, 19s. 6½d.
(13) £40, 18s. 1½d.
(14) £6, 1s. 10½d.
(15) £3, 4s. 9¾d.
(16) Debts £7296 ; Assets £4864
(17) £1798, 9s. 8½d.

(6) 7¼d.
(7) 12s. 11d.
(8) £74, 6s. 1½d.
(9) (1) 1s. 8d.
(2) £137, 10s.
(10) £13500
(11) £1046, 10s.
(12) £631, 10s.
(18) $10\frac{23}{100}$d. ; £438, 4s. 2d.
(19) £2928, 4s.
(20) £1875
(21) £140
(22) 2·06 per cent.
(23) £105, 16s.
(24) £380

EXERCISE 74.

	£	s.	d.
(1)	33	17	8
(2)	21	7	6
(3)	0	5	$5\frac{3}{17}$
(4)	0	4	$5\frac{1}{4}\ \frac{18}{67}$
(5)	0	2	$1\frac{1}{4}\ \frac{3}{112}$

(6) 582 yd. 1⅓ qr.
(7) 25 cwt. 1 qr. $26\frac{4}{61}$ lb.
(8) 242 cwt. 3 qr. 12 lb.
(9) £4, 18s. 4d.
(10) $3\frac{3}{4}$d. $\frac{1}{27}\frac{3}{8}$
(11) £24s, 9s. 4d. gain
(12) £2, 5s. 6d. gain
(13) 7s. gain per cwt.
(14) £33, 7s. 4d. loss
(15) 5s. 6d. gain
(16) £12, 19s. 6d. gain
(17) 2s. 0½d. gain
(18) 14s. 8¼d. gain
(19) 5s. 4d. selling price per yd
(20) 6s. selling price per yd.
(21) £101, 1s. 6d.
(22) £3, 7s. 8d. per cwt
(23) £14, 5s. 5d.
(24) 6¼d. selling price per lb.

EXERCISE 75.

(1) 13 %
(2) 15 %
(3) 25 %
(4) 20 %
(5) £39
(6) £17, 2s.
(7) £1, 4s.
(8) £24
(9) $11\frac{1}{9}$ %
(10) $16\frac{2}{3}$ %
(11) $9\frac{1}{11}$ %
(12) 10 %
(13) $26\frac{6}{15}$ %
(14) $13\frac{7}{11}$ %
(15) $56\frac{1}{2}\frac{2}{5}$ % or £11, 15s. 1d.
(16) $20\frac{8}{6}$ %
(17) $19\frac{2}{6}\frac{7}{7}$ %
(18) 4s.
(19) 13s. 6d.
(20) 12s.
(21) 4s. 9d.
(22) 9d. per lb.
(23) 4s. 10d. per lb.
(24) 18s. 9d.
(25) £11, 17s. 6d.
(26) £4725
(27) 6s. 3d. per yd.
(28) 5s. 6d. per lb.
(29) $5\frac{3}{4}$d. per lb.
(30) 4s. per yd.
(31) 9s. 6d. per yd.
(32) $\frac{4}{7}$ % loss
(33) £896
(34) £872, 10s. and £837, 12s
(35) £5, 4s. $8\frac{8}{57}$d. ; bought £5, 7s. $9\frac{3}{4}$d. $\frac{17}{57}$; sold
(36) 2350 yd.

EXERCISE 76.

(1) 45
(2) $\frac{5}{80}$
(3) 1·025
(4) 146¼
(5) 64·4 ft.
(6) 691
(7) 3s. 4½d.
(8) 1⅞d.
(9) 4s. 4$\frac{4}{7}$d.
(10) £2, 6s. 8$\frac{2}{3}$d.
(11) £4, 0s. 8$\frac{4}{15}$d.
(12) £14
(13) £5
(14) 184
(15) $\frac{37}{42}$
(16) 1s. 0¾d.
(17) 3s. 2d.
(18) 1s. 9d.
(19) 1 at 2½d.; 2 at 4d.
(20) 4 at 2s.; 3 at 3s.; 3 at 3s. 6d.
(21) 7 at 3s. 4d.; 3 at 3s. 9d.; 2 at 4s. 2d.; 7 at 4s. 6d.
or, 3 at ,, ; 7 at ,, ; 7 at ,, ; 2 at ,, &c.
(22) 3 at 12s.; 7 at 15s.; 1 at 18s.; 4 at 19s.; 1 at 21s.
or, 5 at ,, ; 5 at ,, ; 1 at ,, ; 1 at ,, ; 4 at ,,
or, 7 at ,, ; 3 at ,, ; 4 at ,, ; 1 at ,, ; 4 at ,,
or, 8 at ,, ; 2 at ,, ; 1 at ,, ; 4 at ,, ; 4 at ,,
or, 5 at ,, ; 5 at ,, ; 4 at ,, ; 4 at ,, ; 1 at ,,
or, 2 at ,, ; 8 at ,, ; 4 at ,, ; 1 at ,, ; 1 at ,, &c.
(23) 5 gallons to 1
(24) 2; 2; 1; 1
(25) 5 lb. of each
(26) 26 at 42s.; 13 at 56s.; 13 at 60s.
(27) 9 at 7d.; 2½ at 11d.; 2½ at 10d.
(28) 9 gal. water
(29) 15$\frac{3}{4}$ gal. water
(30) 7 at 15s. 3d.; 45½ at 16s. 4d.; 73½ at 17s. 2d.; 28 at 18s. 1d.
45½ at ,, ; 7 at ,, ; 28 at ,, ; 73½ at ,, &c.
(31) 108 oz. gold; 36 oz. copper
(32) 137$\frac{1}{7}$ oz. gold; 12$\frac{5}{7}$ oz. copper
(33) 12 cwt. of each of the first 3 with 37 cwt. of the fourth
(34) 2, 2, and 9; 4, 2, and 15
(35) 60; 160; 220; 120; or, 110; 110; 170; 170, &c.
(36) 20s. 6d.
(37) 11 lb. coffee to 6 of chicory
(38) 5 gal. at 15s. to 1 gal. at 21s.
(39) 16 lb. at 1s. 5d.; 16 at 1s. 6d.; 5 at 3s.
(40) 10, 4, 2, 3; 4, 10, 3, 2; 14, 14, 5, 5.

EXERCISE 77.

(1) 14
21
49

(2) 264
288
312
336

(3) 3
9
15
21

(4) 2$\frac{1}{5}$
22$\frac{2}{5}$
75

(5) 180
120
90

(6) 726
363
242

(7) £1, 12s.
£2, 2s. 8d.
£3, 4s.

(8) £14, 17s. $2\frac{1}{2}$d., A
£14, 17s. $2\frac{1}{2}$d., B
£14, 17s. $2\frac{1}{2}$d., C
£14, 17s. $2\frac{1}{2}$d., D

(9) £106
£265
£318

(10) £144, A
£73, 12s., B
£60, 16s., C
£35, 4s., D

(11) £315
£577, 10s.
£367, 10s.

£29, 14s. $4\frac{8}{2}$d., E
£39, 12s. $6\frac{1}{4}$d., F
£9, 18s. $1\frac{1}{2}$d., G
£19, 16s. $3\frac{1}{4}$d., H

(12) £2, 12s. 6d., G
13s. $1\frac{1}{2}$d., J
£1, 19s. $4\frac{1}{2}$d., H

(13)
cwt
2·794, Water
3·142, Organic Matter
4·326, Sol. Phosphates
2·286, Insol. Phosphates
3·166, Sulphate of Lime
3·126, Sulphuric Acid
·220, Alkaline Salts
·940, Silica

(14)
lb.
$\frac{37}{180}$, H
$9\frac{7}{60}$, O
$9\frac{7}{60}$, S

(15) £270, A
£210, B
£90, C

(16) £9, 9s. $5\frac{3}{4}$d., A
£7, 2s. $1\frac{1}{4}$d., B
£28, 8s. $5\frac{1}{4}$d., C

(17) £375, D
£150, E
£100, F

(18) £540, A
£300, B

(19) £243, A
£198, B
£108, C

(20) £113, 5s. $9\frac{1}{2}$d. $\frac{3}{61}$, A
£88, 19s. $1\frac{1}{2}$d. $\frac{3}{61}$, B
£162, 15s. $0\frac{3}{4}$d. $\frac{5}{61}$, C

(21) 65 days, A
60 days, on 15th July, B
36 days, on 13th Sept., C

(22) 3 men each 11s. 8d., £1, 15s.
4 boys ,, 8s., £1, 12s.

(23) £581, 5s., A
£318, 15s., B
£465 C

(24) £464, 0s. $9\frac{1}{2}$d.
£407, 10s. 8d.
£349, 6s. $3\frac{1}{2}$d.

EXERCISE 78.

	£	s.	d.
(1)	20	0	0
(2)	32	10	0
(3)	70	0	0
(4)	104	0	0
(5)	54	15	0
(6)	13	0	0
(7)	3	0	0
(8)	24	5	0
(9)	25	0	0
(10)	55	0	0
(11)	13	10	0
(12)	144	0	0
(13)	67	10	0
(14)	31	10	0
(15)	35	0	0
(16)	56	0	0
(17)	81	0	0
(18)	33	12	0
(19)	52	0	0
(20)	180	0	0
(21)	70	0	0
(22)	200	0	0
(23)	320	0	0
(24)	30	0	0
(25)	420	0	0
(26)	1	13	4
(27)	2	8	0
(28)	2	10	0
(29)	1	16	0
(30)	3	6	0
(31)	1	16	0
(32)	22	10	3
(33)	9	9	8
(34)	86	7	0
(35)	17	17	3
(36)	6	0	8

EXERCISE 79.

	£	s.	d.
(1)	9	12	0
(2)	25	18	$2\frac{3}{4}$
(3)	28	1	$9\frac{3}{4}$
(4)	32	16	11
(5)	262	10	0
(6)	32	8	8
(7)	80	5	9
(8)	39	10	$6\frac{1}{2}$
(9)	63	6	$0\frac{1}{2}$
(10)	157	9	9
(11)	5	9	4
(12)	19	4	10
(13)	8	5	$5\frac{1}{4}$
(14)	49	6	$3\frac{3}{4}$
(15)	92	2	$8\frac{1}{2}$
(16)	109	7	$1\frac{1}{4}$
(17)	30	3	$9\frac{1}{2}$
(18)	307	18	0
(19)	879	19	$2\frac{1}{2}$
(20)	1030	8	$11\frac{1}{2}$
(21)	8	9	$11\frac{1}{2}$
(22)	4	18	$9\frac{3}{4}$
(23)	1	3	$11\frac{3}{4}$
(24)	3	0	$3\frac{3}{4}$
(25)	408	15	0
(26)	1699	2	9
(27)	644	2	0
(28)	378	0	0
(29)	4349	6	8
(30)	507	0	0
(31)	1600	0	0
(32)	800	0	0
(33)	2664	10	0
(34)	800	0	0
(35)	851	13	6
(36)	705	8	4

(37) $2\frac{1}{2}$ %.

(38) 5 %.

(39) 4 years

(40) 160 days

EXERCISE 80.

	£	s.	d.
(1)	3	17	4
(2)	65	4	1
(3)	18	17	7
(4)	19354	16	0
(5)	7	0	0
(6)	22	1	1
(7)	1268	15	0
(8)	39	0	3
(9)	6	1	$10\frac{1}{2}$
(10)	10	2	6
(11)	23	14	0

(12) $\frac{11}{18}$ %.

EXERCISE 81.

	£	s.	d.
(1)	5	0	0
(2)	3	0	0
(3)	8	18	$10\frac{5}{8}\frac{6}{7}$
(4)	7	2	$5\frac{9}{4}\frac{6}{7}$
(5)	9	8	$0\frac{1}{4}\frac{6}{7}$
(6)	7	9	$5\frac{1}{2}\frac{9}{1}\frac{0}{8}\frac{2}{3}$
(7)	104	19	$8\frac{1}{2}\frac{6}{7}$
(8)	7	1	$0\frac{1}{2}\frac{8}{1}\frac{0}{2}\frac{3}{5}$
(9)	1	14	$3\frac{1}{4}\frac{1}{1}\frac{3}{8}\frac{9}{5}$
(10)	13	8	$2\frac{3}{7}\frac{3}{8}$
(11)	6	0	$11\frac{1}{2}\frac{2}{7}\frac{4}{5}$
(12)	34	2	$1\frac{9}{4}\frac{2}{6}\frac{4}{1}\frac{1}{2}\frac{8}{5}$
(13)	2	15	$10\frac{1}{2}\frac{5}{7}\frac{4}{8}$
(14)	0	8	$1\frac{1}{2}\frac{4}{7}\frac{3}{8}$
(15)	15	18	$2\frac{1}{2}\frac{3}{7}\frac{8}{8}$
(16)	3	1	$1\frac{3}{4}\frac{1}{7}\frac{7}{1}$

EXERCISE 82.

	£	s.	d.
(1)	463	1	0
(2)	720	6	6
(3)	973	6	5
(4)	975	3	10
	976	2	$11\frac{1}{2}$
(5)	97	0	6
(6)	154	15	$3\frac{1}{2}$
(7)	91	18	$11\frac{1}{4}$
(8)	51	13	6
(9)	663	2	1
(10)	128	18	$5\frac{1}{4}$
(11)	236	8	$11\frac{1}{2}$
(12)	252	7	7
(13)	1003	6	$6\frac{1}{4}$
(14)	811	6	$1\frac{3}{4}$
(15)	853	9	$9\frac{3}{4}$
(16)	789	0	$5\frac{3}{4}$
(17)	154	18	$10\frac{1}{4}+$
(18)	323	3	$8\frac{3}{4}$
(19)	12	7	11
(20)	381	13	$2\frac{1}{4}$

EXERCISE 83.

(1) £636·54 Amount
(2) £347·2875 ,,
(3) £900·4070 ,,
(4) £467·9434 ,,
(5) £772·6690 ,,
(6) £990·5304 ,,
(7) £809·1761 ,,
(8) £548·1080 ,,
(9) £294·3657 ,,
(10) £39·0130 ,,
(11) £754·2721 ,,
(12) £364·6543 ,,
(13) £800·0150 ,,
(14) £770·7483 ,,
(15) £166·9963 Comp. Interest
(16) £3·8591 Comp. Interest
(17) £4·6246 Difference
(18) £·1812 ,,
(19) £814·4473 Amount
(20) £3693·6386 ,,
(21) £832·1006 Present Value
(22) £575·8917 ,,
(23) £1066·1845 ,,
(24) £1201·4914 ,,
(25) £95·2113 ,,
(26) £294·4729 ,,
(27) £88·7449 ,,
(28) £200 ,,
(29) £1270 Original Capital
(30) £300 paid by A

EXERCISE 84.

	£	s.	d.			£	s.	d.	
(1)	2	8	0	Common Discount	(12)	2	10	$0\frac{1}{2}$	C.D.
	2	7	$7\frac{1}{2}$	True ,,		2	9	$6\frac{1}{2}$	T.D.
(2)	4	16	0	C.D.	(13)	71	8	$6\frac{3}{4}$	
	4	15	$2\frac{3}{4}$	T.D.	(14)	84	15	$2\frac{3}{4}$	
(3)	1	7	$10\frac{1}{2}$	C.D.	(15)	4	6	8	
	1	7	$8\frac{3}{4}$	T.D.	(16)	1	10	0	
(4)	1	4	$10\frac{1}{4}$	C.D.	(17)	2	0	0	
	1	4	$9\frac{1}{4}$	T.D.	(18)	30	8	5	
(5)	2	15	9	C.D.	(19)	365	0	0	
	2	15	$2\frac{1}{4}$	T.D.	(20)	0	5	$8\frac{1}{4}$	
(6)	2	9	$8\frac{1}{2}$	C.D.	(21)	12	10	0	
	2	9	$6\frac{1}{2}$	T.D.	(22)	25 °/₀			
(7)	0	13	$11\frac{1}{4}$	C.D.	(23)	$2\frac{1}{2}$ years			
	0	13	$10\frac{1}{2}$	T.D.		£	s.	d.	
(8)	1	18	$1\frac{1}{2}$	C.D.	(24)	190	1	0	
	1	17	$10\frac{3}{4}$	T.D.	(25)	5	2	$2\frac{1}{4}$	
(9)	5	0	1	C.D.	(26)	475	18	$6\frac{1}{4}$	
	4	19	$0\frac{1}{2}$	T.D.	(27)	1	12	$2\frac{4}{7}$	
(10)	0	19	$0\frac{3}{4}$	C.D.	(28)	342	3	$2\frac{1}{3}$	
	0	18	$11\frac{1}{4}$	T.D.	(29)	6	10	1	
(11)	0	6	$11\frac{1}{3}$	C.D.	(30)	1239	15	$4\frac{4}{7}$	
	0	6	$11\frac{1}{4}$	T.D.					

EXERCISE 85.

(1) 42 days	(7) 14 days	(13) $283\frac{46}{147}$ days
(2) 34 ,,	(8) 11 ,,	(14) $188\frac{2}{3}$,,
(3) 26 ,,	(9) 46 ,,	(15) $133\frac{99}{105}$,,
(4) $16\frac{2}{3}$,,	(10) $5\frac{1}{2}$ months	(16) 22nd March
(5) 7 months	(11) $77\frac{7}{73}$ days	(17) 21st April
(6) $3\frac{1}{2}$,,	(12) $238\frac{1}{17}$,,	(18) 13th ,,

EXERCISE 86.

(1) £1695, 15s., Annual Revenue
(2) £326, 2s. $10\frac{3}{4}$d. $\frac{1}{8}$, Anl. Income
(3) £761, Value
(4) £396, Value
(5) £264, 7s. 6d., Buying Price
(6) £593, 18s. 9d., Buying Price
(7) £288, 6s. 9d., Selling Price
(8) £538, 13s., Selling Price
(9) £800, Stock
(10) £600, Stock
(11) £800, Stock
(12) £533, 6s. 8d., Stock
(13) £600, Stock
(14) £800, Stock
(15) $3\frac{137}{161}$ °/₀
(16) $3\frac{223}{415}$ °/₀

(17) At 75
(18) At $92\frac{5}{7}$
(19) £26, 5s., Income
(20) £12, 4s. $1\frac{3}{4}$d. $\frac{1}{4}$, Income
(21) £1123, 6s. 8d., invested
(22) £13320, invested
(23) £64, 14s. $11\frac{3}{4}$d. $\frac{3}{7}\frac{3}{7}\frac{3}{9}$, Income
(24) £13, 10s., Gain
(25) At $103\frac{5}{8}$
(26) $\frac{5}{21}$ %, Difference
(27) £15, Gain
(28) £30, Loss
(29) £1573, 6s. 8d.
(30) £8310, 12s.
(31) £1195
(32) £823, 14s. 6d.
(33) £43, 13s. 4d.
(34) India 5 per cents., which yield £4, 15s. 11d. %
(35) £60, 0s. 6d.
(36) £441, 4s. $10\frac{1}{2}$d.
(37) 125 %
(38) 243 %
(39) £13, Gain
(40) £23, 3s. 4d., yearly loss
(41) £10815
(42) $88\frac{5}{8}$ %
(43) £3373, 13s.
(44) £$205\frac{4}{5}\frac{5}{6}\frac{5}{1}$
(45) £389, Income
(46) £3, 16s. $2\frac{1}{4}$d. %
(47) $109\frac{1}{4}$ %
(48) £21, 13s. $3\frac{3}{4}$d., yearly gain

EXERCISE 87.

(1) 9·2
(2) 7·24
(3) 4·07
(4) 2·17
(5) 9·345
(6) 9·345
(7) 9·345
(8) 9·345
(9) 13·024
(10) 9·077
(11) 5·216
(12) 8·019
(13) 23·7
(14) ·346732
(15) ·003007
(16) 2·98
(17) 9·003027
(18) 29·023008
(19) 3007·092
(20) 1700·9
(21) ·0083
(22) 3·02
(23) ·1709
(24) 2·4673
(25) 19·8
(26) ·034
(27) 3·405
(28) ·01706
(29) 21·637
(30) 2·467
(31) 4·309
(32) 6·00043
(33) ·024983
(34) ·036728
(35) 9·16
(36) ·003000007
(37) ·004873
(38) ·170004
(39) 52·984
(40) 14·085006

EXERCISE 88.

(1) 54·246 m.
(2) 5·261 m.
(3) 93·452 Hm.
(4) 774·16 a.
(5) 513·21 Hl.
(6) 104·377 Hg.
(7) 4523·79 Kg.
(8) 33·0518 sq. m.
(9) 874026·129 c.cm.
(10) 5·34 a.
(11) 8·8 s.
(12) 225·2074 Kg.
(13) 1353·999424 Kg.
(14) 17762·309 m.
(15) 3324·6 sq. Dm.
(16) 3·968853 c. m.
(17) 24·11 m.
(18) 1417 mm.
(19) 15·456 Km.
(20) 84·096 g.
(21) 220·7797 Ha.
(22) 335·57 l.
(23) 1323·42 c.cm.
(24) £·35425 = 3 fl. 54$\frac{1}{2}$ mils.

EXERCISE 89.

(1)	(2)	(3)	(4)
512·752 Km.	848·9133 Ha.	3650·280342 g.	798·483 l.
(5)	(6)	(7)	(8)
19304·8423 sq.cm.	28507 c.cm.	120·387 m.	970·1 g.
(9)	(10)	(11)	(12)
171·909 Hl.	8·8047 a.	14277·824 sq. Km.	3·753729 c.m.
(13)	(14)	(15)	(16)
£13 6fl. 5 cents	50·169 Kg.	25 florins	915 g.
(17)	(18)	(19)	(20)
12·570 g.	£2726 6 fl. 7 cents	£4 13·95 cents	31 fl. 13 mils.
(21)	(22)	(23)	(24)
one part = 834 m.	39	24	224

EXERCISE 90.

(1) 650×10^2 chains
(2) 245·8 Km.
(3) 3930 ft.
(4) 184·94 m.
(5) 6·394 ft.
(6) 491 cm.
(7) 22·12 gr.
(8) ·002851
(9) 1085 miles per hour
(10) 1·03 Kg. per sq. cm.
(11) 24·71 fl. per Ha.
(12) ·516 cents per litre

EXERCISE 91.

(1) £149·889
(2) £1·369
(3) £1·225
(4) £1·775
(5) £135·375
(6) £5·675
(7) 125 ounces
(8) £388·875
(9) 202·5 yards
(10) £2·520
(11) £9209·600
(12) 45 men
(13) $3\frac{1}{3}$ years
(14) $\frac{7}{12}$
(15) ·512
(16) £252·350
(17) £90·4995
(18) £6063·242
(19) £32·760
(20) £568·575
(21) £172·500
(22) £9·424
(23) £185
(24) 4 per cent.
(25) 512 days
(26) £12·3375
(27) £878·559
(28) £7541·622
(29) £6658·050
(30) £·192
(31) £323·0400; £262·9440; £381·9456; £390·6624; £416·3520
(32) £36·975
(33) £659·2502
(34) £485·625
(35) £4·443
(36) £692·581
(37) 729 of each
(38) £525·624; £479·865
(39) 9 per cent.
(40) 4 per cent.

EXERCISE 92.

(1) 650 dollars
(2) 21 dollars
(3) £9270, 16s. 8d.
(4) 3198 rupees
(5) £1755
(6) $4105\frac{5}{9}$ dollars
(7) £706, 5s.
(8) £205
(9) $3594·06
(10) $37444
(11) 13253 fr. $74\frac{1}{2}$ c.
(12) 24791 fr. $89\frac{3}{4}$ c.
(13) £400
(14) £1047, 10s.
(15) 17751 fr. $11\frac{1}{4}$ c.
(16) £222
(17) 6138 fr.
(18) 10288 lire 70 c.
(19) £25
(20) £1250
(21) 36029 lire 70 c.
(22) £200
(23) 6800 lire
(24) 14s. lost
(25) £210
(26) £333, 6s. 8d.
(27) 30960 marks
(28) £2, 10s.
(29) 300 marks
(30) 8035 kr. 20 öre
(31) £444, 5s. 1d.
(32) £400
(33) 19329 kr. 20 öre
(34) £998, 0s. $5\frac{2}{3}\frac{5}{1}$d.
(35) 18094 fl. $82\frac{3}{4}$ c.
(36) £782
(37) 363 fl.
(38) 8488 fl. 80 kr.
(39) £1360
(40) £400
(41) 4590 fl.
(42) 5553 roubles 44 k.
(43) £187, 11s. $6\frac{3}{4}$d.
(44) £572, 18s. 9d.
(45) £13, 10s. 10d.
(46) 208 roubles
(47) £233, 6s. 8d.
(48) 9600 milreis
(49) 3780 milreis
(50) £4, 3s. 4d.
(51) 12236 piastres 10 par.
(52) £2 = 56 dr. 30 lp.
(53) 2030 fr.
(54) $1920
(55) 620 kr.
(56) £75, 15s

EXERCISE 93.

(1) 3·97
(2) ·425
(3) 53·75
(4) ·036
(5) 12·345
(6) ·0703
(7) 2·645751
(8) 46·904157
(9) ·158113
(10) ·27
(11) 3·316624
(12) 6·708203
(13) 4·083503
(14) 5·361902
(15) 6·586704
(16) 1·778279
(17) 2·646318
(18) ·001264
(19) ·088543
(20) ·028
(21) ·207
(22) ·284604
(23) $\frac{8}{13}$
(24) $\frac{1}{19}$
(25) ·763762
(26) ·866025
(27) ·707106
(28) $\frac{7}{12}$ or ·583̇
(29) $\frac{3}{5}$ or ·6
(30) $3\frac{3}{17}$

EXERCISE 94.

(1) 6·3
(2) 5·6
(3) 44·4
(4) ·041
(5) ·074
(6) ·099
(7) 12·86
(8) 28·93
(9) ·06463 +
(10) ·04308 +
(11) ·$\dot{4}$
(12) ·714285
(13) ·83947 +
(14) ·75
(15) ·82624 +
(16) 18 in.
(17) 6·52070 + in.
(18) 3·94814 + in.

EXERCISE 95.

(1) 1444 sq. in.
(2) 144 sq. yd.
(3) 390$\frac{1}{8}$ sq. ft.
(4) 324
(5) 5542 sq. yd.
(6) 124 ac. 1 ro. 1 po.
(7) 288
(8) 220
(9) 6 ft. 2$\frac{1}{4}\frac{3}{2}$ in.
(10) £5, 16s. 0$\frac{3}{4}$d.
(11) 2062$\frac{3}{4}$; £48, 6s. 10$\frac{1}{2}$d.
(12) 440 yd.
(13) £71
(14) 3 ac. 1 ro. 20 po.
(15) 780$\frac{1}{2}\frac{5}{3}\frac{1}{3}$ yd.
(16) 24000
(17) £13, 10s.
(18) 26048 sq. yd.
(19) 220 yd.
(20) 349·415 miles
(21) 648 yd.
(22) 600 yd.
(23) 73 po.
(24) 466·56 times.
(25) £21, 15s.
(26) 169·7056 ft.
(27) 120 yd.
(28) 5 yd. 2 ft. 4 in.
(29) £33, 16s. 8d.
(30) 60 sq. yd.
(31) 537 yd. or 2 fur. 17 po. 3$\frac{1}{2}$ yd.
(32) 2 fur. 25 po.
(33) 25$\frac{1}{2}$ sq. yd.
(34) 61$\frac{1}{2}$ yd.; 5s. 1$\frac{1}{2}$d.
(35) 36$\frac{5}{7}$ ft.
(36) £53, 13s.
(37) 185$\frac{7}{8}$ sq. yd.
(38) 201 sq. yd.; £8, 15s. 10$\frac{1}{2}$d.
(39) 7s. 4d.
(40) 5877$\frac{1}{3}$ sq. yd.

EXERCISE 96.

(1) 107$\frac{1}{2}$ sq. yd.
(2) £241, 10s.
(3) 460 sq. ft.
(4) 196$\frac{1}{6}\frac{1}{4}$ sq. yd.
(5) 589 sq. ft. 20 sq. in.; 191 sq. ft. 24 sq. in.
(6) 643 sq. ft. 136 sq. in.
(7) 88 sq. ft. 92 sq. in.
(8) £3, 16s. 6d.
(9) 6s. 5$\frac{3}{4}$d.
(10) 73$\frac{6}{2}\frac{7}{1}\frac{7}{6}$ sq. yd.
(11) 63$\frac{8}{1}\frac{}{4}$ copies
(12) £5, 8s. 6d.
(13) 731$\frac{1}{2}$ sq. ft.
(14) 32$\frac{1}{3}$ sq. ft.
(15) 103$\frac{1}{4}$ yd.

(16) 12 sq. yd. $7\frac{3}{4}$ sq. ft.; 10s. $8\frac{1}{2}$d.
(17) 73 sq. yd. $0\frac{2}{3}$ sq. ft.
(18) $40\frac{1}{5}$ yd.; £6, 15s. $1\frac{1}{2}$d.
(19) £1, 2s. $11\frac{1}{2}$d.
(20) £2, 6s. 9d.
(21) £9, 5s. $9\frac{1}{2}$d.
(22) £6, 18s. 4d.
(23) 8 pieces; 12s.
(24) £11, 8s. 8d.

EXERCISE 97.

(1) $\frac{1\,3\,3\,1}{6\,8\,5\,9}$ cub in.
(2) 64
(3) 162 sq. in.; $121\frac{1}{2}$ cub. in.
(4) $28444\frac{4}{5}$
(5) 18 tons 8 cwt. 2 qr. 7 lb. 10 oz.
(6) 85 tons 18 cwt. 2 qr. 22 lb. $3\frac{5}{16}$ oz.
(7) $28\frac{4}{5}$
(8) 625
(9) 1000
(10) 1036·95 gal.
(11) 9 tons 11 cwt. 1 qr. $17\frac{1}{2}$ lb.
(12) 2 tons 1 cwt. 3 qr. 22 lb.
(13) 30·267 in.
(14) 254·446 ft.
(15) 4·641 ft.
(16) 12·83 ft.
(17) 125 cub. ft.
(18) 3375 cubes
(19) 6 tons 2 qr. 4 lb.
(20) $1711\frac{709}{1110}$ gal.
(21) 4500 gal.
(22) 1 ft. 4 in.
(23) $3046\frac{7}{8}$ cub. ft.
(24) 384
(25) 302 cub. ft. 744 cub. in.
(26) 42 cub. ft. 1476 cub. in.
(27) 9048 cub. ft. 960 cub. in.
(28) 11 tons 12 cwt. 3 qr. 27 lb. $7\frac{1}{3}\frac{1}{2}$ oz
(29) 1 ton 7 cwt. 24 lb. $11\frac{9}{64}$ oz.
(30) £4, 18s. 9½d., nearly

EXERCISE 98.

	Sq. ft.	′	″	‴	⁗
(1)	7	1	6	.	.
(2)	34	6	9	.	.
(3)	69	10	2	.	.
(4)	130	6	0	.	.
(5)	142	5	8	.	.
(6)	240	7	6	.	.
(7)	16	9	3	4	6
(8)	24	3	3	7	6
(9)	83	10	8	5	11
(10)	142	8	1	7	6
(11)	234	5	4	4	6
(12)	322	3	9	1	3

	Sq. ft.	′	″	‴	⁗
(13)	345	8	7	2	1
(14)	530	0	5	2	6
(15)	1270	5	6	2	2
(16)	1345	10	0	2	6
(17)	1084	2	0	10	6
(18)	1684	5	6	11	3
(19)	28	7	9	.	.
(20)	239	6	8	.	.
(21)	321	0	1	.	.
(22)	155	2	6	3	.
(23)	65	4	0		.
(24)	205	6	8	.	.

(25) £1, 15s. $0\frac{3}{4}$d.
(26) £5, 18s. 3d
(27) 3s. $0\frac{3}{4}$d.
(28) £19, 7s. 6d.
(29) £15, 19s. $5\frac{1}{4}$d. $\frac{3}{4}$
(30) 4s. $7\frac{4}{5}\frac{5}{6}$d.

	Cub. ft.	Primes.	Seconds.	Thirds.	
(31)	450	0	6	4	
(32)	262	5	3	0	
(33)	340	8	10	8	
(34)	413	4	6	11	2 fourths
(35)	480	10	2	3	
(36)	172	2	3	6	7 fourths 10 fifths 9 sixths

EXERCISE 99.

(1) 1182 sq. ft. 32 sq. in.
(2) 3984 sq. ft. 126 sq. in.
(3) 37 sq. ft. $6\frac{1}{8}$ sq. in.
(4) $65782\frac{600846}{1338837}$ gal.
(5) $526259\frac{710417}{1338837}$ gal.
(6) 181 cub. ft. 867 cub. in.
(7) 302 sq. ft. 122 sq. in.
(8) 261 sq. ft. 88 sq. in.
(9) 288 sq. ft. 89 sq. in.
(10) 35 cub. ft. $851\frac{29}{36}$ cub. in.
(11) £9, 18s. $0\frac{1}{4}$d. $\frac{47}{64}$
(12) £20, 3s. $9\frac{1}{4}$d. $\frac{1}{8}$
(13) £5, 3s. $2\frac{1}{2}$d. $\frac{3}{28}$
(14) £40, 15s. $7\frac{1}{2}$d.
(15) 2s. $9\frac{3}{4}$d. $\frac{5}{18}$
(16) 262 sq. yd. 4 ft. 72 in.

EXERCISE 101.

Boy Clerks, May 1908.

(1) 63
(2) 2·7 sq. mm.
(3) The second route; by 1 min.
(4) Average, £29,330,000
Percentage, 26
(5) (Omitted)

Boy Clerks, September 1908.

(1) On 19th July 1908
(2) 1s. 8d. per lb.
(3) 13 miles
(4) (A) 38 p.c.; (B) 26 p.c.; (C) 6 p.c.
(5) (Omitted)

Boy Clerks, January 1909.

(1) £6, 12s. $9\frac{1}{2}$d.
(2) 41 p.c. in Ceylon
370 lb. per acre.
(3) (*a*) At 9.28; (*b*) at 9.31
(4) 155
(5) (Omitted)

Girl and Lady Clerks, March 1907.

(1) £1, 11s. $6\frac{3}{4}$d.
(2) 37 centimes per litre (nearest centime)
(3) (Omitted)
(4) 13 yrs. 7 mos.; by a decrease of 4 mos.
(5) 20·9 p.c.; 1s. $11\frac{1}{4}$d. per week for 50 weeks.
(6) 1·5 acres (nearest $\frac{1}{10}$)

Girl and Lady Clerks, September 1908.

(1) £1, 12s. 3d.
(2) (1) 58; (2) 18; (3) 43 p.c.; (4) 55 p.c.
(3) 29·3 sq. in.
(4) 25·08 francs
(5) 1-lb. pot = 24 cub. in.; 3-lb. pot = 69 cub. in.; 0·34 (nearest $\frac{1}{100}$)
(6) (Omitted)

Abstractors, January 1908.

(1) £1, 6s. 1d. (nearest penny)
(2) Bucket weighs 2½ lb.
Water = 2·75 (nearly)
(3) (Omitted)
(4) 28·2 per 1000
(5) Air is 14·4 times as heavy as hydrogen

Check:— $\frac{\text{Air}}{\text{Hydrogen}} = \frac{1{\cdot}2932}{0{\cdot}0896} = 14{\cdot}4$ (nearest $\frac{1}{10}$)

(6) (Omitted)

Abstractors, January 1909.

(1) First man's rate = £0·39 per day
Second ,, ,, = £0·41 ,, ,,
(2) Percentage profit in England = 21·74
,, ,, France = 21·92
∴ 5 fr. a metre gives higher rate of profit.
(3) In London, 38795 per sq. ml.
In Yorkshire, 619 ,, ,,
(4) 4·3 gal.
(5) 1·04
(6) (Omitted)

Royal Society of Arts—Grade I.—1906.

(1) 165
(2) 86·3689345
(3) £2469, 10s. 10½d.
(4) 2·2
(5) 246·90 ft.
(6) 54 days
(7) ·03493
(8) £124, 13s. 9d.
(9) 5·6
(10) (1) 917922
(2) 63627
(3) 139730
(4) 75·6342
(5) £109820, 11s. 6d.
(6) £132565, 5s. 6d.

Royal Society of Arts—Grade I.—1907.

(1) 17⅓
(2) £15·5823; 5·84298 tons. £91·047
(3) 1·0011
(4) 5911·5 lb., or 6142·254 lb., if the proportionate parts are taken by size, instead of by weight
(5) £5964, £6248, £7100; £6049, £6282, £6980
(6) 2 %
(7) ⅛ hr.; 2 miles
(8) 3s. 4d.
(9) 160
(10) (1) 224008, (2) 467571, (3) £337, 8s. 2d. (4) 873·9984, (5) £86184, 18s. 6d., (6) £67981, 9s. 5d.

Royal Society of Arts -Grade I.—1908.

(1) 7 (very nearly)
(2) 2515 dols. 88 cents
(3) 1·76 pints (nearly)
(4) £4
(5) £21, 15s.
(6) 1s. 3½d. or 1s. 4d.
(7) 5 %
(8) 15·43 grains
(9) 9.2 p.m. (very nearly)
(10) (1) 5378, (2) 11214500, (3) £412, 5s. 4d., (4) 246·1685, (5) £53134 16s. 9d., (6) £55113, 18s. 4d.

EXERCISE 102.

(1) £1, 9s. 5⅔d. $\frac{5}{11}$
(2) 936 men
(3) A, 356; B, 440; C, 324 acres
(4) 9 men, 12 women, 18 boys
(5) 55 yd.
(6) £143, 15s.; $\frac{23}{165}$ of the gain
(7) £324, 3s. 4d.
(8) 2203 ft. $1\frac{5}{7}$ in.
(9) $219\frac{7}{48}$ yd.
(10) 75 in front, 25 deep
(11) $9\frac{3}{5}$ qr. of each of the first three; $24\frac{3}{5}$ qr. of each of the other two
(12) 13 ft. 6 in.
(13) 24 ft.
(14) A, £2937, 10s.; B, £2637, 10s.; C, £2387, 10s.; D, £2037, 10s.
(15) £88; £70; £126
(16) 54 pieces
(17) £2799, 16s. 5½d. $\frac{9}{11}$
(18) C's gain, £234; A's stock, £961$\frac{7}{8}$; £20
(19) A, £2112; B, £1760; C, £2640; D, 1980
(20) 217⅛ yd.
(21) £905·974025
(22) £910; £1, 11s. 1⅛d. per gal.
(23) £29, 1s. 3d.
(24) £525, 9s. 9½d.
(25) 102·184134
(26) £19, 11s. 5¾d. %
(27) £1825
(28) £29, 2s. 6¼d. $\frac{17}{102}$
(29) $146\frac{8}{13}\frac{3}{13}$ days
(30) 196 gal.
(31) £297, 10s
(32) 3840 days
(33) 10¾d. $\frac{5}{7}$
(34) $42\frac{1}{13}\frac{49}{13}$ %
(35) £100000; £15000
(36) 10 %
(37) 8d. loss
(38) £1600
(39) 12½ % loss
(40) A, £125; B, £175
(41) £3403, 15s.
(42) 8⅔ years
(43) £130
(44) £48, 2s. 8d.
(45) $5\frac{2}{11}\frac{9}{5}$d. per lb
(46) £636·135
(47) 25 years
(48) 4½ years
(49) £46, 16s.
(50) £640, 16s. 8d.
(51) £118
(52) 376·729 gal.
(53) 112½ lb. copper; 37½ lb tin
(54) $15\frac{1}{13}$ gal.
(55) £5, 11s. 6d.
(56) 40 gal.
(57) 4 %
(58) £482, 12s. 6d.
(59) ·375
(60) £70, 1s. 8d.
(61) £1, 8s. 1½d.

(62) 126 of each
(63) $29\frac{1}{27}$ yd.
(64) £11, 13s. 3d.
(65) 16 ft.
(66) £1331, 15s.
(67) £240, 12s. 6d.
(68) £428·4344
(69) £34, 17s. 6d.
(70) 512
(71) £26·818 gain
(72) £8, 18s. $6\frac{1}{2}$d.
(73) Neither gain nor loss
(74) 2 tons 13 cwt. 1 qr. 25 lb. $9\frac{1}{8}$ oz.
(75) 1 fur.
(76) £650, 11s. $2\frac{1}{2}$d.
(77) £2·465, or £2, 9s. $3\frac{1}{2}$d.
(78) 5 at 21s. and 7 at 15s.
(79) 1200 yd.
(80) £15, 2s. 8d.
(81) 54432
(82) $45\cdot6\dot{3}$ yd.
(83) £3756, 12s. 6d.
(84) £431, 9s. $8\frac{1}{4}$d.
(85) £63357·41171875
(86) 15 yd. at 12s. 6d. and 25 yd. at 15s.
(87) 500 yd.; 17s. 8d.
(88) £30, 4s. $10\frac{3}{4}$d. gain
(89) £14374, 10s.
(90)

£	s.	d.
115	12	6
192	14	2
269	15	10
346	17	6
423	19	2
501	0	10

(91) 398 cub. ft. 636 cub. in.
(92) $1\frac{3}{8}$
(93) $41\frac{1}{4}$ yd.
(94) 248 sq. yd. 8 sq. ft.
(95) 3 at 3s. 4d. to 1 at 4s.
(96) £2400
(97) £1, 5s. $7\frac{3}{4}$d.
(98) 29260 yd.; £792, 9s. 2d.
(99) 184 sq. ft. 36 sq. in.; £7, 9s. $8\frac{1}{2}$d.
(100) 42
(101) 15 men
(102) 6·6 %
(103) £235, 17s. 2d.
(104) $17\frac{3}{5}$ ml. from A's starting-place; $17\frac{2}{5}$ ml. from B's starting-place
(105) 363 tons
(106) $236\cdot0\dot{4}4$; $\frac{233}{990}$
(107) $343\frac{3}{8}$ miles
(108) 5 hr. 36 min. 40 sec.
(109) Length, between 94·50 cm. and 95·49 cm.; breadth, between 14·50 cm. and 15·49 cm.
(110) £310
(111) 10 kilometres
(112) 250
(113) $4\frac{1}{2}$ acres
(114) 53 min. 20 sec. past 4
(115) 619 cub. cm.
(116) £2, 16s.
(117) $35\frac{3}{4}$ ft. per sec., or $24\frac{3}{8}$ ml. an hour
(118) $8\frac{11}{16}$; £3, 6s. 8d.
(119) Mark; 2038 marks
Franc; 2512 francs
Rupee; 1100 rupees
Dollar; 480 dollars
} approximately
(120) £4398; £5288; £7546
(121) $17\cdot\dot{7}$ acres
(122) $8\frac{7}{12}$ tons
(123) £1700
(124) 6·57 %
(125) £396, 10s. 5d.
(126) $\frac{2084}{4995}$
(127) $5\frac{1}{17}$ per cent.; £30, 10s. 2d. less
(128) £896
(129) 1047·62 lire
(130) £4479, 11s. 11d.

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